A missing curvature term in the Liénard-Wiechert retardation equations

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In 3+1 space, the differentions for the first derivativves are just in space, or just in time. There are no space-time cross terms in the first derivatvies, so the non-orthognaly of the coordinaes does not affect them. The Maxwell equations are in terms of the second derivatves, and the chain rule for differentiaon would be required in 3+1 space, as the solutions would otherwise be incomplete. The chain rule is not needed if the differentiaons are performed in 4-space and then projected into 3+1 space. An equivalent proceure is to obtain a potential equation that differentiates correctly in 3+1 space.

The LW retardaion equations represent a projection of 4-space into 3+1 space. The potential solutions are then differentated in 3+1 space to obtain the fields. Due to the non-orthogonality of the coordinates in 4-space, the chain rule for differentaion would be required in 3+1 space. The chain rule is not needed if the derivatives are computed in 4-space in the first place, then projected into 3+1 space. An equivalent procedure is to obtain a potential equation that differentiates correctly in 3+1 space. It is possible to differentiate twice then integrate once, so the first derivatives are not actually as accurate as they seem to be. The solutions obtaned this way are not solutions to the Maxwell equaions. They are solutions to the Proca equations. The Proca equations reduce to the Maxwell equations when the conditions are mild.

Arguments are presented that the LW retardaion equaions are for the wrong particle. The chain rule for differentiaon is then used to obtain the soluton for the right particle.

The Newton equations are the equations of a removable singularity

If the coordinates are first-known at two points on a trajectory, the time for a particle to traverse between the two points goes to infinty as the particle velocty goes to zero. The equatons are singular at zero velocity. The singularty is removable. After is is removed, the equations are not the Newton equtions. There are space-time cross terms in the full solution.

A method for uncoupling the acceleration and angular velocity terms of the Newton equations.

It is shown that the Newton equations are coordinate-dependent unless events at the location of the particle are simulteneous with events at the origin of the coordinate system. The angular velocity and the acceleration terms are coupled when they are not simultaneous, making it impossible to integrate the particle's accelerton until they are separated. After the terms are separated, the solutions of the LW retardaion equations are no longer solutions to the Maxwell equations. They are solutons to the Proca equations.

Arguments are presented that the Newton equations are coordinate-dependent. Coordinate dependencies can exist in either space or time. The anguler velocity and acceleration terms are coupled in the solutions of the equatons, causng the double integral of acceleration to depend on where the origin of the coordinate system is. The Newton series has the form of a Maclaurin series. In representing the derivatives at at displaced point, A Taylor series is more general. By converting the Newton series to a Taylor series, it becomes possible to apply the equations without knowing what time it is at the origin of the coordinate systm.

It is shown that the Newton equations are coordinate-dependent. They cannot be applied without knowing when time t=0 should be. The basis of the limitaion is that there are space-time cross terms that are of no consequence for the first derivaive, but that grow faster than than the first order terms when integrating in time. The solution contains $\boldsymbol{a}\times\boldsymbol{v}$ cross terms that have a connection to the Thomas precession.

An extension of the Newton equations to include angular velocity

Any equation that can be used to integrate around a circle in 3-space can also be used to compute the value of pi. Several nonliner intgrals are known with that capability [].

It is shown that the Newton equations are missing angular velocity terms. The equations would be for the wrong particle even if the speed of light were infinite. The missing terms result in the loss of the precessional terms in orbital solutions. The basis of the shortcomming is that either the chain rule for differentiation or the multivariate Taylor theorm is required when the variables are not independent. The x and y coordinates are not independent variables in the equation for a circle, and orbital solutions are scarcely simpler.

Arguments are presented that the LW retaration equatons are not for one particle. They are for a family of particles. After a potential solution is differentited to obtain the solution for the fields, the fields are for the wrong member of the family. The problem can be avoided by first selecting the coordinates for the right particle, then integrating them in time to obtain the coordines for the right particle at a later time. The retarded potentials are the integral of the fields. They cannot be differentited in time until the coordinates have been integrated in time.

Due to the constantly changing angular reltionships when there is a transverse velocity, the trajectory of an unaccelerated and retarded particle is perceived as being curved. The particle appaers to move along a straight line, but there is a false acceeraton term parallel to the retrded velocciy, which is a form of curvature.

The false curvature term causes the solutions of the LW retardation equations to not be for an inertial particle. The actual retarded time at the particle can be obtainied by integrating the retarded dt interval, then the actual time used for deriving the retardaion equations for charged particles.

The LW retardaion equations do not contain any terms that are identifiable with the transverse doppler effect of the relativistic doppler equation. A light cone equation is obtained that does predict the right doppler frequency.

There are additional terms in the solution if the particle is accelerated, jerked, yanked, or hammered, but the solutos for a hammered partcile is divergent.

It is then used to derive the equaions for the retarded potentils of accelerated charged particles. Like the LW equations, the equations do not contain acceleration terms, but acceleration terms do occur after a global potental solution is differentiated to obtin the fields.

This paper is not finished. Only some of the calculaions are shown in this version.

It is shown that the derivatives of the light cone equaion are singlar at the point on the trajectory where the paticle is at its closest approach to the observer. The singularity can be removed with L'Hôpital's rule, and in other ways. After it is removed, the simultaneous point is no longer where it was thought to be, which accounts for the transvese doppler effect. The correction causes the Liénard-Wiechert retardaion equations to be for the wrong particle. The retardaion solutions for the right particle are no longer solutions to the Maxwell equations. They are solutions to the Proca equations.

It is shown that the equation for the doppler shift predicted by the light cone equation is singular at the point of a particle's closest approach to the observer. Singulries of the form 0/0 are capable of yielding specific solutions, but they need to be removed. This one can be removed by differentiating, then integrating and providing a constant of integration. The constant of integration causes the simulaneouls point on a trajectory to be at a different location than is predicted by undifferentiated vector equatins, which accounts for the transferse doppler effect of the relatifistc doppler equation. Since the simultaneous point is not where is was thought to be, the LW retardation equatios are for the wrong particle. The solution for the right particle is not a sollution to the Maxwell equations. It is a solutions to the Proca equations.

There are no terms in the Liénard-Wiechert retardation equations that are identifiable with the transverse doppler effect. It is shown that the light cone equation contains a removable singularity at the point on the trajectory where the particle is at its closest approach to the observer. The light cone solutios do contain transverse doppler terms after the singularity is removed. This vesion of the paper is in preliminary form. The methods and the reasons for removing the singulariy are not shown, but the consequences of doing so are presented.

The error in representing the curve by a straigh line does vanish as the magnitude of dr goes to zero, but it does not vanish quadradically. The same limitation applies when the tip of dr is moving.

The derivation of the LW equatios assumes that both the location and the retarded time at the particle are already known at some point on the trajectory. The time and space coordints are not independent variables in light cone solutions. When the observatins are delayed by the light time accross the system, it is not possible to instantly determne both where a particle was and when it was there. the LW equations assume that more is known than is knowable. When more realistic constraints are imposed on the light cone equation, the solutions are not solutions to the Maxwell equations. They are solutons to the Proca equations.

A comparison of the LW retardaion equations with the relatistic doppler equation indicates that the retardaion equations are missing a constant of integration.

The vector from an observer to a particle rotates as the particle moves. The vector from the particle to the observer rotates in the opposite direction. While it cannot matter which direction the vector is rotating, it is sometimes necessary to know if the solution is left handed or right handed in order to select the right sign. Vectors that behavew this way are not true vectors. They are pseudo vectors. Treating a pseudo vector as a true vector results in the loss of all multipoles beyond the dipole, which is why there are no symmetric terms in the solutions of the Liénard-Wiechert retardation equations.

With true vector equations, the difference between two closely spaced vectors is itself a vector. There are no quadradic terms in the equations. There are no multipoles beyond the dipole. In more general solutions, the difference between two closely spaced vectors is a pseudo vector. True vectors and pseudo vectors behave differently for rotations and sign inversions. Applying the methods of vector analysis to pseudo vector equations results in the loss of all multipoles beyond the dipole. The derivation of the Liénard-Wiechert retardation equations assumes that pseudo vectors behave in the same way was true vectors, hence they are true vector equations. More genreral solutions are not solutions to the Maxwell equations. They are solutions to the Proca equations. The Proca equations reduce to the Maxwell equations when the conditions are mild.

The gradient of a vector is a tensor of the second rank, which includes both symmetric and anti-symmetric terms, yet the 4-potential solutions of the Liénard-Wiechert retardation equations do not have any symmetric terms. The degeneration appears to be a consequence of treating pseudo vectors as vectors. Pseudo vectors behave differently than true vectors for rotations and sign inversions.

The Liénard-Wiechert retardation equations do not contain acceleration terms, but acccelertion terms do occur after a global potential solution is differentiated to obtain the fields. The computed fields are for accelerated particles. The fields are always solutions to the Maxwell equations. There are no avdot terms in the solutions.

It is possible to directly retard the E and B fields. There are avdot terms in those solutions if the light cone equation is solved with the time at the particle being the independent variable. The solutions are for jerked charged particles. The solutions are not solutions to the Maxwell equations. They are solutions to the Proca equations. The Proca equations reduce to the Maxwell equations when the conditions are mild, and for distances small in relation to the range of the fields. The range of the fields is not yet known. It is assumed to be infinite in the approximate solutions obtained, restricting the solutions to the nearby region of the cosmos.

If a high velocity particle is radially approaching the observer and emits a photon when at a great distance, the particle and the photon then both continue their journey to the observer. At extremely high veloicties, the partile is traveling nearly as fast as the photon, so the differnce in the arrival times of the photon and the particle can be only an instant, yet the travel time can be an eternity. If the partile emits a second photon just before it arrives, there would have been ample time for the acceleation of the particle to deflect it in the transverse direction between the two events.

Consequently, no matter how small dtf is, the bogus curvature terms in Eq – are still present if the partice only barely misses the observer. It is therefore necessary to integrate over an interval that looks like and infinitesimal quantity, but does not behave like one.

This behavior is not contrary to the theorms of Eucludina calculus. It is rather that the chain rule for differentiation is required whenever the variables are not independent. The time and space coordintes depend on each other in light cone solutions, tending to confound appliatons of the chain rule, but it is nevertheless applicable. The chain rule can be applied recursively. It is capable of converting first derivatives into derivatives of any order.

This relationship makes it necessary to integrate the retarded coordinates over the dt interval, no matter how short the dt interval is. However, the equaions are linearly dependent if the the dt interval is subdivided. Curvature is not representable with the first derivative.

This relationship implies that the retardation equations in potential form should be obtained by inegrating the equations for the retarded fields. Not all equations are integrable, so the integral might or might not exist.

... The transformations between these two perspectives are probably more important than which perspective is the right one. The transform is the solution. Each perspective is one aspect of the problem.

When working the retrdaion problem, the observational data points are always stale and incomplete.

This relationship implies that eqaions for the first frame of reference sometimes neeed a factor of gamma even when there is no second frame of reference (that we know of).

A closely related solution is obtainable by computing the first derivative at a displaced point.

The gradient of a vector is a tensor of the second rank, yet there are no symmetric terms in the retarded vector potential. There should be.

There is ample time for quadradic or even cubic terms to affect the trajectory of the particle. The simplest way of retaining the curvature terms is to work the problem in the frame of referene of the particle.

The curvature terms are not representable with the first derivative in the frame of reference of the field point. They are easier to represent in the frame of reference of the particle.

Consequently, aftere differentaion to obtain the fields, the LW retardation equations are missing acceleration terms. The missing terms represent the Thomas precession from the perspective of an observer in the first frame of reference.

There are no space-time cross terms in the first derivatives of 3+1 space. The differentiations are just in space, or just in time. There are cross terms in the first derivative if it is comuted at a point displaced in space or time, but that is one of the contributors to the second derivative. Conversely if the second derivative is needed, the calculaion should begin by computing the first derivative at a displaced point. The Maxwell equaions are in terms of the second derivatives of the potentials. The first derivative of the LW retardation equations at a point displaced in space is not a solution to the Maxwell equations. It is a solution to the Proca equations. The Proca equations reduce to the Maxwell equatins when the conditions are mild.

The gradient of a vector is a tensor of the second rank, which represents both symmetric and anti-symmetric terms. The symmetric terms are irretreviably lost if the derivatives of Liénard-Wiechert retardaion equations are computed as though the retarded space and time coordinates were independent variables. The symmetric terms can be retained by computing the gradient of the vector potential symbolically, then directly retarding the derivatives of the retarded potentials. The solutions are not solutions to the Maxwell equations. They are solutions to the Proca equations. The Proca equations reduce to the Maxwell equations when the conditions are mild. The associated retardation equations are for jerked particles, which reduce to the solutions for accelerated particles. There are angular velocity terms in the solutons that behave like acceleration terms. Consequently, the soutions for unaccelerted particles do not reduce to the Newtonian.

It is shown that the space derivatives of light cone solutions cannot be fully reduced to first order. The solutions can be developed as a series expansion parameterized by the retarded velocity, acceleration, avdot and higher order terms. The first term of the series is the Colulomb solution. The LW solution is the second term.

The gradient of a vector is a contravariant tensor of the second rank, which includes both symmetric and antisymmetric terms, yet the solutions of the Liénard-Wiechert retardation equations do not have any symmetric terms.

The space and time coordinates are not independent variables in light cone solutions, requiring the chain rule for differentiation. There are space-time cross terms in the the second derivatives that are missing if the chain rule is not used. The solutions are not solutions to the Maxwell equations. They are solutions to the Proca equations.

Retardation equations are intrinsically observer dependent, but their basis should be equations that are the same for all observers. The first derivatives of the Liénard-Wiechert equations in the frame of refernce of the particle depend on which observer is watching the particle. The abberation is a consequence of the non-orthogonality of the space and time coordinates in the second frame of reference. The non-orthogonality causes the observer dependent angular velocity of the particle to masquerade as a Newtonian accelertion term that is parallel to the retarded velocity. Consequently, the Liénard-Wiechert equations are not for inertial particles. They are nevertheless adequate when relativistic corrections can be neglected, which is always the case for stationary current carring wires. They are a legitimate term of the retardation series.

The light cone equation can be solved with either the time of an event or the locaton of the event being the independent variable. When a space vector is the independent variable, the angular relationships between a velocity vector and a space vector are not vector equations. They are tensor equations.

The relativistic doppler equation and the Liénard-Wiechert retardation equations are both based on the Lorentz transform, but the doppler equation is in differntial form, while the retardation equations represent an undifferentiated function. The two solutions cannot be directly compared without integrating the doppler equation in both space and time, which is hard to do. It is easier to discover a light cone solution with a first time derivative that is the same as the doppler equation. The comparison indicates that the Liénard-Wiechert equations are are missing a factor of $(1-v^2/c^2)^{-1/2}$, which is ubiquitous in relativistic equations. The doppler equation is thought to be for an inertial particle, implying that the Liénard-Wiechert solution needs a mass term to represent the trajectory of an inertial particle.

In merging the terms for the radial and transverese doppler shift into one term, it is not possible for an observer to determing which partile a solution is for, making it impossible to integrate the equation. The LW equaions have the inverse problem. In not distinguishing beteween the radiaal and trnssverse compoinets of a vecoty vecot, they cannot be differentiated without knowing which obdserver they are for. Neither equaton is for both the right particle and the right observer.

I. OVERVIEW

This paper has not been refereed. Critical comments and suggestions, including grammaticl corrections, are welcome at gary@s-4.com.

Most calculations are shown in more detail in the Supplemental Online Material (SOM). Due to the length of the expressions, some of the calculations are only available in the SOM.

I will submit the material to a journal after some more development. For now, please reference the version at `dx.doi.org/10.6084/m9.figshare.5477056` if you make use of the material. The PDF file is also available at ..., but there is no supplemtal data at that site.

The preferred download site is www.s-4.com/cone, and the version available there may be more recent, but personal web pages are not satisfactory for archive purposes.

This paper is archived at
`http://vixra.org/abs/1707.0344` and
`https://figshare.com/articles/An_approximate_non-quantum_calculation_of_the_Aharonov-Bohm_effect/5477056`. The latest version will always be availale at `www.s-4.com/cone`. There is no supplemental data at the vixra.org site.

The supplemental data is available at www.s-4.com/cone. All of the supplemtal files are in the HTML

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The supplemental data at the figshare.com site is a single .zip file containing many HTML files. The file must be downloaded and decompressed for viewing. The top level file is index.html, and all files are accessible with links from the entry page. There are no subdirectories. Most browsers have the capability of navigating files on a hard disc in the same way that they are viewed on the internet. The files can be stored in any directory on any drive. The supplemental files at `www.s-4.com/cone` are easier to access if they are still available. They can be navigated like any other web page.

II. INTRODUCTION

If the protons and conduction electrons are retarded separately, as must be done, the Liénard–Wiechert[9] (LW) retardation solution for a rotating current loop is the same as for a stationary loop, even when the rim velocity is near c. It seems there should be relativistic corrections for the magnetic field. The magnetic field is of order v^1 and relativistic corrections are of order $(v/c)^2$, so the missing terms should be of order v^3.

The solutions to the LW equations are always solutions to the Maxwell equations. Unless the Maxwell equations are more general than the LW equations, they have the same shortcoming.

There is a connection between retardation equations and the doppler shift. When the time interval at the source is longer than the interval at the field point, the flux becomes concentrated into a shorter interval, enhancing the signal strength.

In our frame of reference, provided the wind is not blowing, similar equations apply to the constantly changing doppler shifted acoustic tone of a passing high speed vehicle. A changing pitch could be due to the transverse velocity of an unaccelerated vehicle, or it could mean that the vehicle is approaching radially and decelerating. Unlike the Newton equations, accelertion and angular velocity are not instantly separable when the data points are delayed by the propagation delay accross the system. Until they are separated, angular velocity is capable of masquerading as acceleration, and conversely. The doppler frequency cannot be integrated until the contributors to its rate of chage are separated.

A closely related relationship is that the angular acoustic location of a low flying jetliner lags noticably behind its visual location.

Interplanetary spacecraft navigation software incorporates corrections from the general theory of relativity when integrating the round trip doppler frequency[22]. We do know how to integrate the doppler frequency.

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In modern times, the four dimensional space is the space we exerience every day.

Because angular velocity and acceleration are not quickly distinguishable, there are ambiguities in observtaions of short duration, regardless of the precision of the measurements. The ambituities are not present for a field of observers. Classical equations represent the perspective of a field of observers. They represent a perspective that we can know, but can only experience with isolated glimpses. A memory of past events is required to connect the glimpses.

The solutons of the Proca equaions [] include exponential terms that represent the range of the fields. There is a static scalar solution and a static vector soluiton[11]. The vector solution will not be considered furthr at this time. The static scalar solution reduces to the Coulomb solution in the nearby region. The static solutions have to be transformed when the particle is in motion.

In the reference, it is stated that the $\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A})$ of the static vector solution is a vector. It is actually a pseudo vector[15].

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If the coordinttes are first-known when a paritle is at the simulteneious point, the angle between the simultaneous point and the retarded point can be large. The computed angular velocity, relative to the simultaneos point, does not depend on how far away the particle is. This relationship causes the behavior of retarded angular velocity to be unintuitive. Curvature terms that would otherwise vanish at great distances do not vanish at all.

The non-vanishing curvature terms result in some similarites to the problem of reducing 3-space rotations to first order. The radius vector rotates as the particle moves, but it rotates even when the particle is moving along a straight line. Retardation equations are a more general form of the equations for circular motion. They would also work for an elliptical orbit.

There is no known way of reducing 3-space circular motion to first order with one linear equation unless it contains π, which is obtained with a nonlinear integral[23]. In being capable of representing more general forms of periodic motion, it is indicated that exact linear retardation equations do not exist.

3-space rotations do not include the speed of light, but including it is scarcely a simplificaion.

later *************

An equation can be reduced to a system of several first order equations[2], but not in one step. For example, the two linear equations for integrating around a circular Newtonian orbit are $\boldsymbol{r}_{n+1} = \boldsymbol{r}_n + \boldsymbol{v}_n\ dt$ and $\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \boldsymbol{a}_n\ dt$. The integral becomes exact as dt becomes arbitrarily small. The equations are of first order, suggesting that the light cone equation could also be reduced to first order with a sufficient number of linear equations.

III. PSEUDO VECTORS

When the speed of light does not matter, the difference between an approaching and receding particle can be represented by inverting the sign of the velocity vector. Rotating the vector by 180 degrees and translating it would have the same effect.

The equations for approaching and receding particles are not symmetrical when the speed of light matters. Vectors and pseudo vectors behave differently for sign inversions and rotations[15].

There are no symmetric terms in true vector equations, but the Lorentz condition, $\boldsymbol{\nabla}\cdot\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, is a symmetric pseudo vector equation (however, it is always zero for solutions in the Lorentz gauge). $\boldsymbol{\nabla}^2\boldsymbol{A} = \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\boldsymbol{A})$ is a pseudo vector, while $\boldsymbol{\nabla}\boldsymbol{A}$ is a tensor of the second rank.

The magnetic field, b=del cross av, is another psuedo vecotor. It is related to del2a by the identiy del2a = ... div av is zero in the static current loop soluion. There are no symmetric terms in the solutions of the Maxwell equations.

The decompositon products of the contravariaant tensors include pseudo vectors, which are of the dipole symmetry, along with other multipoles []. The tensor theorm is in a static 3-space. Two or more closely spaced (and retarded) snapshots are required when the vectors are in motion, but a space rotation is still a space rotation.

[later] The Legendre polynomials represent the scalar multipoles. The vector spherical harmonics[2,21] represent the vector multipoles. Both are irreducible. They cannot be syntesized by any linear combination of lower order multipoles.

There are no multipoles beyond the dipole in the solutions of the LW equations. They are true vector equations. The contravariant tensor of the second rank represnts the first derivatives []. Technically, the LW equations are not differntiable.

[?] Vector equations are not differentiable by the methods of vector analysis unless the solution represents three independent copies of the equations for one space dimension. Correctness of the equaions for one space coordinate does not imply that the differntial angular relationships are correct.

This relationship remains true even for non-relativistic 3-space rotations. There is no known way of integrating around a circle with one linear equaion unless it contains π, which is obtrained with a nonliner integral[23]. Regardless of its complexity, the problem cannot be reduced to first order with one linear vector equation.

IV. CURVATURE

When integrating around a unit circle in 3-space, the first infinitesimal step is

$$y = \sin d\theta$$
$$x = \cos d\theta.$$

To third order

$$y = d\theta - d\theta^3/6$$
$$x = 1 - d\theta^2/2.$$

Then

$$dy/d\theta = 1 - d\theta^2/6$$
$$dx/d\theta = -d\theta/2.$$

θ is a function of time in a circular orbit, but the orbital equations in 3+1 space are otherwise the same as for 3-space rotations.

If the magnitude of dy is reduced by a factor of two then the error in dx is also reduced by a factor of two, but then twice as many steps are required to integrate around a circle, so nothing is accomplished. There is no value for dy that is small enough to reduce the problem to first order.

Several nonlinear equations are known for integrating around a circle [], but there is no known way of doing with one linear equation unless it contains pi, which is obtained with a nonlinear integral. (Complex exponentials implicity contain pi).

It is nevertheless possible to accurately extrapolate part way around a circle with the Taylor theorem for one variable, which is a linear equation. The curvature represented this way is not exact, but it becomes more accurate if more terms are carried. The series expansion would work equally well for elliptical orbits. With this approach, dtheta maps into an arbitraritly short line segment, but the segment remains curved, no matter how small dtheta is.

... In the figure, no matter how small dth is, the line segment is curved. The curvature is of no consequence for the first derivative, but it cannot be neglected when integrating along a curve.

When reducing a problem to first order, terms must either vanish quadradically or be carried. The basis of this limitation is that the chain rule for differentiation is generally required when the variables are not independent.

For light cone solutons, Rv and t are not independent variables, even when the particle is unaccelerated.

θ is a function of time in circular orbits, but non-relatistic orbital equations are otherwise the same as for 3-space rotations. The angular relationships for elliptical orbits are necesssrily more elaborate.

If an observer in in free fall, the angular velocity of a nearby particle depends on how long it has been in free fall. The solution should probably be integrated in time if the observer has a memory of past events, even when the velcity is so low that the speed of light does not matter. Observations of short duration are ambiguous.

An equation can be reduced to a system of several first order equations[2], so it is possible to integrate around a circle with more than one linear equation. For example, the two linear equations for integrating around a circular Newtonian orbit are $\boldsymbol{r}_{n+1} = \boldsymbol{r}_n + \boldsymbol{v}_n\ dt$ and $\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \boldsymbol{a}_n\ dt$, which is a differential equation.

The equatio dv=av dt is not the total differential of the Newton equations. It is only one of the terms.

Pseudo vectors are so ubiquitous and have been around for so long that they are usually just called vectors. The difference frequently does not matter until the equations are integrated. The retarded potentials are the integral of the fields. Because terms must either vanish quadradically or be carried, terms that are of no consequence for field equations can be vital for potential equations. However, the same terms are no less important if the solutions of field equaions are integrated in space and time.

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When integrating the gradient of the Newton equations, for what reasons are the symmetric terms neglected?

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xxx The Maxwell equations are sometimes inapproatriately associated with the tensor of the second rank. It is the tensor of the third rank that represents the second derivatives, such as $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{A}) = \boldsymbol{\nabla}\times\boldsymbol{B}$. Its decomposition products are a scalar, three pseudo vectors, two quadrupoles, and an octupole [].

xxx However, the solutions of the LW equations are degnerate. There are no multipoles beyond the dipole in the solutions. They are true vector equations.

xxx It has been established that acceleration is not representable with true vector equations[8].

The tensor irreducibily theorm[6] is in 3-space, but a space rotation is still a space rotation when the vectors are in motion. More accurately, it is because of the motion that there are multipoles beyond the dipole. Two consectutive static 3-space solutions do not become a 4-space solution just because the 3-space coordintates are different in the two solutions. The two 3-space solutions are disconnected. To some extent, they can be connected with the Newton equations, but they are still disconnected unless it is shown that both solutons are for the same particle.

The equations are ill behaved when the particle is nearly on a collistion course with the oberver at extremely high velocites. In that case, the observer is the target and the particle it traveling nearly as fast as the photon. No matter how far away the particle was when the photon was emitted, the difference in the arrival times of the photon an the particle can be only and instant, yet the travel time can be an eternity. There is ample time for the acceleration of the partcile to deflect it in the transverse direction. The two vectors from the observer to the locations of the particle at times $t_f + 0$ and $t_f + dt_f$ are disconnected. The soluion is not for one particle. It is for a family of particles with different histories that pass through two widely separated points. That would not matter unless the equation needs to be differentiated.

When a particle is at its closest appoaach to the observer, the observer cannot tell from the doppler frequency itself whether the partice is moving from the left to the right or from the right to the left. Rotating the observer 180 degrees about the line of sight would exchange left and right. The equatios have to work in the same way for any orintation of the observer, but the two doppler solutions are disconnected. The disconnected solutions for two particles traveling in opposite directions could be connected by requireing that the soluton for one particle be rotaionally invariant with respect to the orientation of the observer. The calculation would require the space derivatives unless the observer is a point.

The nature of the connection between the two light cone solutions for particles traveling in opposite directions is of no consequence unless the equation needs to be differentited. The doppler frequency is a derivative.

The light cone equation is not a true vectore equation. The time and space coordinats are not independent variabvles. They depend on each other. It is difficult to determine both where a particle was and when it was there. It can be done, but not instantly.

The theorens of Eucludian calculcus do not apply unless two nearby points are connected. When there is only one independent variable, being nearby is a sufficent condition for them to be conneced. But when the variables are not free and independent quantites, the chain rule for differentiation and the tensor irreduciblity theorem are essential parts of modern Euclidian geometry.

In the equation f(dx) = 1/(dx0 + dx), where dx0 is an arbitrarly small constant, f(dx) is a straight line. But the equaion $f(dx, dy) = 1(dx0 + dx)^2 + (dy0 + dy^2))^(1/2)$ represnts a family of solutions. This relationship would be of no consequence if we knew where we were in space and time. Equations of physical significance are a subset of the equations that can be written. We have to select the right particle from a family of partiles. A Taylor series always represents a family of particles.

Even if the speed of light were infinite, the Newton equaions would still be for the wrong particle unless the angular velocity of the particle is zero. Classical equaions are observer- free, but angular velocity depends on where the obserer is, which is a delima for the phylosiphers to ponder, along with the question of whether or not an event without an observer has a meaning.

As illustrated by Ol.. paradox [], inverse square law relationshiips are ill behaved at infinity. Curvature equations are singular at infinity, and the singularity is not necessarily removable. It is mostly necessary to restrict the attention to lesser distances.

When integrating around a unit circle in 3-space, the first infinitesimal step is

$$y = \sin d\theta$$
$$x = \cos d\theta.$$

To third order

$$y = d\theta - d\theta^3/6$$
$$x = 1 - d\theta^2/2.$$

Then

$$dy/d\theta = 1 - d\theta^2/6$$
$$dx/d\theta = -d\theta/2.$$

When integrating numerically in n steps, terms must either vanish as $1/n^2$ or be carried. After integration, terms that vanish as $1/n$, while usually small, are in the same order as the solution. The equation is not of first order unless the residual vanishes as $1/n^2$ in each step of the n steps of the integration. The $dx/d\theta$ term is usually small in other problems, but it does not vanish as $1/n^2$ when dy is the independent variable.

An equation can be reduced to a system of several first order equations[2], but not in one step. For example, the two equations for integrating numerically around a circular Newtonian orbit are $\boldsymbol{r}_{n+1} = \boldsymbol{r}_n + \boldsymbol{v}_n\ dt$ and $\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \boldsymbol{a}_n\ dt$. The integral seems to become exact as dt becomes arbitrarily small, although the behaviour is degenerage. In a circular orbit, the avdot vector is antiparallel to the veloty vector, concealing the fact that the obital velocity contains an error of order ω^2. A third equation would be required to take out the error.

In Fig. –, drv1 is not a true vector. It is a pseudo vector. The error in reprsesnting a short section of the curve by dr vanishes as dr becomes small, but it does not vanish quadradically. The same limiation applies if the tip of drv is moving.

When the tip of drv is moveing, the Coulomb equation is a familar example of this behavior. ...

When the tip of drv is moving, 1/r term in this soluton is associted with angular velocity and the magnetic field. There is no hope for linear 4-space equations until th 3-space equatios are right.

Angular velocity is associated with the magnetice field of an orbiting charged particle. There are curvature terns in 3-space that should not be neglected when extending the equations to particles that are in motion, as doing so would result in the loss of the magnetic field. 3-space is no flatter than 4-space unless terms are dropped that should not be dropped.

There are no 1/r terms in the first derivative if the trajectory is represented by a Taylor series for one variable, no matter how many terms are carried. The x and y coordinates in Fig – are not independent varibvles. The Taylor series for one independent variable is the wrong equation. There is a multivariate extension of the theorem[2].

While the first derivavtie of the Taylor series for one variable is too slipprery for efffectively constrainng the place and time of the particle, representing a prolonged portion of the trajectory with a series expansion does provide effective constraints, because the first derivavie at a displaced point is one of the contributors to the second derivative, while the first derivatie at a twice dispaced point is one of the contributors to the third derivative. Consequently, depending on the method of calculaion, it is not necessaruil true that the multivariate version of the theorm is required. However, it is required if only two nearby points on the trajectory are known, as there would otherwise be no magnetic field in the solutions. For the same reason, there would be no multipoles beyond the dipole in the relativisic doppler equation.

There are cross terms in the multivariate version that are not present in the solution for one independent variable. In 3-space, the cross terms are between x and y. When the partice is moving, they are bewteen r and t.

Since the infitesimal diffence between vectors rv1 and rv2 in Fig – cannot be reduced to first order with linear equations, and retardation equations are linear equations, it is necessary to develop the problem as a series expansion.

There are several nonlinear formulas for integrating around a circle in 3-space[23], but they do not apply to retardation equations. The Einsten tensor[8] is the basis of a nonlinear formula for integrating around a circle in 4-space. All of the nonlinear formulas provide a means of computing the value of π, which in 4-space depends on the circumstances.

It is not possible to integrate around a circle with one linear equaion unless it contains π. The only reason it is possilbe at all is that the nonliner 3-space integral was discovered long ago. (Complex exponentials implicity contain π.) The calculations of the Thomas precession[7] attempt to integrate around a circle with one linear equation.

The advantage of this approach is that the solutions look and behave like vector equations, even though the solutons are not obtainabe by the methods of vector analysis. That is possilbe, becuause vector equations are slipper than they seem to be. A Taylor series always represents a family of solutions and a family of particles. A smaller family of particles can be selected by carrying more terms in the series. If only two nearby points on a trajectory are known, the family of particle is so large that the derivagives are unusable. The multivarite Taylor does impose more constraints. It is better suited to obtaining field equations.

Retardation equations do not form a complete representation. They are just one aspect of a much larger problem. They are the solution for a distant and detached observer and one particle. The solutions are unphysical if the momentum constraints for systems of two or more particles are not developed separately. Radiation transfers momentum, so the far field radiative terms would probably provide useful clues of unphysical behavior, but the relationships would still require a separate development. For example, there is a possibilty that the integral of the acceleration vector over the surfece of a sphere would reveal the mass in the interior region, even in radiative systems.

****

In representing the starting and ending points of a definite integral between the location of the particle at time $t_f + 0$ and its location at time $t_f + dt_f$, the vector beween the two points is not a true vector. It is a pseudo vector. Since there are quadradic terms in the integral, pseudo vectors do not add and subtract like true vectors. There are space-time cross terms in the equatons.

No matter how small dtf is, the two points on the trajectroy can differ by an eternity. They are disconnected. They can be connected by integration. In representing the beginning and ending points of a definite integral, the differene between the two vectors is a pseudo vector.

Under these circumstance, the coordinates of the particle at the times tf+0 and tf+dtf are disconnected. The two points have to be connected by a path integral. Since the integral contains qudradic terms, the vector pointng from the first point to the second point is not a true vector. It is a pseudo vector. The integral contains quadradic term, so pseudo vectors do not add and subtrct like true vectors. There are space-time cross terms in the equations.

Pseudo vectors reduce to true vectors when the path is short enough that it can be represented by a straigt line. Integration degenerates to multiplication in that case. With the Newton equations, the path integral of velocity times time degenerates to multiplication.

If the midoint of a straigh line is not at the middle then the trajectory is curved in a way that is not representable with the roots of Eucludian geometry. That is because straight lines in 3-space do not have tick marks at regularly spaced intervals representing the ticks of a moving clock. This form of curvature occurs with the Newton equaions when the accelertion and velcoty vectors are parallel, such as for a particle in radial free fall. The Newton equatons are in 4-space, but in not containing the speed of light as a parameter, they are introductory equations.

The modern form of Eucludian geometry includes the chain rule for differentaion and the tensor irreducibly theorm, which are needed when integration does not degenerate to multiplication.

The actual meaning of the infitesiam is not that a quantiy is small. The acctual meanis that the equatains are of first order. There is no known way of reducing 4-space to first order, making it necessary to proceed one step at a time. The velocity terms come first.

****

Due to the non-orthogonalty of the space and time coordintes, there are always space-time cross terms in dynamic solutons. In being first order in space and first order in time, the cross terms vanish quadradrically in as small region of space-time, while the space and time terms are each of first order. For the same reason, the cross terms grow quadracially when integrating in space and time, so they cannot be neglected if the equation is to be integrated.

The cross terms do not actually eithr vanish quadradically or grow quadracically. It only seems that way with vector equatins. Their actual basis is angular relations that do not depend on the size of a small region. But since the quadradic growth of terms that vanish quadradically is not representable with vector equations, a constant of integration is required. In being the integral of the fields, potential equations implicity contain a constant of integration.

In implicityl containga a constant of interation that is not present in field equaions, potentaial equations have more flexibility than field equations. However, the retarded potentials exist only for the purpose of being differentiated. it is only their derivatives that are locallaay measurable.

It is possible to differentiate field equaions, then integrat the derivatves twice and pofvid two constants of integration to obtain potential equations. After differentaion, the two constants of integration provide additional degrees of freedom.

With retardaion equations, the constant of integration is only a constant for a specific trajectory and a specifice observer. In more general solutons, the constant of intgration varies from place to place, and from time to time.

A constant of integration is not needed for a definite integral. However, a deffinite integral is not an absolute quantity. The whole integral can be offet by an amount that is not discernable until the equaion is differentiated again. It is possible, even easy, for a local observer to overlook an offset that a distant observer would require, especially in radiative solutions.

An example of an offsect for a definite integral in the outbound direction is the minimum radius at the Schwarzschild singularity. A distant observer is not the one who needs a constant of integration.

If a test mass is at rest inside a mass shell and a string attached to the mass is jerked, then, according to Mach's principle[12], there should be a frame dragging effect on the mass shell. With true vector equations, space-time cross terms vanish in the infitesimal and have to be dropped. The light cone equation is not a true vector equation. Cross terms cannot necessarily be dropped, especially if they are radiative. Radiation is capable of transfering momentum from the test particle to the mass shell.

****

The cross terms vanish as 1/n, so they cannot be dropped if the equation is to be integrated.

When integrating numerically in n steps, the relative weigth of the cross term is 1/n times eithr of the other terms, so it cannot be dropped.

The Coulomb equation is a familar example of the behavior. It is usable in quasi-static solutons, but because the nagnetic field is of order v/c, the solutons contain a small error of the first order, no matter how low the velocity is. This behavior makes it impossible to reduce 4-space to first order, making it necessary to proceed one step at a time.

While it is not possible to fully redcuce the problem to first order, the tensor irredduciblity theorm[6] provides useful clues for the next step, and cautions agaist assuming that there is no next step. It is always possible to differentant again, then ingegrate again and provide a constant of integraton that was not noticed before. It is the nature of retadation equations that the constant of integrataion varies from place to place, and from time to time. The role played by the constants of integration is similar to that of the angular relationships that are not representable with vector equations.

****

[later] An incomplete solution for the seven multipoles of the third rank tensor with 4-potential paramterizeon is shown in Ref.[13]. The solution is incomplete because the scalar of the solution is $\boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{E}$. The first time derivatve of the Lorentz condition is also a scalar, so it should be included in the solution. It seems that vector equations in the first frame of reference are always missing terms. The basis of the calculations is the tensor decomposition equation in Ref.[6].

The vector spherical harmonics[2,21] represent the multipoles of static pseudo vector solutions. Two closely spaced snapshots are required to represent the first time derivatives.

The Lorentz transform only transforms true vectors. It is not capable of transforming pseudo vectors. If it could, it would be all right to choose the origin of the coordinate system to be at the observer and the time shown by a nearby clock to be zero, then not bother with transforming the tail of the vector. The shortcut may seem justifiable when the time at the particle and the time at the observer are both zero, but it assumes that we already know where the particle is. We cannot know where a particle is until waiting for the light time accross the system. The shortcut is unphysical.

In 3-space, pseudo vectors sometimes seem to have only one end, enabling shortcuts in some cases. In 4-space, both ends of a vector should always be transformed unless a solution is wanted that is represnetbale with a true vector equation.

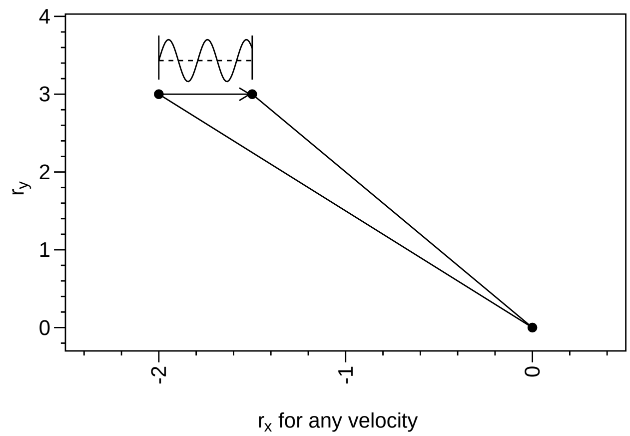

FIG. 1. The doppler shift.

V. THE RELATIVISTIC DOPPLER EQUATION

The relativistic dopper equation[20] is

$$\frac{f_f}{f_s} = \frac{\lambda_s}{\lambda_f} = \frac{dt_s}{dt_f} = \frac{1}{\gamma(1+\frac{v}{c}\cos\theta)}. \tag{1}$$

γ is $(1-v^2/c^2)^{-1/2}$. θ is the angle between the line of sight and the velocity vector. θ is 0 for a radially receding particle. It is $\pi/2$ at the point of closest approach, and π for a radially approaching particle.

The signal is blueshifted for an approaching particle. The frequency shift vanishes slightly before the point of closest approach, then becomes redshifted to $f_f/f_s = 1/\gamma$ at the point of closest approach, which represents the transverse doppler effect.

The $\hat{\boldsymbol{R}}\boldsymbol{\cdot}\hat{\boldsymbol{v}}$ terms of the following calculaions are the same as the $\cos\theta$ in this equation.

A directional antenna does not point to the retarded location of a monochromatic transmitter. It points in a direction perpendicular to the received wavefront.

Thus, the predictded moment of impact depends on how far away the particle was. ... The oberver in 1+1 space is using the wrong equation. It wouldl be the rigth equatin if the time and space coordintes were independent variables, but they are not. Eq – is the right equation.

****

The derivation of the LW equations [] assumes that we know when the paricle is at the simultaneous point befor performing any canculations. As illustrated by the Einstein tensor [], the relationships of space and time are not instantly separable, making it difficult to quickly determine both where a particle was and when it was there. This equation does not assume that we know the answer yet.

On the other hand, when applying retardaion equaions to engineering problems, it is nevertheless necessary to assume that the location of the simultaneous point is already known in order to compute the retarded location.

With true vector equations, these relationships are contradictory. The light cone equation is not a true vector equation, so questions are not so easily resolved.

This interpretation (which is from the reference) assumes that we know when the partcile is at the simultaneous point before performing any calculations. As illustrated by the Einstein tensor [], the relationships of space and time are not instantly separable, making it difficult to quickly determine both where a particle was and when it was there.

On the other hand, when applying retardaion equaions to engineering problems, it is nevertheless necessary to assume that the location of the simultaneous point is already known in order to compute the retarded location.

With true vector equations, these relationships are contradictory. The light cone equation is not a true vector equation, so the relationships are not so easily resolved.

This interpretaion assumes that we already know when the partcile is at the simulteneous point. There can be a stream of particles passing by the simultaneous point. How do we know which particle is the right particle?

In being the integral of the fields, potential equations represent a global perspective. The 4-potential transforms in the same way as the coordinates, implying that the coordinates can be viewed as being a potential representaion.

There can be more than one partilce in the same orbit. Even with the Newton equations, prolonged astronomical snapshots are required to insure that all of the data points are for the same particle, especially when the angle between the plane of the orbit and the line of sight is not already known.

Since the doppler shift can be due to either the radial and transverse velocity, the doppler phase cannot be integrated unless the angle of the trajectory with respect to the line of sight is already known. Interplanetary spacecraft navigation software incorporates corrections from the general theory of relativity. The accuracy of the corrections has been demonstrated [], confirming that the doppler equation cannot be integrated without additional considerations. (Spacecraft navigation usually uses round trip doppler, which would complicate an analysis.)

Similarly, the discussion in §xx indicates that angular velocity and Newtonian acceleration cannot be separated if only three closely spaced doppler samples are available, complicating the integraion unless the angle between the trajectory and the line of sight is already known.

The simulteneous point is where the partcle is now. We have to wait for the light time accross the system before we can find out where it is now. An assistant near the trajectory could report the time tha the particl passes by an at-rest clock, but that would not allow us to find out where it is any sooner.

But since we can see the particle after waiting for the light time accross the system, it is possible to begin at the simulteaneous point.

Conceptually, the light cone equation is still an integral of the doppler frequency, but since there are no longer any quadradic terms in the equation, the integration would not serve any purpose.

If the $\hat{\boldsymbol{R}}\cdot\boldsymbol{v}$ terms in Eqs. (2) are written as $v\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{v}}$, then they should be the same as $v\cos\beta$. The magnitude of the radius vector is at a quadradic minimum at the point of closest approach. The radial velociy of the particle is zero at that point, so there is no transverse doppler term.

... but not in 4-space. The doppler equation would require both the time and space derivatives befor it can be integrated. .. wavefront angle.

The absence of a transverse doppler term in the light cone equation is therefore inconclusive until the constant of integration is determined.

On the other hand, the absence of a transverse doppler term in the light cone equaton is conclusive. The doppler equaion has been experimentally confirmed, so there should be one.

There do not appear to be any terms in the retardaion equations that are identifiable with the transverse doppler effect, although the impression is inconclusive until a constant of integration is determined.

The doppler eqation does not specify how far away the partile was, while the distance is required for the potential equations, hence the need for a constant of integration when evaluating a connection between the two equations.

At extremely high velocites, locaion of the simulatienous point can be almost anywhere. In that regime, eq – is not usable unless the locaion of the simulenos point is already known. If its location is already known then we already have a global soluton and do not need to know how to integrate the doppler equation.

An important special case occurs when the particle is at the point of closest approach to the obsrver. In that case, the point of closest approach and the location of the simulteneous point are the same. The transverse term of the doppler equation has been experimentally verified for this configuration. That does not imply that there are no multipoles beyond the dipole in other configurations.

$\cos 2\beta$ terms represent solutions of the quadrupole symmetry, $cos3\beta$ terms are contributors to an octupole, and so on. The Legenere polynomals represent the scalar terms. The vector spherical harmonics [] represent the vector terms. In containing onle $\cos\beta$ terms, the relativistic doppler equation is a true vector equation. 4-vector equations are also true vector equations unless the solutions contain multipoles beyond the dipole.

Except for a constant of integration, one aspect of the problem can also be investigated by discovering a light cone equation that differentiates correctly. The doppler equation has been experimntally confirmed, so the equation does need to differentiate correctly.

This solution is interprttable in two ways. The tradinoanl interpretino is that the solutions of the Lorntzt transform do not exhibit absolution simulatniy []. The interptethino in this paper is the the simulteniros point is not where we though it was, in which case the LW equations are for the wrong particle.

After discovering a soluton for just one power of velocity, the procedure was then repeated for the next power. After a few terms in the series are obtained this way, it is not too difficult to guess at the closed form solution, which is

The doppler shifted wavelength is identical to that of Eq –.

At the point of closest approach, ts and sin tau are both zero. The solution at that point is then 0/0. The singularity is not just for one point. When it occurs depends on what time it is. We do not know what time it is.

It is mathematically possible for equaions of the form 0/0 to yield specific solutons. However, it is usually better to avoid singularites in the first place. The light cone equation is not a true vector equation. Applying the methos of vector analysis to it pushes them beyond their limits.

The potentials are the integral of the fields, implying that the constant of integration depends on the history of the particle. It is only a constant for a specific trajectory and a specific observer. At any given instant, it is possible for particles with different histories to have the same retarded fields, as it is only the recent history of the partile that matters. A longer history is required if the equations are to be differentiated more times.

Regardless of the point on the trajectory selected for deriving the LW equations, the case where Ru dot vu is zero included in the solution set.

Due to the prescnce of vx vy cross terms in the soluiton, the light cone equation is not a true vector equation. It is a tensor equation. Being a tensor equation does not necessarily preclude the need for a constant of interaton. The constant of integration might take the form of a dislacement for the starting pont of a definite integral. The simulteneous point is not a physical quantity. Choosing an unreal point as an anchor point could lead to unreal solutions.

The light cone equaion can also be viewed as being a pseudo-vector equation, provided that the methods of vector analysis are not unconditionally applied to it.

.... the absence of a transvers dopper term in the light cone equation is therefore inconclusive until the constant of integration is determined.

The constant of integration can be evaluated indirectly by discovering a light cone equation that differentiates correctly.

Since the doppler equation does not specify how far away the particle was, but the distance is required for the retardaion equations, the intgral will require a constant of integration. Eq – provides another perspective of why the doppler equation needs to be integrated.

The doppler equaion has been experimentally confirmed, while the LW equations assume that the constant of integration is zero

[later] The basis of the calculations in this section is that the loop integral of the derivative of a potential should be zero.

[later] There is probably a way of determining where the particle is, but not with these figures. They povide two different solutions, and they are both wrong.

[later] This solution should be zero. The bogus terms are a consequence of angular velocity masquerading as an acceleration term, as discussed in section §xx. The bogus terms make it impossible to integrate the retarded Newtonian accelertion in kinematic solutions.

The equations can be reduced to first order by the same method used in §–.

The LW equations represent an undifferentiated function, whereas the doppler equation is in differntial form, making direct comparisons inappropriate until the doppler equation is integrated in space and time. (The doppler equation does not specify how far away the particle was, but the distance is required for the retardaion equations.) There nevertheless do not appear to be any terms in the retardation equations that are identifiable with the transverse doppler effect.

As illustrated by the Einstein tensor[8], the relationships of space and time are generally not separable. The doppler equation needs both time and space derivatives to be complete. Either the time or the space derivatives would be sufficent in 3+1 space, but not in 4-space. The response of a directional antenna is at a maximum in a direction perpendicular to the wavefront angle. The space derivatives are required for computing the wavefront angle.

Interplanetary spececraft navigation software incorporates corrections from the general theory of relativity when integrating the doppler frequency. The accuracy of the corrections has been demonstrated. It may be possible to infer the full form of the doppler equation for accelerated particles from those solutions. Barycenter corrected solutions would be easier to interpret. The calculation would be relevant, but it would probably not help in reconciling the LW equations with the doppler equation.

VI. THE FIRST FRAME OF REFERENCE

There is always a point of closest approach, so there is a possibilty that the more general ill behaved soluton could be transformed into a well behaved solution. There is always more than one way of working a problem. The essential consideration is that there are angular relatioships that are important for the lighe cone equation but that have no special significaen for true vector equations.

gamma is more difficult to identify when the simulteneouls point is not at the point of closesst approach, but it is still present in the equaions for the first frame of reference.

Due to the presence of kx2 and kx3 terms in the solution, the partile is not half way to any destination in half of the computed travel time. The particle seems to be accelerated in a directo parallel to the retarded velocity. The acceleration is not real, but that is why it needs to be considered when integrating the trajectory of an inertial particle. The retarded potnetials also need to be for inertial particles rather than ghost particles.

Acceleratin parallel to the velocity vector is a form of curvature. It occurs for a partile in radial free fall, for example. The essential relationships are no differnt than for the case where the vectors are not parallel.

The simulteaneous point is rarely at the point of closest approach, but it can be, and the equations are simpler in that case. ru dot vu is zero at that point. The retarded point, the simultaneous point, and the field point define a right triange.

If the velocity vector is parallel to the x axis, then gamma appears in the solution for x, but not in the solution for dx/dt. The connection between the function and the derivative of the function is not the same as it is for true vector equations. The equations for the point of closest approach should not be overgeneralized, although there is always a point of closest approach.

The basis of this relationship is that the time and space coordintes are not indepent varialbes in light cone solutions. The chain rule for differentiaon, or an equivaltnt method, is generally required when the variablses are not independent. Equivalent methods may be preferred in some cases, but the theoretical basis is still the chain rule.

Applying a relativeis correction to a Eucludian soluton for the angular velocity would also work.

A solution can be obtained by integating the derivatives, or by differentiating correctly in the first place. It is easier to differeentiate than to integrate, which is why there are the retarded potentials.

****

Since the acceleration and velocity terms are not instantly separable, the equation is difficult to integrate unless the angle between the trajectory and the line of sight is already known. The accelertion and velocity terms are orthogonal with the Newton equatios. They are generally not orthogonal when the speed of light (or sound) matters. There are cross terms in the equations.

****

The perceived angular location of electromagnetic radiation also lags behind the current location of the particle. The acoustic wave behaves differently when there is a wind, so this analogy is incomplete. The aether does not have a wind, allowing the electromagnetic retardation equations to be simpler.

It is sometimes too easy to assume that we know more than is measurable. As illustrated by the Einstein tensor [], the relationships of space and time are not instantly separable. Prolonged observations are required to determine both where a particle was and when it was there.

****

The doppler frequency is a constaint high frequencey when an unaccelerated source is radially approaching the observer. It instantly switches to a constant low frequancy after it passes the observer. The observer cannot determine from either signal alone when the time of closest approach is. Determining the time of closest approac when the souce is not on a collision course with the observer is no easier. It is sometimes too easy to assume that more is known than is measurable.

VII. THE LIENARD–WIECHERT EQUATIONS

In obtaining light cone solutions, there can be one observer and many particles, or one particle and many observers. The equtions for one observer are[9]

$$\begin{aligned} \boldsymbol{A} &= \frac{\mu_0 q}{4\pi R}\frac{\boldsymbol{v}}{1+\hat{\boldsymbol{R}}\cdot\boldsymbol{v}/c} \\ \psi &= \frac{q}{4\pi\epsilon_0 R}\frac{1}{1+\hat{\boldsymbol{R}}\cdot\boldsymbol{v}/c}. \end{aligned} \tag{2}$$

$\boldsymbol{A}$ is the vector potential and ψ is the scalar potential. $\boldsymbol{R}$ is the retarded location of the particle; $\boldsymbol{v}$ is the retarded velocity. $\boldsymbol{R}$ points from the field point to the source with this sign choice.

The identities $\mu_0 = 1/(\epsilon_0 c^2)$ and $\epsilon_0 = 1/(\mu_0 c^2)$ can be applied to the solutions as appropriate. μ_0 is traditionally associated with the magnetic field, but since the magnetic field is a transformed E field, there are usually options in choosing the parameterization of the solutions.

The potential equations do not contain accelertion terms, but accelerations terms do occur after the velocity of the particle is differentiated to obtain the fields. The LW equations are known to work well for accelerated particles.

The Lorentz transform in vector form is

$$\begin{aligned} \boldsymbol{r}' &= \gamma(\boldsymbol{r}-\boldsymbol{v}t)-(\gamma-1)(\boldsymbol{r}-\boldsymbol{v}\boldsymbol{v}\cdot\boldsymbol{r}/v^2) \\ t' &= \gamma(t-\boldsymbol{v}\cdot\boldsymbol{r}/c^2), \end{aligned} \tag{3}$$

with $\gamma = (1-v^2/c^2)^{-1/2}$. Except for a power of c, the 4-potential transforms in the same way as the coordinates.

The quantity $\boldsymbol{r}\cdot\boldsymbol{r} - c^2t^2$ is invariant with the Lorentz transform. The invariant quantiy is zero for light cone solutions. They are special cases of more general solutions.

Light cone solutions can be obtained by selecting one of the roots of a polynomial. There is no equation for the fifth root of a polynomial, implying that there is something different about high order light cone solutions. Lower order equations tend to be difficult to simplify when there are acceleration terms. The method of successive approximation is frequently a more convenient way of obtaining solutions.

As illustrated by the calculaions of the Thomas precession[7], two consecutive Lorentz transforms are not representable with a single Lorentz transform. Two consecutive transforms are equivalent to a single transform followed by a space rotation. The Lorentz transform is therefore not the most general invariant coordinate transform. The rotations of the Lorentz group[6,10] are still more general.

VIII. AN ALTERNATIVE FORMULATION OF THE LW EQUATIONS

For light cone solutons only, the retarded location of a particle can be expanded as a Taylor series in $\boldsymbol{R}$

$$\boldsymbol{R} = \boldsymbol{R}_0 + \dot{\boldsymbol{R}}_0 t_f + \frac{1}{2}\ddot{\boldsymbol{R}}_0 t_f^2 + \frac{1}{6}\dddot{\boldsymbol{R}}_0 t_f^3 + \ldots\ .$$

$\boldsymbol{R}$ and the retarded $\boldsymbol{r}$ are the same, but $\dot{\boldsymbol{r}}$ is likely to be interpreted as a Newtonian quantity.

This series is differentiated with respect the time at the the tail of the vector from from the observer to the particle, while the Newton series is differentitated with respect to the time at the head of the vector.

The series represents the perceived location of a particle. It is observer-dependent. The Newton series is usually better suited to kinematic calculations.

Vectors and pseudo-vectors behave differently for sign inversions[15]. $\boldsymbol{R}$ is a pseudo-vector. It can be rotated, but it cannot be inverted without transforming to the other frame of reference, in which case it points at us. (A vector can be taken as pointing in either direction, but the usage has to be consistent.)

The advanced potentials for a charge that is at rest in our frame of reference are the retarded potentals for an obeserver in the other frame of reference. A dual series exists for the advanced potentials.

The light cone constraint is

$$t_s = t_f - \frac{1}{c}(\boldsymbol{R}\cdot\boldsymbol{R})^{1/2},$$

where t_s is the time at the source and t_f is the time at the field point. Differentiating with respect to t_f

$$dt_s/dt_f = 1 - \frac{1}{c}(\boldsymbol{R}\cdot\dot{\boldsymbol{R}})/(\boldsymbol{R}\cdot\boldsymbol{R})^{1/2}$$

It can be seen by inspection that

$$\hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}} = \dot{R} = \frac{\boldsymbol{R}}{R}\cdot\dot{\boldsymbol{R}} \tag{4}$$

Substituting $\boldsymbol{R}\cdot\boldsymbol{R} = R^2$ and applying the idendities in Eq 4

$$dt_s/dt_f = 1 - \hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}}/c = 1 - \dot{R}/c \tag{5}$$

and

$$dt_f/dt_s = 1/(1 - \hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}}/c) = 1/(1 - \dot{R}/c). \tag{6}$$

The retarded velocity is $d\boldsymbol{R}/dt_{\mathrm{s}} = d\boldsymbol{R}/dt_{\mathrm{f}}\ dt_{\mathrm{f}}/dt_{\mathrm{s}}$. Substituting for $dt_{\mathrm{f}}/dt_{\mathrm{s}}$ from Eq 6

$$\boldsymbol{v} = \dot{\boldsymbol{R}}/(1 - \hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}}/c) = \dot{\boldsymbol{R}}/(1 - \dot{R}/c). \tag{7}$$

After substituging for $\boldsymbol{v}$ from Eq (7) into Eqs. (2) and simplifying, the LW equations become

$$\begin{aligned} \boldsymbol{A} &= \frac{\mu_0 q}{4\pi}\frac{\dot{\boldsymbol{R}}}{R} \\ \psi &= \frac{q}{4\pi\epsilon_0}\frac{1 - \dot{R}/c}{R}. \end{aligned} \tag{8}$$

$\dot{R}/c$ is less than +1 for radially receding particles. It is unbounded for approaching particles. This asymmetry is one of the relationships that account for the superluminal jets of some astrophysical objects (they only seem to be superluminal).

The effort required to obtain solutions to practical problems with this parameterization is about the same as with the Newtonian parameterizaton. The solutions are exactly the same either way.

A non-zero value for $\ddot{\boldsymbol{R}}$ can mean that the retarded Newtonian accelertion is not zero, or it can be due to the angular velocity of an unaccelerated particle.

While $\ddot{\boldsymbol{R}}$ is ambiguous in this respect, a charged particle does not radiate power unless the retarded Newtonian acceleration is not zero. Retardation equations are intrinsically observer-dependent. The terms in the equations do not have an unambiguous meaning for other observers until a specific global solution is obtained.

The LW equations are form invariant for extraplations in time. Eq. (8) can be written as

$$\boldsymbol{A}_0 = \frac{\mu_0 q}{4\pi}\ \frac{\mathrm{d}\boldsymbol{R}_0}{\mathrm{d}t}\frac{1}{(\boldsymbol{R}_0\cdot\boldsymbol{R}_0)^{1/2}}. \tag{9}$$

The solution at time $t + dt$ is

$$\begin{aligned} \boldsymbol{A}_1 &= \boldsymbol{A}_0 + \frac{\mathrm{d}\boldsymbol{A}_0}{\mathrm{d}t}\ dt \\ &= \frac{\mu_0 q}{4\pi}\Big[\frac{\mathrm{d}\boldsymbol{R}_0}{\mathrm{d}t}\frac{1}{(\boldsymbol{R}_0\cdot\boldsymbol{R}_0)^{1/2}} \\ &\quad -\Big(\boldsymbol{R}_0\cdot\frac{\mathrm{d}\boldsymbol{R}_0}{\mathrm{d}t}\Big)\frac{\mathrm{d}\boldsymbol{R}_0}{\mathrm{d}t}\frac{1}{(\boldsymbol{R}_0\cdot\boldsymbol{R}_0)^{3/2}} \\ &\quad +\frac{\mathrm{d}^2\boldsymbol{R}_0}{\mathrm{d}t^2}\frac{1}{(\boldsymbol{R}_0\cdot\boldsymbol{R}_0)^{1/2}}\Big]dt. \end{aligned} \tag{10}$$

After substituting $\boldsymbol{R}_0 = \boldsymbol{R}_1 - (\mathrm{d}\boldsymbol{R}_1/\mathrm{d}t)\ dt$ and expanding in a series in dt, keeping only the first power of dt, the solution is the same as Eq.(8). The scalar equation works the same way. Thus, when the coordinates are first-known at time t, the LW equations look the same at time $t + dt$ as they do at time t. The solution could then be extrapolated and reparameterized a second time, and the equations would still look the same. Equations that behave this way are rare. For example, the Coulomb equation does not. Retardation equations have to work this way, because they have to work in the same way for the same particle at a later time.

This calculation assumes that Eq. (10) represents the total differential. The calculations in § LIV indicate that it is not the total differntial, although it is close enough for many practical problems when the particle velocity is low. The LW equations are a legtimate term of the retardaion series. They are frequently the only term that is needed.

IX. THE RETARDED EAND BFIELDS

The usual way of applying retardaion equations is to first obtain a global potential solution, then differentiate it to obtain the soluton for the fields. While the method is not of much practical interest, it is possilbe to directly retard the fields in the first place.

The symmetric cross terms in Eq – are important, but they cannot be locally represented with the first derivatives of true vector equations. In being first order in space and first order in time, the cross terms always vanish quadradically in a small region of space-time, while the space and time terms are each of first order.

The cross terms are required if the equation is to be differentited, or if the second derivatives are to be integrated twice, yet they vanish. That seems to be a delima, but not really. If terms vanish quadradically then, for the same reason, they also grow quadradically when integrating in space and time. It becomes necessary to differente, then integrate and provie a constant of integration. The cross terms seem to vanish becasue the LW equations are missing a constant of integration.

The constant is only a constant for a specific trajectory and a specific observer. It is the nature of retardaion equations that the constant of integration varies from place to place and time to time.

The symmetric terms are nevertheless required for computing the second derivatives, or if the second derivatives are to be integrated twice, which poses a delima for true vector equations.

Potential equations represent a global perspective. They are capable of representing terms that are difficult to obtain locally unless a constant of integration can be locally evaluated.

[needs work] The same delima occurs for an observer in radial free fall. Due to tidal forces and other reltionships, the vector from the observer to a nearby marble is changing with time, which should result in antisymmetric space-time cross terms. However, with true vecotor equations, the cross terms always vanish in the infitesimal. The vecor to the nearby marble is not actually a true vector. It is a pseudo vector. In representing a potential relationship, the solution would have to be obtained by integrating from infinity unless a constant of integration can be locally determined.

Despite the missing symmetric terms, the LW equations are accurate at low velocities. They are frequently the only retardation equations that are needed. Indeed, the velocity of conduction electrons in stationary copper wire is so low that relativeisc curreections are completely undtecttalbe, even at currens high enough to melt the wire. They appear to be the correctly determied retardation equations for the tensor of the first rank, a vector.

The retardation equation for the tensor of the zeroth rank is the Coulomb solution, which is usable in quasistatic solutions when the magnetic field is weak enough that it can be neglected.

[delete] In being the gradient of a static scalar, Newtonian gravity is in the same order as the Coulomb solution. If we expect more from electrical electrical equations, it is only because the gravitational field is weak on the laboratory scale of things, and even in our solar system. The roles are reversed in extreme astrophyscal systems.

In this section, the retarded potentials are computed for a nearby observer, then the solution is reparameterized by the coordinates first-known at the nearby place. Since we have no way of knowing where we are in space and time, the nearby observer could be us.

It is therefore plausible that, even when the retarded velocity is zero, the retarded potentials do depend on the acceleration of the particle.

X. THE FIRST DERIVATIVE

There are some mathematical similaries between the light cone equation and 3-space rotations.

When integrating around a unit circle in 3-space, the first infinitesimal step is

$$
\begin{aligned}
y &= \sin d\theta \\
x &= \cos d\theta .
\end{aligned}
$$

To third order

$$
\begin{aligned}
y &= d\theta - d\theta^3/6 \\
x &= 1 - d\theta^2/2.
\end{aligned}
$$

Then

$$
\begin{aligned}
dy/d\theta &= 1 - d\theta^2/6 \\
dx/d\theta &= -d\theta/2.
\end{aligned}
$$

When integrating numerically in n steps, terms must either vanish as $1/n^2$ or be carried. After integration, terms that vanish as $1/n$, while usually small, are in the same order as the solution. The equation is not of first order unless the residual vanishes as $1/n^2$ in each step of the n steps of the integration. The $dx/d\theta$ term is usually small in other problems, but it does not vanish as $1/n^2$ when dy is the independent variable. The equations are not linear if dx and dy are the independent variables [].

Consequently, if the equation is linear, it cannot be reduced to first order by choosing an arbitrarily small value for dy. If the magnitude of dy is reduced by a factor of two then the error is also reduced by a factor of two, but then twice as many steps are required to integrate around a circle, so nothing is accomplished. The basis of this limitation is that x and y are not independent variables in circular motion. The chain rule for differentiation is generally required when the variables are not independent.

An equation can be reduced to a system of several first order equations[2], but not in one step. For example, the two linear equations for integrating around a circular Newtonian orbit are $\boldsymbol{r}_{n+1} = \boldsymbol{r}_n + \boldsymbol{v}_n\ dt$ and $\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \boldsymbol{a}_n\ dt$. The integral becomes exact as dt becomes arbitrarily small. The equations are of first order. It mitht be possible to reduce the light cone equation to first order with more than two linear equations.

The equations are different, but the time and space coordinates are also not independent in light cone solutions. It is difficult to determine both where a particle was and when it was there.

...

In 3+1 space, the quadradic terms are not representalbe with the first derivative at the field point, making it necessary to integrate before differentiating. The solution does not have to be in 3+1 space, but it can be.

It may be possible to apply these relationships recursively, in which cas a double integratl would be requited to reduce the light cone equation to first order.

It may be possible to apply these relationships recursively, in which cas a double integratl would be requited to reduce the light cone equation to first order.

The acceleration terms will need further evaluation, but there is no hope for them until the velocity terms are right.

... Consequenly, even though the cross terms vanish in the infinitesmal, it is not possilbe to choose values for dr and dt that are small enough to reduce the problem to first order (with linear equations).

The equation $1/2\boldsymbol{a}t^2$ is not linear, but it does not have any essential advantages, as the same solution can be obtained by the integration of linear equations. Furthermore, solutions obtained by integration sometimes need a constant of integration that would oterwise be difficult to discern.

XI. SOME INCONSISTENCIES

In Fig –, the time at the field point is the independent varialbe in the light cone equation, which has the effect of artifically pre-compensating the doppler shift so that a constant frequency is received.

As can be seen in the figure, the signal is blueshifted at the time tf=ry/c. According to Eq –, it should be redshifted.

Eqs – and – are both for a particle that is on the light cone, but the solutions are not for the same particle. So which particle should be used for deriving the retardation equations? According to the LW equations, it does not matter. The solution is the same either way. The calculains are shown in the SOM.

From one perspective, this result makes sense. The retardaion equaions for a partcile should not depend on when it reaches the simulteneouls point. On the other hand, there are two differnt particles that reach the simultineouls point at different times, yet the equations are the same for both particles.

****

In this way, the nonlinear equation $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}_0 t + \boldsymbol{a}_0 t^2/2$ is reduced to two linear equations.

When integrating from standstill, a particle acquires the velocity $d\boldsymbol{v} = \boldsymbol{a}\ dt$ at time dt. There are no relativistic corrections that are first order in v. The equations for accelerated particles are vector equations at time dt.

In deriving the LW equations, the Lorentz transform takes out the velocity terms. The particle is initially at rest in the second frame of reference. The first derivatives of the LW equations are for accelerated particles. They are vector equations.

There are relativistic corrections at time $dt_1 + dt_2$, causing symmetric terms to become important in the second infinitesimal step. True vector equantions cannot represent symmetric terms, but pseudo-vector equations can. The Lorentz condition, $\boldsymbol{\nabla \cdot A} + 1/c^2\ \partial\psi/\partial t$, is a symmetric pseudo-vector equation. It vanishes for the first infinitesimal step, but 4-space cannot be reduced to first order in one step.

The derivatives are always exact – until they are differentiated again. The completeness required of the first derivative depends on how many times it is to be differentiated.

The equations would not behave this way if the first dervative represented the total differential in the first place. Obtaining the total differential is more difficult when the chain rule for differentiation is required.

XII. THE RETARDED FIRST DERIVATIVE

There are no space-time cross terms in the first derivatives of 3+1 space. There are cross term in the equations, but the first derivtive at a point displace in either space or time is one of the contribtors to the second derivave. In 3+1 space, it is not possible for there to be space-time cross terms in the first derivative. They always vanish quadradically.

There are nevertheless cross terms in the second derivatives. In potential form, the term $\partial/\partial t\ \boldsymbol{\nabla}\psi$ is one of the contributors to the Maxwell equations. It is a space-time cross term.

The light cone equation is not a true vector equation, so its derivatves are not necessarily computed in the same way as they are for vector equations. The solution for the sum of two velocity vecotors is generally not in the same direction as the vector sum of the velocites. The additional velocity $d\boldsymbol{v} = \boldsymbol{a}\ dt$ acquired by an accelerted particle at time dt counts as a second velocity vector. The behavior of the solutions does approach that of vector equations at low velocites.

While vector equations are a good approxmation at low velocities, they can nevertheless contain an error of the first order. For example, the Coulomb solution is usable in quasi-static solutions, but since the magnetic field is of order v^1, they contain a small error of the first order, no matter how low the velocity is.

This simplification causes the $\mathrm{d}^3\boldsymbol{R}/\mathrm{d}t^3$ terms to be lost. Once they are lost, then cannot be recovered by differentiting the solution.

XIII. THE RETARDED SECOND DERIVATIVE

There are $\mathrm{d}^5\boldsymbol{R}/\mathrm{d}t^5$ terms in the solution for five consecutive infitesimal steps in time. Three steps in time and two and space are required to compute the second derivatvies of the LW equations in a fully general way. There are also $\mathrm{d}^5\boldsymbol{R}/\mathrm{d}t^5$ terms in the retardaion solution. Thus, there are similarites between a step in time and a step in space, but there are also differences, partly because there is only one direction in time – forward.

There are no $\mathrm{d}^6\boldsymbol{R}/\mathrm{d}t^6$ terms in the solution for the second derivative. The soltion is shown in the SOM, but solutions of this complexity should be recomputed on an as-needed basis to insure that there are no software compatibilty issues.

The tensor of each rank is irreducible[6], implying that the solution for three steps in space will be different than this solution.

The ... and ... terms have no effect at all on the final solution for the second derivative, so it would be all right to drop them from the beginning. It seems that we are always one step behind in 3+1 space – sometimes two steps.

These would be our retardation equations when we are at the location drv. The equations would be different if we were at the origin of the coordinate system. But since we have no why of knowing when we are at the origin, we would have to use the other equation. There is this differnce between mathematics and physics.

XIV. XXX THE FIRST DERIVATIVE

Clocks do not show absolute time. They show elapsed time. The time shown by a clock is not usable until it has been synchronized.

As the flying clock experiments show[7], the time shown by a clock depends on its history. A moving clock runs slow by a factor of γ. Unless the history of the clock is known, we do not know what time it is in the second frame of reference.

The Lorentz transform implicity assumes that the clocks have been satisfactorily syncjronized. It will hold the quantity $\boldsymbol{r}\cdot\boldsymbol{r} - c^2t^2$ invaraint regardless of the accuracy of the synchronization. The transfom does not exhibit absolute simultaniety[1,16], so there are no indications that the clocks are inherently synchronized.

It is not physically possible to synchronize a clock when it is at the simultenous point. The clock is visible at the field point at the time $t_s + r/c$, but by then the clock has moved on to a new location. Since the clock is visible at the time r/c, it could in priciple be synchronized at that time, but because the space and time derivatives are not separable, an observer in the second frame of reference might not agree on when that time is. The clock rate does not matter at first if both clocks are reset to zero when at a time when the moving clock is visible (the clock is permanently visible, but the equations can be for times when it is not visible.)

The second derivtive is required for computing the first derivtive at a displaced point. The retarded point is displaced, relative to the simulteneousl point. For this reason, terms that would be first derivatvies in a simulteneous system can look like second derivatives at the retarded point.

XV. THE FIRST FRAME OF REFERENCE

(Eq – was selected for illustrative purposes. It is not necessarily the right equation.)

It is always possible to rotate the coordintes about the observer so that the velociy vector has only one component at the point of closest approach. A velocity vector with one component is mathematically equivalant to a scalar, so the equations are simpler. The coordinates can be roatated back their original angles after the calculaions, so there is no loss of generality in choosing a particular coordinate system.

This method of constructin reduces to the basis of vector equations when the particle is on a collision course with the observer.

****

True vector equaions cannot represent multipoles beyond the dipole. Multipoles beyond the dipole exist even in 3-space. The spherical vector harmonics[2] are an example. A true multipole is irreducible. It cannot be represented by any linear combination of dipole solutions.

The tensor irreduciblty theorm[6] represents the differntial anglular relationships between the tips of two or more closely spaced vectors in 3-space. There are other terms in 3+1 space, but a space rotation is still a space rotation. The theorem would not apply if the light cone equation were a true vector equation, but it is not.

****

The radial and transverse velocites are orthgoal with the Newton equaions, but they are coupled in light cone solutions. In being coupled, Newtonian methods of vector analysis are capable of produceing misleading results. They assume an orthobonalty of the four coordinates that does not exist in 4-space.

By rotating the coordeinates to this point, the radial and transverse compoinents become orthonalized and Newtonian methods of vector analysis become applicable to some extent, but never completely.

There can be a particle at that place and time, and the light cone equaion works just as well for it as for any other particle, but it is the wrong particle.

The soltuon xsim =0 is the rigth soluton for vector equaions. There is nothing wrong with vector equaions. However, the gradient of a vector is a tensor of rhe second rank, which includes symmetrice terms. The othr soluton is required if the symmteric terms are to be carried.

The integral of time dialation is a time offset. In being the integral of the fields, potential would be expectec to contain offsets rather than rates.

Thus, even though the two solutions look idential, there is a possibity that they do not have the same meaning. The role of absolute simulatney in the solutons needs further investigation.

****

There are vectors and there are pseudo-vectors. The tensor irreducibilty theorm[6] represnts the differential angular relationships of 3-space pseudo-vectors. There are other terms in 3+1 space, but a space rotaion is still a space rotaion.

If a particle is perceived as radially receding now then it was approaching from the rear at an earler time. The former observer was the target. Vectors and pseudo-vectors do not work in the same way.

****

From the persectie of an observer in the frame of reference of the particle, the doppler equation is for one particle. But at any given instant, an observer at the field point cannot tell whether the doppler shift is due to the radial or transverse component of the velocity vector. The solution is for a family of particles with various radial and transverse velocites. One member of the family can be selected by rotating the coordiant system so that the velocity vector has only one component at the point of closest approach.

*****

In implcity assming the existence of absolute simultainey, the LW equaions apper to be for the wrong particle. There are other equaions, such as the calculaions of the Thomas precession[7], that implicityly assume the existence of absolute simultanaity. It sould be possible to apply correctons to reduce the ambiguity in the equations.

****

The transvers compinetn of the velocity vector is the only component at the point of closest approach, effetively uncoupling the two contibutors. The particle acuires a longtiulandl component at othr times.

It should prove easier to integrate the doppler equaion if the coordinates are rotated to this point, as the radial and transverx compoinent becme orthonalized.

Equations can be confusing when they are for the wrong particle. They tend to defy the methods of reason.

When the equations are for the wrong particle, that can have the unfortuneate effect of leading to the conclusion that the methods of reason no longer work.

****

The particle is on a collision course with an unfortunate second observer at the point of closest approach, but the motion is purely radial from the other observer's perspective. The radial and transvers componets of a velocit vector become orthoganlized for that special observer, because there is no transverse component, which makes the equations easier to solve.

Other observers in the same frame of refernce would agree on when the partcile is at its closest approach to our location in space and time, making it possible for us to obtain the soution for them. We need to do that, because one of the other observers could be us next time.

****

Dipoles are the only multipole terms that vector equations can repreent. It is always possible to rotate the coordinates so that the velocity vecotr has only one componint at the point of closest approach. The particle is then on a collision course with an unfortuante second observer located at that point, but the multipole terms drop out. A velocity with one compoent is mathematicly equivalelnt to a scalar, which does not have multipoles. After the soluion is obtained in that particular coordinte system, the multipoles beyond the dipole can be recovered by rotating the coordinates back to their original location.

Because of Olbers' paradox, we know from the study of geometry that we cannot see to infinity, indicating that the expansion facor of the cosmos needs to be included in the scalar term. It is unintuitive, but mathematical singularites at infinity are also possible. It is not a satisfacory reference point for equations. The cosmologial term will be neglected for now, but it may need further considration. The expansion factor stretches the coordinates isotropiclly.

The the first multipole beyond the dipole becomes

There are probably better ways of representing it.

t_f can be set to zero at the start of an experiment. It does not matter when the clock is reset, but it cannot be reset again during the course of a given experiment.

XVI. THE LW EQUATIONS

The virtual acceleration term is not real. The linear momentum of a particle is independent of its angular velocity.

The particle is visible forever. Obtaining a solution that is valid forever may be overly ambitous.

The magnitude of the radius vecor is at a quadradic minimum at the point of closest approch, so the light cone equation cannot precisely define the time of the event. Consequentlly, light cone solutions can be for the wrong partile if they are based on an extrapoltion from the time of closest approach.

It is possible to work the retardaion problem without assuming that the time of closest approach is already known.

There can be several obserevers of the same particle. The solution would be unphysical if location of a particle in the second frame of reference depended on which observer is watching. The LW solution appears to be the right equation for the wrong particle.

The radial and transverse velocityies components are obserere-dependent, leading to ambiguites in the interpretation of the projection of the retarded potentials into 3+1 space.

The transverse doppler term at this point on the trajectory is approximately twice the value given by Eq –. ... The transverse doppler term vanishis at this point on the trajectory. The doppler term would be appoximaely correect midway betweean the two points, but equatons of that form are not usable, becauaes we do not know what time it is.

The virtual acceleration term is not real. The linear momentum of a particle is independent of its angular velocity. The solution is not for an inertial particle. The solution is the right equation for the wrong particle.

The trajectory is not for an inertial particle. Any one observer does percieive the trajectory as being curved. However ,the inertial momentum of a particle is unrelated to its angular velocity.

Even thought the retarded potentials are intrinsically observer-dependent, they have to work in the same way without knowing wherther the equation of radial or transverse velocity should be used, becuse the contributions of the two terms constantly change roles in orbital motion.

The kx2 terms represent a false curvature term that has to be taken out. Accelertion terms are important, but they cannot be developed until it is possible to write the equation for a straight line. Actually, the line is stratight, but it is not uniformly stretched, which is a form of curvature that is not representable with straight lines unless they have uniformly spaced tic marks. It is only in 4-space that straight lines have tick marks indicatint the time shown by a clock moving along the line.

****

The time required for a particle to reach a specified marker goes to infinity as the velocity of the particle goes to zero. The singularity at zero velocity can cause the equatons to be ill behaved even at low velocites. The singlarity can be exlicit in potential equatons. It tends to be hidden in the derivatives, but the inverse relationship is also present in them.

An equaion can be reduced to a system of servral first ordee equaions[2], but not in one step. For example, the two linear equaions An equaion has to be of first order before it can be integrted. The first derivative is free of error, but there are not many equations can be reduced to first order in one step.

Consequently, no matter how small the velocity is, the integral will contain an error of the first order unless the equation is of first order. The error is usually small, but when integrating numerically in n steps, terms must eithr vanish as 1/nx2 or be carried.

The transverse doppler is, which is approximately twice the value shown in Eq –. The basis of the discrepancy is that the LW equatios are missng a factor of γ. the redshift is visible in the bottom panel of Fig – for the slanted line that is coincident with the locaton of the particle at the point of closest approach.

There are no indications that this soluton is the last term of the series.

With other ways of deriving the LW equations, it is possible to interpret the transverse doppler term as being zero. There are ambiguites in interpretng the roles of the radial and transvese velocitys with vector equations. The ambiguites are not resolvble with vector equaions, so an equally valid interpretion is that the transvere doppler term is zero. In any case, the equaions do not correctly represent the term.

In the figure, R0 and R1 are not parallel, but the angle between them should be computed from the space coordiates rather than from the time coordinate.

The distortion does not affect the LW equations – until they are differentiaed.

The endpoint of each line is at an oberver. One of the other observers could be us next time. Or maybe this is the next time.

XVII. ROTATIONAL INVARIANCE

By carrying more terms in the series expansions, the dropped cross terms can be made arbitrarily small. However in being of higer order, the dropped terms grow faster than lower order terms when integrating in time. No matter how small they are, they eventually become dominate. When computing the differnce between two Talor exansions, it can be necessary to carry more terms than seem to be needed if the terms are not orthogonal and difference is to be integrated.

?these relationships are from the persective of an observer in the first frame of reference. An observer in the other frame of reference would perceive the same relationships differntly.

Rotational and translational invariance are not independent concepts. The transverse velocity for one observer can be the radial velocity for another observer at a different location.

Working the retardaion problem implicity requires two appliations of the Loretnz transform, although shortcuts can be taken that should not be taken. It is only the differnce between the two transforms that matters. The absolute value of a coordinate is not of physical significance.

No matter how many terms are carried, the neglected cross terms, in being of higher order, grow faster than lower order terms when integraging in time, and eventually become dominate.

Due to the non-orthogonalit of the Newton series in 4-space, one more power of velocity must somtimes be carried than will be required in the final solution. The Taylor theorm does not work this way in an orthogonal system. When in doubt, carry one more power. After dropping the excess terms in the final solution, if a more accurate calculation makes any difference at all, then it is required.

In an orghogoal system, the diffrence between two Taylor extrapolations is just as accurate as either extrapolation. That is not necessarily true when there are cross terms. The cross terms can be of higher order than the terms of either extrapolation. This behavior is rare, but the Taylor theorm does contains the assumption that the terms are orthogonal.

The flaw tends to be invisible if the variables are assumed to be orthogonal when they are not. No matter how many terms are carried, the missing cross terms are of still higher order, but they grow faster than lower order terms and ultimately become dominate. The requirement for rotaional invariance is one of the few ways of nailing the elusive terms.

The vx3 terms in this solution are incomplete. Due to the non-orthogonality of rhe Newton series in 4-space, they must nevertheless be carried to obtain usable accuracy. In 3+1 space, we are always one step behind – sometimes two steps.

The order of an equation is difficult to judge by its appearance. It is the highest order multipole in the solutions that matters.

The vx3 terms are actually in the order of the octupole. There are no octupoles in the solutions of this order, but due to the non-orthogoality of the Newton series in 4-space, that does not imply that the vx3 terms can be dropped. We are always one step behind.

dx and dy have dropped out. If the solution is valid for the location y+dy then it is also valid for the location y+0. The inverse relatinship is not necessarily true.

The vx3 terms are actually in the order of the octupole. For reasons of mathematical self-consistency, they nevertheless need to be carried in lower order solutions.

It follows that the vx2 terms in the space derivatvis of the LW equations are not rotationally invariant. However, due to the non-orthogonality of the Newton series in 4-space, powers of velocity beyond the first have to be carried to obtain usable accuracy in the LW solutions. The additonal powers are incomplete but harmless.

However the series expansion for 1/(1+v/c) contains powers of velocty in all orders. For reasons of mathematical self-consistency, powers of velocity beyond the first should usually be carried when applying the equations.

The absolute value of a coordiate is not a measurable quantity. Only coordinte differences are of physical significance, so there can be no preferred orientaion for a coordinate system. The equaions have to be rotationally invariant.

From either perspective, it is only coordinate differences that represent measurable relationships. Transformng the coordnates back to the first frame of reference does not help, becuase the Lorentz transform will restore the original coordinates regardless of the accuracy of the solution in the other frame of reference.

The retarded potentials provide a means of resolveing one aspect of the enigma. After thansforming the coordintes to the second frame of referce, the are used to parameterize the potentials in that frame of refernce. The scalar potential has no angular sensitivity, so angular reaionships are lost when computing it. Howver, the same rotation occurs with the other sign when transformn the potentials back to the field point, making it possible to determine if the soltion is rotationally invariant. By proceeding in this way, it becmes possilbe to know what what cannot be seen.

There is a possiblity that the calculaion should be extended to the third frame of reference, so the following calcualons may be incomplete in more ways than not carrying enough powers of velocity.

When working the retardaion problem, there are implicity two transforms, one for the simulteneous point and one for the retarded point.

Since an explicit delta t and delta r are not visible in the second frame of refernce, assumng the the differene between two separate Lorentz transforms is not coordinate dependent would require justification, especially when the two events are widely separated in time.

While coordinte absolutes clearly could play a role in the solution, it is difficutl to be sure that it does. The followng calculations show that the solutons are not rotatinally invariant, which represents the intruston of coordinate absoluteness in a different but related way.

It may be possible to apply a corretion for the behavour, but it seems better to choose the right particle in the first place, especially for kinematic calculaions.

As shown in the SOM, the light cone equatin is rotaionally invariant for a single rotaion about the z axis, but it is not rotationally invariant after transforming with the Lorentz transform. It would not be appropriate to blame the Lorentz transform until the light cone equaion works in the right way. However, the problems are scarcely independent.

It is not sufficint that equations hold the speed of light invariant. They must also be rotationally invariant, as they would otherwise contain an unphysical coordinate dependency.

Since the light cone solution is rotationally invariant, it might seem that it is the Lorentz transform is not, but the conclusion is not justifible. The problem is that the solution is for the wrong particle.

To assume that the locaion of the partile at the simultenouls point is already known it to assume that the solution is known before working the problem. The Lorentz transform does not exhibit absolute simultaney cite, so it does not help in determing which partile is the right one.

There is an octupole in the solution for the angle phi+dph1+dph2. The octopole vanishes unless the vx4 terms are carried. It will not be considered at this time.

There would not be any point in carrying the vx4 terms in these calculations. Those terms belong in the third frame of refernce. We cannot be sure that the third frame of reference is not our frame of reference.

There are special cases where a moving clock can be synchronized and then read at a later time, but we usually do not know the history of the clock, so we do not know what time it is in the other frame of reference.

We do not know what time it is in our frame of reference either, but that is not a problem. The Lorentz transform can be applied independently at two times and it is only the coordnate differences that matter. However, the coordinates are not orthogonal in the second frame of refeerence. The time difference becomes contaminated by an absolute space coordiante that is not yet known, or conversely. Consequently, the solution depends on what time it is, but we do not know what time it is.

The particle is visible forever, so it needs to be permanently on the light cone. We cannot see forever, so the equation is missing terms that will need further attention.

The solution is the LW solution. However, retardaion equaions are intrinsically observer dependent. The soltution is the right equaion for the wrong observer.

The solutin is the LW solution. The LW equaions are very adequate when the particle velocity is low, which is always the case for stationary current carrying wires.

That allows the absoluteness of the coordintes to contaminate the solution, causing it to be unphysical.

By proceedng this way, it is only time differences that appear in the equaions, so we no longer care what time it is in the other frame of reference.

The consequences of the the lack of absolute simultaney are somewhat different for retardaion equations. If the simultaneous point is not where it is thought to be then the computed retarded location will be for the wrong particle.

This approach also suffers from logial circularity, because the actual retarded location of the partilcle was not known in the first place.

Consequently, the absolute value of the coordinates pervades the solutions, causeing them to be unphysical. The problem could be corrected by orthogoalizeing the coordiantes. That would require the location of the particle, and the objective was to compute its location. The problem is logially circular. The disconnect between the time intervals is manefested in the solutions of the Lorntz transform by the absence of absolute simulteniery cite.

There are four ways for two observers to observe two consecutivve events from the same particle. It might be possible to impose some additional constraints by requiring that all four observations be for the same particle. The consequences of the absence of absolte simultety should be included in the calculions, as the solution would otherwise be logically circular and self-fullfilling.

The problem is somewhat different for the retarded potentials.

This disconnect is manifested in the solutions of the Lorentz transform by the absence of absoute simultanty cite.

This approach also suffers from logical circularity, because the actual retarded location of the particle was not known in the first place.

The consequences of the absence of absolute simultanie has to be included in the calcuions, as the result is otherwise logially circular.

This calculation also sufferes from logial circularity, because we did not know where the particle was in the first place. But when applying retadion equains, the location of the particle is assumed to be already availalbe, then the derivtives are computed. The retarded potentials exist only for the purpose of being differentiated. The actual locaion of the particle is of secondary importance. The considerations for orbit determination would be different.

This location is where the field point will be when the signl arrives there. It is in the other observer's future. The other observer now has to compute when the sigal should be transmitted so that it will arrive at the expected place at the expected time.

The corrction is large when the velocity is high. It makes the application of vector equaions at the Hubble distance inappropriate.

But since the objctive is to compute the loaion of the particle, there ia an element of logical circular in the problem.

retardaion equaions offer an advantage in this respect. If the location of the retarded point is taken as a reference point, the magnitude of the disconnect at the simultaneous point is computable, making possible to apply a correction for it.

The magnitue of the disconnect at the simpultanes point, considered in relation to the discaonect at the retarded point, is computable, making it possible to apply a correction for it.

When woring the retardion problem, the magnitude of the disconnet between the retarded and simultenous points becomes substantial at high velocitys

The kx2 terms represent a false curvature term that needs to be taken out. An unaccelerated inertial particle should be half way to the destination in half of the travel time.

Any one observer does not perceive the particale as being half way to the destination when k is 1/2, but a field of obervers would conclude that it really is half way there. Retardation eqations are intrinsically oberver dependent, but their basis should be equations that are the same for all observers.

The most distant objects visible to us are about 13 billion light years away and receding at nearly c, so they must be around 26 billion light years away now, if they still exist. Large galaxies can survive for 13 billion years, so they are probably still there, but we will have to wait another 13 billion years to see where they are now. That is not on the agenda.

It is probably not possible to prove that it works that way, but there no solution without assumptions.

We cannot see the particle when it is at the simultinouls point, so we did not know where it is in the first place. There is no contradiction in the solution. It is rather that the guess was not very very accurate.

This choice for the locaton of the simultaneous point appears to be the right choice for vector euations. There is nothing wrong with vector equaions, however the gradent of a vector is a tensor of the second rank, which inludes both symmetric and antisymmtric terms. The solutions of the LW equation do not include any symmetric terms, indicating that the solutions are incomplete.

A more accurate estimate of the location of the simultaneous point is needed if the symmetric terms are to be carried.

The particle is not on a collision course with the observer in this solution, which is why it contradicts itself. The soltuion is not obtainable with true vector equations. There are vectors and there are pseudo vectors. Their behavior with sign inversions is different, meaning that pseudo-vector solutions have to be obtained by rotations. The solution for the observer's backside cannot be obtained by inverting signs.

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The absolute value of a coordinte does not represent a measuragle relationship. It is only coordinate differnces that are measurable. The difference in the readings between a clock our frame of refernce and the other clock is a coordinate difference, but the solution is usually for the wrong particle. As the flying clock experiments show[7], the time shown by a clock depends on its history. When the history of the clock is unknown, the only option is to connect the diffrence between two times in the other frame of refence with two times in ours.

If neither transform is for the right particle, then assuming that the differnce is for the same particle would require justificaion.

The solutions of the Lorentz transform do not exhibit absolute simultaineyl cite. If the simultanos point is not where it is thought to be then the computed reatarded location will be for the wrong particle, and conversely.

4-space equations tend to be logically circular.

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The soluton for the special case should be obtained by a limiting process from a more general global solution.

The radial velocity for one observer is a transverse velocity for another observer at a different place. Retardaion equations are intricsially observer dependent, yet they have to work in the same way without knowing whether the equaion for radial or transverse velocity should be used. They require more information than the doppler equation provides, although the doppler equation would provide more information if the space derivatives were obtained.

Because the equation merges the contributions of the transverse and radial components of the velcoity vector into one term, it cannot be integrated until the contributors are separated. Obtaining the space derivativs of the equation would help, although the space and time derivaties are generally not uncoupled either.

The simultaneous point is where the particle is now. The partcile velocity cannot exceed c, so an unacceleted radially receding particle had to be at least half way to where it is now when it emitted the signal. When the particle is radially approaching with a velocity near c, it is traveling nearly as fast as the signal, so the simultaneous point is nearby (we think). The Lorentz transform does not exhibit absolute simultenity cite, so it is difficult to be sure of where it is. It is plausible that the location of the simultaneouls point in the other frame of refernce is more important than it is in ours, which could provide another approach to working the problem. In any case, the absolute value of a coordinate does not represent a measurable quantity.

By proceeding in this way, it becomes posssibl to compute where the simultaneous pont is without knowing what time it is in the other frame of reference. That is important, because we do not know what time it is in our frame of refrence either.

If the radial velocity were the only velocity that mattered then the time in the other frame of refecnes could be computed with the equains ts=rf-r0/c, thereby avoiding absolute coordiate values. As the doppler equaion shows, the radial velociy is not the only velocty that mattes. It is only when the partice is on a collision course with the obsevr, eigher in the future or in the past, that the radial velocity is the only one that matters. In other cases the Lorentz transform assumes a coordinate absoluteness that is not measurable.

Yet, if the calcualions are to be perfomed with the Lorentz transorm, there is no alternative to assuming that the soluton is known before working the problem.

Retardatin equations are intrinsically observer dependent, yet they have to work in the same way at differnt times and at differnt places.

The solutions are coordinate-dependent and therefore unphysical, even for unaccelerated particles.

It is nearly impossilbe to show that there is anything wrong with coordinate-dependent equaions. The requirement is a physical constraint that mathematial theorms do not necessarely consider.

The observer at this location has no way of knowing when the time dtf=0 should be. It has to drop out of the solution if the equaions are to be of physical significance.

When working the retardaion problem, the partile emits a signal at the retarded location. The signal propogates to the field point with the velcity c. In the meantime, the particle proceeds on to the simultaneous point. It arrives at the simultaneous at the same time that the signal arrives at the field point.

We cannot see where the particle is when it is at the simultaneous point. The particle is at rest in the other frame of refence. The other observer does know where the particle is then, but the other observer cannot see where we are.

A similar disconnect is present in the solutons of the Lorentz transform. They do not exhibit absolute simultanety cite.

The speed of light is the same in all frames of refernce, but the two frames of reference are are disconnected from each other. Similarly, the solutns of the Lorentz transform do not exhibit absolute simultenety cite As the flying clock experiments show[7], the the time shown by a clock depends on its history, so it is doubtful that any transform exhibts absolute simultenety.

It has to be possible to perform calculaions without knowing what time it is in the other frame of reference. It is only coordinate diffences that represent measurable relationships. The absolute value of a coordinte is not of physical signifiance.

Comparing the absolute time in the other frame of refernce to the absolute time in our frame of reference is not meaningful in a physical sense. We do not know what time it is in either frame of reference.

The equations are less logially circular if the requirement is imposed that soluton be for the same particle at two different times in its history. The constraint is related to the requirement that the solution be for one inertial particle rather than two ghost particles. The solution is still logially circular, but less so.

The solution in the other frame of referen is timeless. The particle is always in the same place so it is not necessary to compute when it was there.

Due to the absence of absolute simultanety, the location of the partile at the simultaneouls point has to be computed. The locaion of the particle at that point is of little interest in its own right. It is rather that the location of the retarded point is not computable until the simultaineouls point becomes know.

This solution is only for the first derivative. More observers and more events would be needed for a more complete solution. There is always more than one way of working a problem. There are probably better methods. The equations from the rotaions of the Lorentz group cite are suggestive.

There are four ways for two observers to observe two consectutive events from the same particle. The four perspectives are not independent.

The unknown offset would not matter if the transvers velocity did not matter, but the doppler equaion shows that the radial and transvers veloctites are not instantly sparable.

Comparisons of the absolute time in the other frame of reference to the absolute time in our frame of refernce have no physical meaning. We do not know what time it is in either frame of reference.

It becomes necesary to compute the locaion of the simultineos point withoot knowing what time it is. Actually, we do not know what time it is in our frame of reference either.

The simultaineous point is not in our future, but it is in the particle's future. Since it is not possible to predict the future perfectly, it is not possible to predict exactly where the partile will be when the signal reached the field point. The simultaineouls point is required for compting the retarded location, so there is a strong element of logial circularity in the problem. There is no solution without making assumptions. Assumptions can be either wrong or incomplete. It is not likely that there are other options.

Since the retarded potentials are first-known in the other frame of refernce, a plausible assumption is that the locaton of the simultaneousl point in the other frame of refernece is more imporatant than it is in ours. The assumption will lead to contradictions if it is wrong. It is inevitable that contradictions will arise, for the tensor of each rank is irreducible[6].

The connection to the Thomas precession is that those calculations implicitly assume the existence of absolute simultaneity. These calculations take out the Thomase precession. Howver, taking a term out of one equaion can have the effect of putting it into another equation. The Thomas rotation survives in clearly recognizable form as a rotation of the retarded vector potential. The Thomas rotaion is of ordr vx1 ax1. The vectro potential is of order vx1, so the rotatio in potential form is of order vx2 ax1.

While the paticle is not visible when it is at the simultaneols point, it is nevertheless an essential mathematicl anchor poiint for projecting the solutin into 3+1 space. The solutions would defy the methods of reason if we already knew where the partilce was when it is at the simultaneoos point, but we do not. The other observer does, but we do not.

In 3+1 space, the location of the simultaineouls point has to be computed. Logical contradictions will arise if the the time shown by a clock in the other fame of referne is assumed to be known before working the problem. It is only coordinate differnces that are measurable. The absolute time shown by the other clock is not knowable. It is only the difference in the displayed clock numbers in its journey from the retarded and simultaneous points on the trajectory is knowable.

The absolute numbers that the clock displayed at eithr point on its journey from the retarded point to the simultanols point is not knowable in a general way. It is only coordinate differences that are knowable in a physical sense.

The Lorentz transform does not exhibit absolute simultanety. If the simultaineous point in the other frame of refernce is not where it is thought to be then the computed retarded location will be for the wrong particle. The derivation of the LW equatons assumes the existence of absolute simultanety, which is why they are for the wrong particle.

The location of the simultainous point has to be computed. The solultions will defy the methods of reason if the location is assumed to be known before working the problem.

It is only the relatonship between the two displayed clock numbers that has a meaning. The absolute time shown by the clock at either point on the trajectory has no general meaning.

The methods of reason will fail, and properly so, if the location of the particle at the simultaineousl point is assumed to be known before working the problem. All things measurable are relative, but it is not the clock in our frame of reference that they are relative to.

Eq – suggests that the light cone equaion in the first frame of refernce should be the inverse of the equaion for when the coordinates are first-known in unretarded form.

If the particle is at the location $x = 0$, $y = r_y$ at the point of closest approach and the velocity vector is in the $+x$ directon, the angle between the line of sight and the velocity vector is

$$\theta = \arcsin \frac{r_y}{(r_x^2 + r_y^2)^{\frac{1}{2}}}. \tag{11}$$

Then

$$\cos\theta = \pm\big(1 - \frac{r_y^2}{(r_x^2 + r_y^2)^{\frac{1}{2}}}\big). \tag{12}$$

The minus sign is selected for approaching particles.

In our frame of reference, the particle can be thought of as moving along a marked course. There is a synctronized clock at each grid intersection. A second observer near the trajectory could report the time that the particle passes by a clock, but that would not allow us to find out where it was any sooner. We can find out where it was, but not until after waiting for the light time accross the system.

It may not be obvious, but retardaion equations and the calculations associated with them have to predict the future of the particle, based on past events. The identiy of the particle becomes establihed when it reaches the simultaneojus point. The simultaineous point is not in our future, but it is in the particle's future. The particle is not visible when it is at the simultanus point, but it is nevertheless an essential mathematical anchor point. It is not possible to predict perfectly when the particle will reach the simultaneous point, making it difficult to be sure of which particle the equation is for in the first place. Because it is not possible to predict the future perfectly, light cone solutions are not necessarily for the right particle.

If a solution seems to defy the methods of reason, it may be because it is for the wrong particle (or the wrong observer).

As noted in the soluton for Eq –, it is important that Eq – not be overgeneralized.

This behavior can be difficut to interpret, because if the solution is for the wrong particle in the first frame of reference it will still be for the wrong partile in the second system.

There could be other reasons for doubting that the soluton actually is for the right particle.

These relationships are not independent of the concept of absolute simultanty. If the simultaneneous point in the other frame of reference is not where it is thought to be then the computed location of the retarded location will be for the wrong particle. The solutions of the Lorentz transform do not exhibit absolute simultanety[1,16]. It should be possible to apply a correction for the effect, but there would still be some uncertainty in the computed retarded location of the particle if a truncated Taylor series is used to extrapolate the trajectory to the simultaneous point.

From another perspective, if the soluton is for the wrong particle in the first frame of refernce then it will still be for the wrong partile in the second frame of reference. These relationships are all part of the same system. They cannot be considered on at a time.

The integral of time dialation is a time offset. Consequently, retardaion equations to not include rate terms until the solutions are differntiated.

From the perspective of an observer located elsewhere, this method of construction reduces to the basis of vector equations when the point of closest approach is arbitrarily close to the origin of the coordinate system. However, the anglar velocity goes to infinity as the miss distance goes to zero. The singularity at the origin is invisible in 3-space, but a singularity is not a good place for an obsever to be when working the retardaion proglem.

Actually, the singularity is not perfectly invisible in 3-space. Inverting the sign of the radius vector in sperical coordinets is usually not the right way to rotate it by 180 degrees.

More terms are required to reliably identify a series. More terms are shown in the SOM, and the two solutions are the same.

Even in our frame of refernce, it is not possible to quickly determine where a particle was. We can find out where it was, but not until after waiting for the light time accross the system. Even in our frame of reference, it is possible to write light cone equations for relationships that predicting the future of the particle. The simultaneous point is not in our future, but it is in the future of the particle. There are limitaions on how accurately the future of the partice can be predicted, even when the events are in our past.

Events simultaneouls in one frame of refernce are not necessarily simultaneous in a different frame of refernce[?] . If the time t=0 at the field point does not correspond precisely to the time t=0 in the frame of refernce of the particlel, there is no obvious justifican for assuming the time ts=tf-r0/c is precisely known either. The transvers term may be explainable as being due to the absence of absolute simultaneaity. On the other hand, there is no possibltu of computing the time in the other frame of refernce if it is not known in our frame of refrence.

XVIII. ABSOLUTE SIMULTANEITY

Since we cannot see where the particle is when it is at the simultaieous point, it is probably not a satisfactory refrence point for equations of physical significnce.

Some interpretaions of the transverse doppler effect assume the existence of absolute simultaneity, which is not thought to exist[1]. If absolute simulteneory does not exist, then the time when the other observer transmitted a signal has to be computed in relation to somthing else. All things measurable are relative, but it is not the clock in our frame of reference that they are relative to.

In not establishing a connection between the simulteneous points in the two frames of refernce, there are no assurenace that the potentials in the LW equations in the frame of refencce of the particle and in the frame of refeence are for the same partice.

Most interpretaions of the Thomas precession[7] implicitly assume the existence of absolute simultcnousy. The thomnas calculaions do not have to be for light cone events, but they can be.

The dervaion of the doppler does not make simeny assumptions, but the transverse doppler efffect would defy reason if it did exist.

All things measurable are relative, but it is not the clock in our frame of refernce that they are relative to.

[delete] The discussions throughout are for equations in the real number system. The methods and the perspectives for complex number equations may be different.

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The potential solutoin obtained this way is not necessarily unique, because potential equations are subject to gauge transformations. In not including mass terms, the solution is also incomplete.

More complete comparsisons could be performed by computing the retarded space derivtives of the doppler shift.

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Under ideal circumstances, the particle in this solution is the same partcle that the LW equaions are for, but the solution is for a different point in the history of the particle. The identity of the particle has to be determned by extrapolating the trajectory to the simultaneous point.

It is not true that we always need to know the identity of a particle. There can be more than one particle in the same orbit. It is usually sufficient to know the shape of the orbit rather than which of the paricles the soltion is for.

Good estimates of where the partice will be at the simulteneos point are achievable, but exact solutons for future events are not possible. The simulteneous point is not in our future, but it is in the particle's future. Our inability to predict the future perfectly results in some uncertainty in where the particle was.

The LW equatons have the same problem, because they assume that the location at the simultenous point is known before working the problem. That has the effect of hiding the uncertainty.

The identity of a particle becomes established when the time that it is at the simultaneous point becomes known. If the particle is on an inertal trajectory, it can be determined if a light cone event for a different time is for the same particle.

The particle in this solution is not the same particle that the LW equations are for. We do not yet know which particle it is. If the retarded location and velocity is known, its location at the simultaneous can be predicted. The location of the particle at the simultaneous point will be about the same as with the LW equatins, but predictions of what will happen in the future are never exact. The simultaneous point is not in our future, but it is in the particle's future. The prediction is accurate at low velocities, but it becomes more difficult at high velocitys or for accelerated particles. The $\dot{\boldsymbol{a}}$ and $\ddot{\boldsymbol{a}}$ terms also make a small contribution.

While there is some uncertianty in the identity of the particle at the start of an integration, the integration iteself is accurate if enough terms are carried. However, there is still nebulosity in predicting where the particle will be when the signal arives at the field point. The nebulosity is not cumulatve in periodic solutions, since it follows approximately the same cycle in the next orbit. The neboulosity is no worse for a distant orbit than it is for a nearby orbit.

It is always possible to rotate a coordinate system about the field point so that the velocity vector has only one component at the point of closest approach. If required, the coordinates could be rotated back to the orignal angles after the calculaions are performed, so there is no loss of generality in choosing a particlar coordiate system.

Because the velocity vector can always be reduced to one component, there is the possiblity that fully general vector equations are too general.

But since it is difficult to be sure of precisely when a partile is at the simultaneous point, there is some uncertainty in the identity of a partile at a light cone event.

The LW equaton is relative to the simultaineous point. This equation is relative to the retarded location. The solution is now for an inertial particle on the light cone, but we no longer know which particle it is. We did not know in the the first place. At low velocities, the light cone equations does provide a good estimate of where the simultaines point is, but the solution is not exact, because it predicts the future. The simultaneousl point is not in our future, but it is in the particle's future.

The solution is now for an inertial paraticle that is on the light cone, but we no longer know which particle it is. We did not know in the first place.

The uncertainty only affects the initial conditions. Therafter, the integration is accurate if enough terms are carried. However, there is still some nebulosity in predicting where the particle will be when the signal arives at the field point. The nebulosity is not cumulatve in periodic solutions, since it follows approximately the same cycle in the next orbit.

The LW equations do not include acceleration terms. There is a family of accelerted particles with different histories that all have the same retarded velcity at a given place and time. The LW equaions are not for one partilcle. They are for a family of particles. The member of the family that the solution is for is not known until a specific global soluton is obtained. In practice, the problem is worked backwards by assuming that a specific global solution is already available, such for an antenna configuration, then the derivatives are computed.

[delete] This solution could be obtained with the doppler equation alone, but that approach could be interpreted as meaning that our frame of reference is special. It actually is special, because it is the only one we can ever measure anything in. Equations that work in our frame of refernce are vital. However, inertial relationships are not relative to us.

From one pespective, the solution looks like the solution for an inertial particle. From a differnt perspective, an unaccelerated inertial particle should be half way to the destination in half of the travel time. There should not be any k^2 terms. They cause the trajectory to seem to be curved when it is not. The k^2 terms vanish for radial motion, where $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{v}}$ is ± 1. They are obsever-dependent. A transverse velocity for one observer is a radial velocity for an observer at a different location. Retardation equations are observer-dependent, but they should be based on equations that are the same for all observers.

One obsever cannot see that the eqations are obsever dependent, because the observer cannot tell the difference between the solutons for one observer and two particles or two observers and one particle. The ambiguity in the strength of the magnetic field is of order $(v/c)^2$, so it can be neglected at low velocites.

It is possible for one oberver to distinguuish between the two cases if all of the events are known to be for the same particle, but it is difficult to be sure that they are.

It is indicated that the relativistic doppler equation is for an inertial particle, whereas the LW equations are not.

XIX. DISCUSSION

The problem is different for the retarded potentials, since the trajectory is assumed to be already known, but the same ambiguity is present.

A transverse velocty for one observer can be a radial velocity for a different observer. One obserer cannot distinguish between the solutions for one observer and two particles or two observers and one particle unless all of the events are known to be for the same particle. It is not possible to be sure that they are unless the future locaton of the simulenous point can be predicted perfectly. Consequently, light cone solutons are not necessarily for inertial particles.

XX. THE METHOD OF RETARDATION

When integrating in time, the importance of the assumed initial conditions fades into history as the integration proceeds. Eventually, the derivatives dominate the integral.

However, there are many particles that were at various locations long ago that behave in about the same way for a short time at the retarded intersection. It is only the derivatives of the retarded potentials that are of interest in the laboratory, because we cannot quicky determine which particle in a family of particles the integral is for. A smaller family can be selected by obtaining more derivatives, but we cannot be sure which particle is the right particle until a global solution is obtained.

In periodic orbital solutions, the integral of the derivatives changes sign on the other side of the orbit, so the integral does not accumulate indefinately. But for short times, the double integral of the $\dot{\boldsymbol{a}}$ terms grows faster than the single integral of the $\boldsymbol{a}$ terms.

Integration in the infinitesimal degenerates to multiplication. There are no curvature terms in the infinitsimal. That does not imply that curvature can be neglected in the integral.

The retarded potentials are the integral of the retarded fields. They are arbitrary to within a constant of integration. The constant of integration depends on the history of the particle. It is not uniquely defined until a global solution is obtained. Since the retarded potentils exist only for the purpose of being differentited, it is not necessary to know what the constant of integration is, but the equations are nevertheless subject to gauge transformations. Potential equations are not necessarily unique.
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Potential equations do not represent locally measurable relationships. They represent a global perspective. A global perspective can be acquired by a field of observers, or by one observer with prolonged observations. When the solution is obtained by a field of observers, the propagation delay in communicating the data points from one observer to another is part of the problem. Regardless of the precision of the measurements, there are ambiguities in solutions based on delayed observations of short duration.

An observer in an orbit about a distant star would have the same problem in obtaining the solution at our location. The retarded potentials would exist if we were not even here. The barycenter correction in our frame of reference would still be required.

Zero doppler shift does not occur at the point of closest apporach. It occurs slightly before then. (There would be other terms for a circular orbit.)

Doppler measurements within our solar system have the same problem to a lessor extent, which is why interplanetary spacecraft navigation software incorporates corrections from the general theory of relativity.

It should be possibe to perform a second infitesimal transform from our orbital velocity at time t to our velocity at time t+dt. Because we are in an approximately circular orbit, these corrections do not accummulate indefinately. They change signs later in the orbit.
****

Astrophysical doppler measurements require a correction to the barycenter of our solar system. A short data record of a distant pulsar would be impossible to interpret if we did not already know where we were.

not know where we were in the first place. An observer at the barycenter could perform a Lorentz transform to our frame of reference, and then perform a second Lorentz transform to the velocity of the pulsar. In that way, our velocity and the velocity of the pulsar would both be accomodated. Two consecutive Lorentz transforms are not representable with a single Lorentz transform. They are equivalent to a single transform followed by a space rotation[7]. Consequently, if we use a single Lorentz transform in our frame of reference, the angular relationships are wrong. We do not seem to be accelerated, but we are in free fall about the sun.

By normalizing observations made at various points in the Earth's orbit to the barycenter, the observations can be directly compared.

Our velocity is normally very small in relation to that of an orbing pulsar, in which case an infinitsimal Lorentz transform to our frame of reference is adequate.

If we were not in free fall, then an orbiting pulsar would be. The problem is the same from a different perspective.
****

With the Newton equations, there would be a false velocity term if an attempt were made to separate acceleration and velocity with two points on the trajectory. The retarded location of a particle depends on both its location and its velocity. There is a false acceleration term if an attemt is made to separate acceleration and velocity with three points on the trajectory. There would be a false $\dot{\boldsymbol{a}}$ term in the solution for four points on the trajectory. We are always one step behind. That is because the tensor of each rank is irreducible[6].

An assistant near the source could record the time that the particle passes each clock in a field of synchronized clocks. We usually do not have access to those numbers.
****

The gradient of a vector is a tensor of the second rank. This relatinship can be misleading, because there is no point in extrapolating a vector if a global vector solution is already known. That is why there are potential equations. Equations that look like vector equations are not necessarily obtainable with vector equations.
****

This limitation is plausible. After transforming to the second frame of reference with the Lorentz transform, the acceleration and velocity terms are coupled. We have no way of knowing whether we are in the first or second frame of reference. That does not matter when we assume our velocity is zero, but two observers cannot make the same assumption at the same time.

A tensor of the second rank is required to extraplate a vector to a nearby point unless a global vector solution is already known. If a global vector solution is already known then there is no point in extrapolating a vector, but it is not implied that the solutin is obtainable with the methods of vector analysis. The differnce between any two points in the solution is nevertheless a vecotor. However, the differential angular relationshops between two or more clolsely spaced vectors are represented by the contravariant tensors.

The tensor of rank n+1 is the gradient of the tensor of rank n. The tensor of the first rank is a vector. Regardless of how many integrations are performed to obtain a global vector solution, the difference between any two points in the solution is a vector, although it is not necessarily a vector that we are adept at measuring.

In any case, the the derivative of the integral is the function.

Actually, there are vectors and there are pseudovectors. They are not locally distinguishable, but their behavior with sign inversions is different.

The potentials are the integral of the fields. A global vector solution is a potential solution. Potentials are not locally measurable. In a physical sense, they are not real. Or is it that the potentials are real and our ablity to perceive them is limited?. We might never know.
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The terms in this series have the same form as the Newton series, but the Newton equations do not contain c, so the terms do not have the same meaning.

There would be transverse terms in Eq – if the partile is not at the origin at the time -r0/c. When ... is zero, the light cone equaton does not impose a first order constraint on the location of the particle. It is therefore possible for the LW solution to be the right equation for the wrong particle.

This solution is for a particle that is on the light cone, but we cannot tell which particle it is until a global solution is obtained.

XXI. THE RETARDAION EQUATIONS

From the perspective of the observer at the barycenter, the time interval at the field point is a dependent variable. The chain rule for differention is required if it is treated as an independent varialbe. An example of applying the chain rule to the retardaion problem is given in the derivation of the electromagnetic tensor in Ref –. The chain rule can be applied recursively for computing derivatives beyond the first.

It is possible to integrate the derivatives to synthesize retardaion equations that can be differentiated without the chain rule.

ts0 is a free parameter in the solution. If its value is zero then the particle is approaching the distant observer at the point of closest approach, which is a contradiction.

An unaccelerated particle does not have a barycenter, but there could be a second observer anyway. The transverse terms of the light cone equation are better constrained when there are two observers.

rvf has dropped out, so it does not matter whether the obserever is near the particle or at a distance, although there is still some slipperyness in the equations.

The partile seems to be approaching us when it is at the point of closest approach, which is a contradiction. We perceive the particle at an earlier time in its history when it is on the other side of its barycenter from us, and there was a radial component of velocity at that time. An unaccelerted particle does not have a barycenter, but the effect is still present when there is one.

The particle seems to be approaching us. We perceive the particle earlier in its history when it is on the other side of its barycenter from us. It was approaching us then, but not when ru dor vu is zero. Constant velocity particles do not have a barycenter, so there are additional considerations orbiting particles, but the velocity terms are of lower order.

While it should be possible to obtain the solutions with the chain rule, the other observer could be us next time, so it is sometimes simpler to explore the perspectives.

The followng calculations are closely related to the calculations of the Thomas precession[7]. The difference is that the radius vector is initally from the barycenter to the particle rather from the observer to the particle. The difference is important unless the particle is orbiting the observer.

The distant observer is not intrinsically impaired. It is rather than thetax2 terms are not computable with two points on the trajectory. Since the radius vector rotates as the particle moves, ru dot vv cannot be zero at both times t and t+dt for an unaccelerated particle. The equatons have to work for unaccelerated particles before there any hope for accelerated particle.

In Cartesian coordinates, this degree freedom becomes visible when one of the coordinates is parallel to the trajectory. The only effect of a coordinate rotation is to distribute the ambiguity amongst the three coordinaes.

The radius vector rotates as the particle moves. It is less likely that ru dot vv is zero for two nearby points than it is for one point, although it is still possible if the particle is accelerated.

The light cone equation does not impose a first order constraint on the transverse location of the particle when ru dot vv is zero. An infitesimal rotaton of the radius vector does not affect a first order solution. A light cone solution could be for the wrong particele and we would not know it.

This degree of freedom is more visible in Cartesian coordinates when one of the coordintes is parallel to the retarded trajectory of the particle. The ambiguity is less visible with other orientaions, but a coordinate rotation only distributes it amonst the three coordinats.

dt has dropped out of the solution. If this is the right equation at time t+dt then it is also the right equation at time t+0. On the othr hand, an equaion that works at the time t will also work at the time t+dt, but the solutions are not necessarily for the same particle.

From the perseective of an observer at the barycenter, the dtf interval of a distant observer is a dependent variable, making the chain rule for differentiaon necessary. No mattr how small the dtf intereval is, the obserer at the barycenter can subdivide its projection onto a small distant orbit into two segements that are not parallel to each other, in that way rpresenting the thetax2 terms. The thetax2 terms drop out of the solutin for the distant observer. That would not affect the distant observer if they were not radiative, but they are. Dipole radiation is the only kind of radiatin that is representable with one infinitesmal rotaion by the distant observer.

The solution is for a family of particles at the retarded intersection. All members of the family are at the same place and have the same velocity at two nearby points on the trajectory. One member of the family has to be selected by applying the equations to a global solution. Since the equatins do not specify the acceleration of the particle, its identity is not determinable until a global solution is obtained.

When the trajectory of the particle is artifically constrained, such as in the solutions for rotating electrical equipment, the global solution and the identity of the particle is already known, but there are usually additional considerations for insuring that the orbit is physically realizable.

The retardion equaions do not specify where the barycenter is, but its location does have to be known before they can be applied. Retardaion equations do not form a complete representaion. They are just one slice of a much larger problem.

The 4-potential and the coordinates transform in the same way, however the anglular informaton was lost in computing the magnitude of R in Eq –. There is this differnce between the behavor of the 4-potential and the coordinates. The gradient of a vector is a tensor of the second rank, while the gradient of a scalar is a vector. For this reason, potential equations can be simpler.

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The derivaties constrain the location of the particle more accurately, but if the light cone soluton is for the wrong particle in the first place then differentinag it without imposing the light cone constraint at each step in the differentiaons would not accomplish anything. The chain rule is not used in the following calculaons, but it should be possible to perform them with it. An example of applying the chain rule to the retardaion problem is shown in the derivation of the electromagnetic tensor in Ref –.

With the Newton equatons, the location of the particle at time t+dt1+dt2 is computable from the location at time t. The trajectory is not necesssarily Newtonian. Eq – looks like the Newton equaions, but it is a Taylor expanson of the trajectory of the particle. The Taylor theorm has a more general meaning than the Newton equations.

We can know our velocity relative to the barycenter of an orbiting particle, but we cannot detrmine where the barycenter is with two points on the trajectory.

What seems to be one solution is actually a family of solutions. It is not possible to determine which member of the family the retardaion equations are for until they are applied to a global solution. Once that is done, we will know whether we are orbitin the particle or the particlie is orbing us. We can and should know which way it is, but it cannot be done with two points on the trajectory.

The location of the particel at time t=0 is not yet known. We do need to know where the particle was then, but that depends on the acceleration of the particle, and the equatinos do not contain acceleration terms. The acceleration terms are not computable until the equations are applied to a specific global problemm. The retardaion equations represent a family of solutions. We have to select one member of the family, and it cannot be done when only two points on the trajectory are known. The retardation equations do not specify where the barycenter is, but its location must nevertheless be known before they can be applied. Retardaion equations do not form a complete representation. They are just one slice of a much larger problem.

It may be possible to select a smaller family of particles by including the $\dot{\boldsymbol{a}}$ terms in the calculains. The equations would not be so slippery that way.

No mattr how small dtf is, the obserever at the brarycenter can subdivide its projection onto a small distant orbit into two short segments that are not parallel to each other, and in that way represent the θ^2 terms. That would not affect the distant observer if the θ^2 terms were not raditive, but they are.

The avdot terms can be neglected in observations of short duration, but they grow faster than lower order terms when integrating in time. They will be neglected in these prelimminary calculations.

Determining where the barycenter is is not easy, especially for an observer in free fall. We are always in free fall.

The first derivative becomes exact as the inteval goes to zero. This principle only applies to independent variables. The chain rule for differentiaon is required when the variables are not independnt. From the perspective of an observer at the barycenter, the dtf interval of a distant observer is a dependent variable in light cone solutions.

The deriviatves impose tighter constraints on the transverse location of the particle, but differenting the equation does not help if the light cone solution is for the wrong particle. The LW solution is the right equation for the wrong particle.

In being closer to the particle, the angle from the observer to two points on the orbit is larger than for a distant observer. An infitesimal rotation of the radius vector maps into a shorter section of the orbit, although there is still some slipperyness in the equations.

The solution is for two points on the trajectory at times t+dt1 and t+dt1+dt2. We cannot determine where the particle was at time t until a global solution is obtained. Potential equations do not represent locally measurable relationships. We can and should know where the particle was at time t, but not until after we know whether we are orbiting the particle or the the partice is orbiting us. It could be either way if we are in free fall and only two points on the trajectory are known. The retarded potentials do not form a complete representaion. They are just one slice of a much larger problem.

No matter how small the dtf interval is, an observer at the barycenter can subdivide its projection onto a small orbit into two short segments that are not parallel to each other. That would not affect the distant observer if the curvature terms were not radiative, but they are. The curvature terms are also visible to the distant observer in other ways, provided that the observer knows where the partice was.

The trajector of an accelerated particle is curved. It has been established by Newton and others that there are no velocity-dependent curvature terms when the speed of light does not matter. Quadradic terms are required to represent curvature. The incremental velocity of a particle at time t is $\dot{\boldsymbol{a}}t^2/2$, so the $\dot{\boldsymbol{a}}$ terms have to be carried to represent velocity-dependent curvature. The equation $\boldsymbol{a} = \ddot{\boldsymbol{a}}t^2/2$ is another quadradic term. It can be neglected at first when the orbital radius is large, but it grows faster than lower order terms when integrating in time.

In applying retardation equations, the location of the particle is assumed to be already known, then the derivatives are computed. On the other hand, the derivativres must be intgrated to obtain the trajectory of a particle. The perspectives are very different.

This soluton would be the same as the LW solution if the location of the particle is assumed to be already known at time t=0, but the ligth cone equation is too slippery for accurate calculations at that point on the trajectory. The trajectory of the particle is better constained by including the $\dot{\boldsymbol{a}}$ terms, although there are still some ambiguites in the solutions.

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The tensor irreduciblity theorm[6] represents the differential angular relationships between the tips of two or more closely spaced vectors. Many of the theorms of Eucludian calculus are for one independent variable, in which case the chain rule for differentiaon may be needed first, especially for problems involving angular relationships.

Many advances in the study of the geometry of rotations have been made during the past century. The Thomas precession is one of them, however those terms appear to incomplete. Incompleteness does not have the same meaning as being wrong. It is our destiny to use incomplete equations.

XXII. AN INTRODUCTION TO EUCLUDIAN GEOMETRY

The modern form of Eucludian geometry is Eucludian calculus. Euclid did not know about the chain rule for differentiaon.

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The vector from the field point to the particle rotates as the particle moves. During the full traverse of an unacclerted particle, the angle ranges from from $-\pi/2$ to $+\pi/2$. $\hat{\boldsymbol{R}}$ rotates though half of a circle. It depends on where the particle was, not where it is. The equation is Euclidian, but it is not a true vector equation.

This example is for one velocity, but in other problems the light cone solution for the sum of two velocity vectors is generally not in the same direction as the vector sum of the velocites. When a particle is accelerated, $\dot{\boldsymbol{v}}\ dt$ is a second velocity vector.

The contravariant tensors represent the differential angular relationships between the tips of two or more closely spaced vectors[2]. The tensor of each rank is irreducible[6], as manefested by the multipole order of the solutions increasing with the rank of the tensor. The highst order multipole of the tensor of the fourth rank is a hexadecapole (4 lobes). The decomposition products of the fourth rank tensor include six vectors. The tensor of the first rank is a vector, but tensors of any rank can be the basis of equatons that look like vector equations. As exemplified by the the Lorentz condition, $\boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, the equations include symmetric terms. The gradient of the Lorentz condition is another vector, and its divergence is one of the scalars represented by the tensor of the fourth rank. The highest order multipole of the LW equations is a dipole. (The sum of two dipole solutions is sometimes referred to as a quadrupole, but it is reducible to two dipoles.).

Actually, there are vectors and there are pseudovectors. They are not locally distinguishable, but their behavior with sign inversions is different.

The tensor irreducility theorm only applies to linear equations. Retardation equations are linear equations. The eqations of the general theory of relativey are not linear equations[8].

Retardaion equations represent the perspective of a distant observer. They are not usable in an accelerted frame of reference, except that the earth's gravity is weak enough that it can usually be neglected.

XXIII. THE LIGHT CONE EQUATION

The light cone equation, $\boldsymbol{R}\boldsymbol{\cdot}\boldsymbol{R} - c^2t^2 = 0$, can be solved by the method of successive approximation or by selecting one of the roots of a polynomial. Due to the presence of quadradic terms in the equation, the solution for the sum of two velocity vectors is generally not in the same direction as the vector sum of the velocities. $\dot{\boldsymbol{v}}\ dt$ is a second velocity, so the cross terms between a velocity vector and its derivatives are present in light cone solutions. The Lorentz transform has the same characteriscic.

As shown in the SOM, two of the cross terms are

$$\dot{\boldsymbol{R}} = -\frac{1}{c}(\boldsymbol{v}\hat{\boldsymbol{R}}\boldsymbol{\cdot}\dot{\boldsymbol{v}} + 2\dot{\boldsymbol{v}}\hat{\boldsymbol{R}}\boldsymbol{\cdot}\boldsymbol{v})\ dt$$

There are other terms in the full solution that are not cross terms. Despite the nonlinearity of the light cone equation, there are no cross terms between the components of one velocity vector. A single velocity vector is rotationally invariant, allowing the light cone equation to behave like a vector equation in more ways than it is.

There would be more cross terms if the $\ddot{\boldsymbol{v}}$ terms were carried. Unlike the Newton equations, the solution depends on the angles between the derivatives. The contravarient tensors represent the differential angular relationsips between the tips of two or more closely spaced vectors[2,6]. The difference between two closely spaced vectors is itself a vector, but in general the two vectors are connected by a tensor of the second rank.

The decomposition products of the tensor of each rank include vectors[6]. The tensor of the first rank is a vector, but tensors of any rank can be the basis of equations that look like vector equations. As exemplified by the the Lorentz condition, $\boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, the equations include symmetric terms.

There are vectors and there are pseudo-vectors. They are not locally distinguishable, but their behavior with sign inversions is different.

XXIV. SOME LIGHT CONE SOLUTIONS

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the solutions are not affected by translating the coordinates in either space or time.

In this solution the particle emits a signal at the time $r_0/c + t_f = 0$. The signal arrives at the field point at the time $t_f = 0$, with the total time being $+r_0/c$. In the meantime, the particle proceeds on to the simultaneous point, still on the light cone. It arrives at the simultaneous point at the same time the signal arrives at the field point. If it emits a second signal when at the simultaneous point it will be received at the field point at the time $+r_0/c$. The events are shown in Fig. 2 for the case where the motion is radial.

The slanted lines in the figure could represent periodic pulses transmitted by the source. The time interval is longer at the source than at the field point, so the pulses are compressed in time and doppler shifted to a higher frequency. The period of all the pulses is the same, so the received frequency is constant. The transfer function is linear in time.

2+1 space light cone solution when the transverse velocity is low. The field point is at the location $z = r_z$. The particle moves left to right along the y axis. $c = 1$. The top of each slanted line is the scalar distance from the field point to the particle when it emitted the signal.

The figure assumes that the particle is at the tip of the r_0 vector when $t_f = r_0/c$, but it is difficult to be sure of where the particle actually is then without a nearby assistant to report the time of the conincidence.

The LW solution is unaffected by translating the Eq – coordinates in time by the amount $-r_0/c$. The equations can also be parameterized by the value of the radius vector at the time $t_f = -r_0/c$, and it makes no difference in the final solution. It also does not matter whether the light cone equaion is solved in the first or second frame

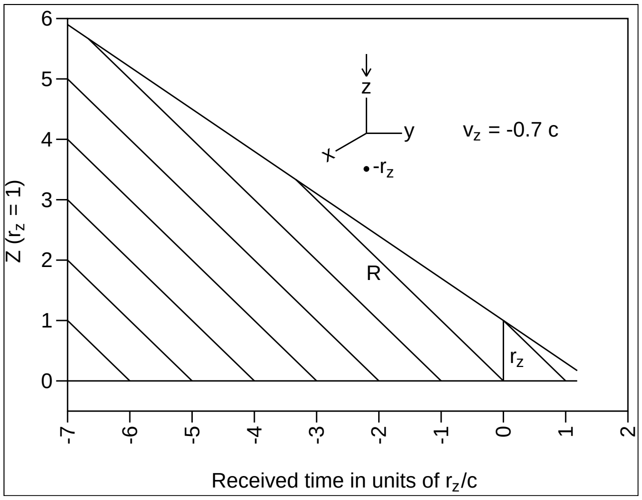

FIG. 2. 1+1 space light cone solution for retarded radial motion. The top line is the trajectory of a particle approaching the field point along the z axis. The slanted lines from the trajectory are light rays propagating downward to the field point. The field point is at the location $z = r_z$, with $r_x = r_y = 0$. $c = 1$. All calculations shown in the figures are based on an exact root of a polynomial. The series calculaions used elsewhere are not accurate enough.

of reference. The LW equations are the same either way. There are endless ways of deriving the LW equations.

The coupling of angular and translational velocites is unique to the four dimensinal space. Doppler shifted acoustic signals are also in 4-space, so the light cone equation would work equally well for them, with c being the speed of sound rather than the speed of light. The coupling is a consequence of the propogation delay, which has no meaning in 3-space. Straight lines in 3-space do not have tick marks equally spaced along their length, so we cannot tell when they have been stretched. In 4-space, straight lines are curved when the stretch is not uniform along their length. Some of the stretch is due to the cosmological expansion factor, which exists within a mass shell. The expansion factor affects the time and space coordinates equally, so the locally measured speed of light is not affected].

The coupling of the angular and translational velocites can be neglected when the velocity is low. But when the relativistic corrections are already of order $(v/c)^2$ times the momentum of the particle, the coupling is in the same order as the corrections, in small systems.

The equations are parametersized by rv1 instead of rf0, but they look same at either time. They are form invariant for translations in time. Form invariance has about the same meaning as the requirement that the equations should not contain a coordinate dependency. Coordinate dependencies can exist in either space or time.

When there is more than one observer in a problem, each observer is not free to choose their own coordinate system. Both observers must use the same system. This is what the solution looks like when the coordinates are first-known at time $t_{off} = -r_0/(2c)$ by the other observer.

The retarded Newtonian acceleration is zero in the figure, but, for observations of short duation, the solution would look the same if the particle had a different retarded velocity and a non-zero retarded acceleration parallel to the retarded velocity. One observer cannot quickly tell the difference between Newtonian acceleration and angular velocity. Two observers would know the difference. One observer evaluating a system at two widely separated times counts as two observers.

In being of order v^3, the midpoint shift depends so strongly on velocity that the LW equations are usable even at moderately high velocities. The v^3 terms also occur with acoustic signals. The propogation delay causes the acoustic location of a low flying high speed aircraft to lag behind its visual location. The doppler shifted tone of a passing vehicle is constantly changing. At low velocities, the rate of change of the tone is at a maximum at the closest approach. As illustrated by the top panel of Fig.**??**, the v^3 terms cause the maximum to occur earlier in the trajectory when the velocity is high.

In the figure, Rvdot is shown aligned with the time at the source, but the effects of Rvdot are not perceived at the field point until one light time later. In Eq (), the solution for Rvdot is not delayed. The LW equations include only the radial aspect of the propagtion delay. The consequences of the propagation delay on the transverse component are not included.

An isolated and unaccelerated inertial particle will reach half way to the destination in half of the travel time. The LW equations assume that light cone solutions behave the same way. They usually do not. The LW equations are not for inertial particles.

This calculation was not performed the right way, so it is difficult to be sure of what it means. It does indicate that the vx2 terms of the scalar equation and the vx3 terms of the vector equation are not meaningful.

The basis of the inconsistency is that the LW equatons assume that an unaccelerated particle reaches half way to the destination in half of the travel time. Isolated and unacclerated inertial particles do behave that way. Light cone solutions usually do not.

The basis of the inconsistency is that the LW equations are not form invariant when there is a transverse velocity component.

The equations are still form invariant if the v^2 terms are dropped from the scalar equaion and the v^3 terms from the vector equaton. The vector equation is already of order v^1, so a $(v/c)^2$ correction to the LW equaions is needed. That does not imply that there is anything wrong with the equations if they are applied within their range of validity. There is no possiblilty of detecting the shortcoming with currents in stationary wires.

The solution becomes form-invariant if the v^2 and v^3 terms are dropped. The velocity of conduction electrons in copper wire is so low that the LW equations are more than adequate in those applications. They are the

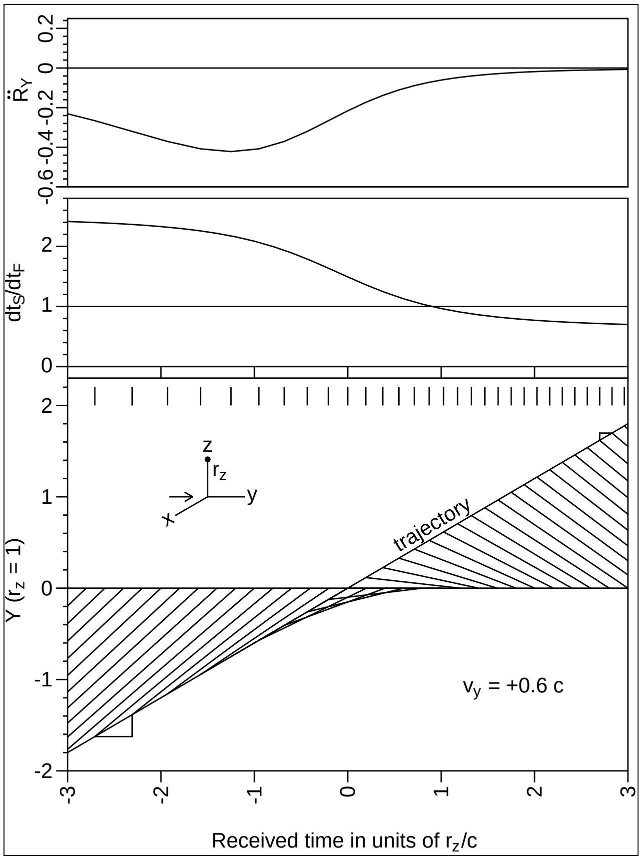

FIG. 3. One slice of the 2+1 space light cone geometry. The transmitter is moving left to right along the y axis. The receiver is at $x = y = 0, z = r_z$. The tick marks along the top of the bottom panel show when each pulse was transmitted. The pulses are not sent at regular intervals, but they are received at regular intervals. The dt_s/dt_t curve in the middle panel represents the doppler shift. The $\ddot{R}_y$ curve in the top panel is shown aligned with the time at the tip of $\boldsymbol{R}$. $\ddot{R}_x$ and $\ddot{R}_z$ are zero. $c = 1$.

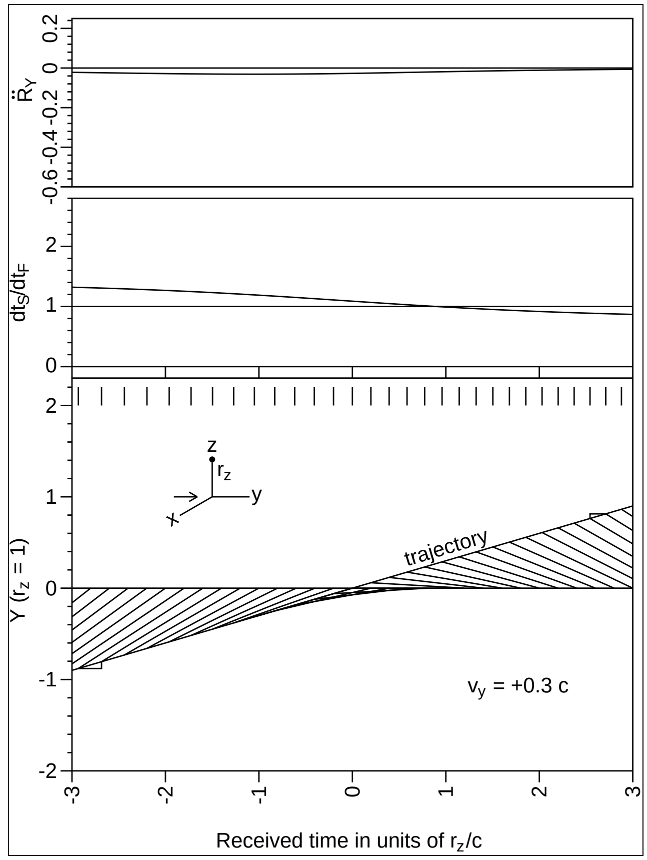

FIG. 4. The scales in this figure are the same as in Fig. 3, but the velocity is only half. The perceived acceleration depends so strongly on the velocity of the particle that it can usually be neglected.

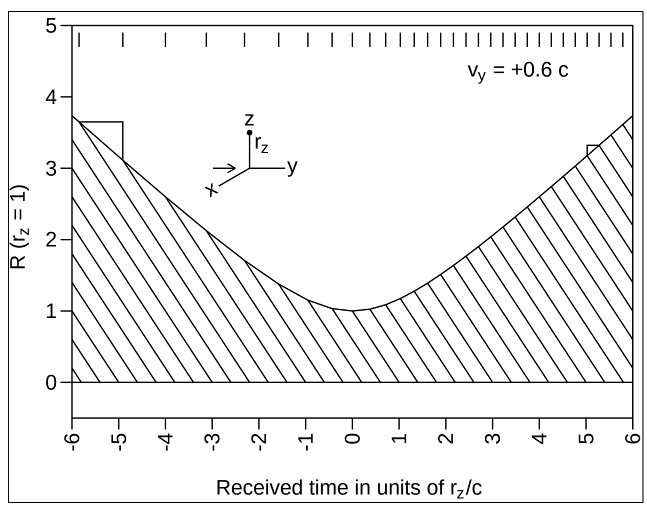

FIG. 5. This sketch shows the same data as in Fig. 3, except that the magnitude of ΔR is shown instead of the y component of the position vector.

valid second term of the retardation series. The static Coulomb solution is the first term of the series. Newtonian gravity is in the same order as the Coulomb solution. The Newton equations may seem to be more accurate, but that is mostly because the gravitational field is weak on the laboratory scale of things.

The time when $t_f = 0$ can be chosen arbitrarily. Once the time is selected, the equations cannot be readjusted as needed if there is, or could be, another observer in the problem. There could be another observer at the particle, and it could be us.

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The basis of the inconsistency is that the midpoint of a time interval at the particle does not map into the midpoint of the corresponding time intervatl at the field point when there is a transverse velocity component. The transfer function is not linear. The duration of the two intervals is also different. The frequency of a doppler shifted frequency is constantly changing when the motion

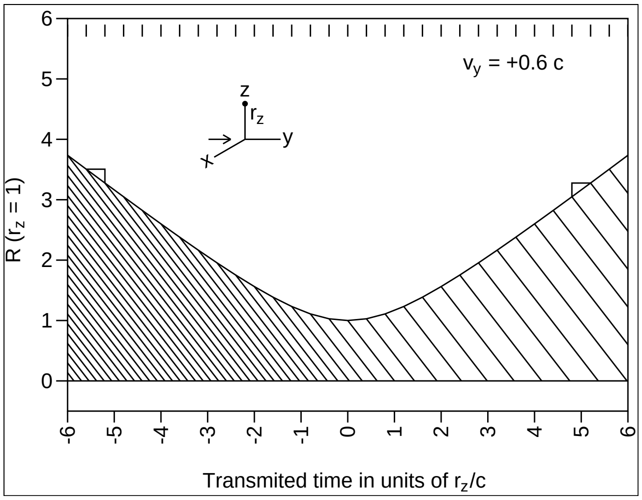

FIG. 6. This sketch shows the same data as in Fig. 3, except that the light cone equation was solved for t_s as the independent variable. It represents the solution from the perspective of the other observer. The pulses are transmitted at a uniform rate, but they are doppler shifted to a higher frequency when the source is approaching the field point.

is not radial. That makes it difficult to determine what the number displayed by the other clock was, and it is more difficult when the midpoint of one time intrval does not map into the midpoint of the other.

Since an unaccelerated clock does run at a constant rate, the time shown by the other clock should be obtainable in prolonged observations by requiring that its computed rate be constant after correcting for the propation delay.

The vector r0 in Fig 1 is easily identifiable in a global solution, but it has to be possible to apply retardaion equations without knowing when the time tf=0 will occur.

The perceived velocity is greater when the particle is nearby. The perceived angular and translational velocites are coupled, making it difficult to separate them unless the retarded velocity is already known, which is usually not the case.

****

The basis of this relationship is the delay from the particle to the field point. A change in the velocity of the particle is not perceived at the field point until one light time later. In the meantime, cycles of radiation are piling up along the propogation path, and the cycles do not lie along a straight line when there is a transverse velocity component. Simplistic doppler equations are not valid when the angular relationships are constantly changing. The angular relationships causes a transverse velocity term to look like a Newtonian acceleration term that is parallel to the retarded velocity. Thus there are two different effects that determine the frequency of a doppler shifted signal, and they are difficult to separate. In being of order $(v/c)^3$, the angular terms are small enough that they can usually be neglected.

The LW equations are for the time tf=0. Other choices would be equally valid, but translating the coordinates in time has no effect on the solution, so there is no point in choosing a different value.

****

In being the integral of the fields, potential solutions are arbitrary to within a constant of integration. It is the nature of retardaion equations that the constant of integration depends on where the particle is, relative to the observer. Unlike simpler equations, the derivatives of a function will not be consistent with the function if the constant is not correctly determined. Other equations also have this characterisc if they represent a double integral and the first constant of integrataion is included in the second integration.

Only the derivatives of the retarded potentials are measurable. Local physical arguments should not be applied to potential equations until they are suitably differentiated, as they do not represent locally measurable relationships.

In being representabla as a 4-vector, the 4-potential transforms in the same way as the coordintes, implying that the coordinates can be viewed as being a potential representaion. The coordinates in 3+1 space have an absoluteness that leads to contradictions if there is more than one observer in the problem. One observer evaluating a system at two widely separated times counts as two observers.

XXV. THE LORENTZ TRANSFORM

The curvature term vanishes when t_f is zero. The LW retardaion equations are for the time $t_f = 0$.

In 3+1 space, we are always free to choose $t_f = 0$. Other choices would be equally valid, but choosing other values does not accomplish anything, because curvature is not representable in 3+1 space.

****

In 3+1 space, the derivatives of $\boldsymbol{R}$ would be exact in all orders, because the four coordinates are mutually orthogonal. It has been established that the four dimensional space cannot be reduced to first order so easily[8]. It is doubtful that it can even be fully reduced to first order.

A more accurate solution can be obtained by representing the trajectory by two or more straight line segments. Two segments are sufficint if the v^4 terms are not carried. Subdividing the time interval at the field point does not accomplish anything. It is necessary to take the interval at the particle as being the independent variable. It is necessary to integrate in the frame of refernce of the particle before differentaing in the frame of reference of the field point, except that a full integral is not required. When working in series form, only a few straight-line segments are needed.

An observer at the field point would not know what to do with this solution, because time does not progress at a uniform rate.

In 3+1 space, this derivative would be exact. Indeed, in 3+1 space, the derivatives in all orders would be exact, because the four coordintes are mutually orthogoanl.

If the light cone equation is solved for one specific instant, the derivatives in 3+1 space are uniquely defined in all orders, because the four coordinates are mutually orthogonal. The four dimensional space is not so easily reduced to first order[8]. The derivatives in 3+1 space are degenerate.

Being degenerate does not have the same meaning as being wrong. The Coulomb solution is degenerate, but it is not wrong.

An observer watching a moving particle does not glimpse it at one instant. It is continuously visible. It is permanently on the light cone.

The light cone equation is $(\Delta\boldsymbol{R}){\cdot}(\Delta\boldsymbol{R}) - c^2(\Delta t)^2 = 0$. The equation can be solved by selecting one of the roots of a polynomial. One of the roots is for a signal propagting from the particle to the field point. The other is for a signal propagaing from the field point to the particle. Depending on the application, both roots are meaningful. There are more than two roots if the $\boldsymbol{a}$ and $\dot{\boldsymbol{a}}$ terms are carried. The method of successive approximation is usually better when working in series form. The constant velocity solution is obtained in the SOM.

One of the terms of the velcity of the retarded intersection is

The velocty of the retarded intersection is not the same as the retarded velocity. The retarded velocity is

$$\frac{\boldsymbol{R}(dt_f) - \boldsymbol{R}(0)}{t_s(dt_f) - t_s(0)} = \frac{\partial \boldsymbol{R}}{\partial t_f}\frac{\partial t_f}{\partial t_s} = \boldsymbol{v}_0 \tag{13}$$

The retarded velocity cannot be measured without having an assistant near the source. The assistant could take various forms, but in most cases the retarded Newtonian velocity is not measurable from afar. It is usually only the percived velocity that is measurable, such as with a doppler shifted monochromatic signal. The perceived velociy is delayed by the propagation time from the source.

The retarded velocity is obtained by differentiting with respect to the retarded time at the source. The percevied velocity is obtaind by differentianting with respct to the time at the field point.

Assuming that the coordinates at the simutaneous point are already known, which is usually the case when working the retardation problem, the trajectory parametrized by R and its derivative can be expanded as the Taylor series

Since the location in this solutin is not where the particle is known to be, a correction is required to account for the propogation delay.

Both soluions are correct, depending on how the measurements are made. However, if the particle is known to be on an inertial trajectory, and independent measuremnts provided by a distant observer are not availale, a correction must be applied to the measurements of Rv and its derivatives to account for the propagaion delay. Since the trajectory is perceived as being curved, a more accurate soluton can be obtained by approximeating the trajectory with two straight line segments rather than just one. That has the effect of refining a constant of integration.

Eq – is a solution for a particle on the light cone, but it is not the solution for an inertial particle. An isolated and unaccelerated particle would be half way to the destination in half of the travel time. The particle is perceived as having an accelertion component in the same direction as the retarded velocity. The perception is real, but a relativistic correction is required if the objective is to reconstruct the trajectory of the particle.

We are not free to choose a coordinate system that follows us as we move about in space and time if there is, or could be, another observer in the problem. such solutions are coordinte-dependent. Our frame of reference is nevertheless the only one we can ever measure anything in, and it is possible to obtain equations that work in our frame of reference. However, they require transformation for any other frame of frame of refence. They have no absolute significance.

a correction to Rvdot measuremts at the midpoint of the trajectory is required. A similar correction is required when tf=0.

In increasing linearly with time, the $\dot{\boldsymbol{R}}$ term looks like an acceleration term, but it is present for unaccelerated particles. It vanishs when $\boldsymbol{v}$ and $\boldsymbol{R}$ are parallel or antiparallel. When $\boldsymbol{v}$ and $\boldsymbol{R}$ are perpendicular, it has the form of the angular velocity of the particle, multiplied by a $(v/c)^2$ relativistic correction. For unaccelerated particles, there is no curvature without the relativistic correction.

Due to the constantly changing propogation delay from a moving clock to the observer, the trajectory of an unaccelerated clock is perceivied as being curved when there is a transverse velocity component. The relativistic correction to the angular velocity represents the curvature. For unaccelerted particles, there is no curvature without the relativistic correction.

The curvature is more than just a visual effect. It will influence the interactions of the particle with other particles.

The particle velocity cannot exceed c, but angular velocity is unbounded. It becomes progressively more difficult to obtain accurate solutions as the size of the system becomes smaller.

The curvature term vanishes when t_f is zero. The LW equations are for the time $t_f = 0$. Once the term is lost, differntiating the potential solutions cannot reinstate it.

XXVI. COORDINATE DEPENDENCY

The solution is for a particle on the light cone, but it is not the solution for a particle on an inertial trajectory. An unaccelerated inertial particle should be half way to the destination in half of the travel time.

For each observer, the solution is for a particle on the light cone, but it is not the solution for a particle on an inertial trajectory. The solution is observer-dependent. At time -r0/(2 c), no two observers can agree on where the particle is.

The solution is for a particle that is permanently on the light cone, but it is not for the right particle.

The solution is coordinate-dependent. Equations of physical significance have to work in the same way at different places and at different times. If C1 is zero, then after the data measurments are processed, no two observers could agree on what the trajectory of the particle was.

XXVII. INERTIAL PARTICLES

The solution in Eq – is for a particle on the light cone, but it is not the right particle unless it has an independent means of propulsion.

In the calculation of this section, the particle is on the light cone for an instant at the retarded intersection. It emits a signal at that point that propogates at c to the field point. In the meantime, the particle proceeds on to the simultaneous point. The particle arrives at the simultaneouls point at the same time that the signal arrives at the field point.

An observer can never know what is happening at the simultaneous point while it is happening, which poses a problem in establishing a reference point. The equations are slippery. There are no absolute reference points that are accessible for us.

An isolated inertial particle should move with a constant velocity unless it has an independent means of propulsion.

For an isolated and unaccelerted inertial particle, both halves of the journey should take the same time. The trajectory seems to be curved, but that is because the solution is coordintate-dependent. Coordinate-dependent solutions are not of physical significance, for we can never know were we are in space and time.

XXVIII. THE SECOND INFINITESIMAL STEP

Finite difference equations are widely used in digital engineering applications, where they are used to approximate analog transfer functions. The second and third difFernces are closely related to the second and third derivatives.

The extrapolation formulas for the n th differnce are elementry. They are derived at .. The extrapolation formula is simply $\delta_k(t_f)n^k$. k is 2 for the second difference, 3 for the third difference, and n is the number of straight-line segments used to approximate the trajectory. This extrapolation formula cannot be used unless all of the $\delta_{k+1}(t_f)$ differences are zero. When they are zero, integration degenerates to multiplication for the sum of the $\delta_k(t_f)$ terms. In effect, it is necessary to integrate the light cone equation before differntiating it, except that a full integral is not required. Only are few straight-line segments are needed to approximate the trajectory.

Subdividing the trajectory into more segments makes no diffrence at all in the solution, showing that the solution exists in the first order of the infinitesimal. However, the third difference would be required if the vx4 terms are carried. The extapolation formula for the third differnce is $\delta_3(t_f)n^3$.

The two distances should be the same for an unaccelerated inertial particle.

For quantities that are of order v^1, such as the magnetic field and the momentum of a particle, the refined v^3 terms represent a $(v/c)^2$ relativistic correction.

The trajectory would have to be divided into three segments if the v^4 terms were carried, but it will be necessary to venture into the four dimensional space one step at a time.

Eq – is a solution for a particle on the light cone, but it is not the solution for an inertial particle. The solution depends on where the observer is. The coordinate system for one observer does not work for a different observer.

It contains a coordinate dependency. No two observers can agree on what the trajectory is if each observer is free to choose a coordinate system that is centered upon themselves.

In reconstructing a Newtonian trajectory for an isolated and unaccelerated source from light cone events, such as a doppler shifted monochromatic signal, the quantity of interest is not where the source was when it emitted the signal. With the Newton equations, it is where it will be when the signal is received at the field point that matters. The objective is to accurately reconstruct the trajectory in 3+1 space, despite the propagation delay from the source to the field point. The calculation requires a relativistic correction.

The solution in Eq – is for a partticle that is on the light cone, but it is not the right partitcle. The computed trajectory is not that of an inertial particle. The reconstructed trajectory depends on where the observer is. No two observers can agree on what the trajectory was. Solutions that follow us as we move about, such as Eq –, contain a coordinte dependency. Equations of physical significance must not depend on the choice for a coordinate system, for we can never know were we are in space and time. We are not free to choose a coordinate system that follows us as we move about if there is, or could be, another obserer in the problem.

The invariance of the speed of light is a necessary but not sufficient condition.

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The soluton is not right, as the retarded Newtonian velocity of the particle was already known. The problem is to measure the retarded velocity correctly despite the propagation delay from the particle. *If* the location of the particle is assumed to be already known when the signal reaches the field point, which is usually the case when working the retardaion problem, then its location at the retardation intersection is computalbe.

The observer at the field point can not know what is happening at the simultaneous point while it is happening, but it is necessary to make an assumption to obtain a solution. The assumption does leave a loophole.

Equations that look like they are exact are convenient, however there would be other terms in this soluton if the vx4 terms were carried from the beginning.

The vx4 terms would have to be carried from the beginning if they are needed, which would require four nearby light cone events.

The compactness of equations that look like they are exact is convenient, but this solution is an approximation. The vx4 terms would have to be carried from the beginning if they are needed, which would require four nearby light cone events.

These relationships look like vector equations, and they are, but the underlyig relationships are actually tensor equations. As shown in the SOM, the same solution is obtainable more elegantly with tensor equatios, which is not to say that the calcuations in the SOM are elegant. The essential relationship is that the gradient of a vector is a tensor of the second rank.

We still do not know where the particle was when it emitted the signals. In computing the first derivtives of the retarded potentails, which are the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields, it is not necessarily true that we have to know precisely where it was. The constraints would be stronger if the first derivatives of the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are needed.

Four light cone events would be required to compute the differential position relationships for three points. We are always one step behind, but that is all right if the vx4 terms are not carried.

This solution is only accurate to order $(v/c)^3$. Three nearby points on the trajectory would be required if the v^4 terms are to be carried.

XXIX. THE FIRST INFINITESIMAL STEP

A more complete solution could be obtained by carrying the v^4 terms from the beginning.

The light cone equation represents an undifferentiated vector function. The gradient of a vector is a tensor of the second rank. The tensor irreducibity theorm [] is in 3-space, but a space rotation in 3-space is still a space rotation in 4-space. The rotations of the Lorentz group [,] represent the angular relationships in a more general way.

The Lorentz transform takes out the velocity terms. It is the acceleration terms that represent the first derivative. We have to carry extra terms in the equations to represent the first derivative in our frame of reference.

XXX. FORM INVARIANCE

Retardaion equations have to work in the same way when applied in different places and at different times.

This solution looks like the LW solution in series form, but it is not, because the location of the particle is different at time dt.

dt has not dropped out, showing that the LW equations do not look like the same equations at time $t+t_f+dt$. The first derivative at a later time is one of the contributors to the second derivative.

Form invariance has the same meaning as the requirement that equations of physical significance should not contain a coordinate dependency. The invariance of the speed of light invariant is a necessary but not sufficient condition. For this reason, we are not free to choose a coordinate system that remains centered upon ourselves. It is necessary to consider how an observer in the other frame of reference would perceive us. We can never be sure that we are not the other observer.

Coordiante systems are not portable. They do not follow us as we move about in space and time.

XXXI. SOME PREIMINARIES

The second differences are all the same, so integration degenerates to multiplicaion.

The tx2 terms drop out if the vx3 terms are not carried, then the solution is the same as the LW solution. The vx4 terms do not drop out of the second difference, but the do drop out of the third difference. The extrapolation coefficnt for the third differnce is nx3.

The Lorentz transform takes out the velocity terms. The particle is initally at rest in the the seconed frame of reference, where the acceleration terms represent the first derivative. The tensor of the second rank represent the first derivatives[6]. We need additinal terms to represent the first derivative in our frame of reference.

A retardation solution has no special significance for other observers until it is transformed. For the same reason, a solution in our frame of reference does not apply to a sensor that is moving in our frame of reference until it is transformed. The equation for the transverse force on a charged particle, $\boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B}$, is an example of a transformed solution.

There are no velocity terms in the Maxwell equations. They apply in a frame of reference where the velocity is zero, in which case we are in the second frame of reference without knowing it. How would we know?

There would be terms quadradic in t_f if the v^4 terms were carried, so they have to be set aside for future study. The first order of the infinitesimal is free of error, but that does not mean that any equation can be reduced to first order in one step.

Comparing the value of computed contants to laboratory measurements provides a way to test our understanding.

The particle is not where it would be in 3+1 space, but it is still on the light cone. The r dot r in eq – is the same as in Eq ().

r dor r is the same as in eq –, so the particle is still on the light cone.

The $\ddot{\boldsymbol{R}}$ contribution to the location of particle could be obtained with a double integral, but the Taylor theorm provides a more convienent way of integrating all of the derivatvies in this solution. There are derivatives in all orders, depending on how many powers of velocity are carried.

of integratin can be determined with a definite integral from infinity. Most potental equations can be viewed as representing the perspective of an observer at infinity, however that is a perspective that we can never know.

The particle is at the center of a sphere of radious r_0 at the time $t_s = 0$. A signal emitted by that particle at that instant reaches all points on the surface of the sphere at the same time, even if the particle is accelerated. For $k = 1$, the time at the the field point is $r0/c$ and a light cone solution is obtained without calculation. When k is 0 the time at the field point is 0 and the paricle is at the locatin Rv. The time at the source is then -—Rv + rv—/c.

The acceleration and the velocty of the particle both contribute to the perceived curvature of the trajecory, so they are not separable in observations of short duration, but they are separable in prolonged observations.

The accelereation of a particle does not affect its location at the time dt, so it is not possilbe to tell from this solution which terms are accelertion terms. The Newtonain velocity and acceleration of the particle are nevertheless separable in prolonged observations.

The number displayed by an acceleromter attached to a particle can be read by observers in any frame of reference. In the frame of refence of an accelerated particle, the partcile acquires the velociy $\boldsymbol{a}\ dt$ at time dt. There are no relativistic corrections of order v^1, so the number displayed by the accelerometer is an invariant quantity. There are relativistic corrections at time $dt_1 + dt_2$, so the Newton equations no longer apply, even in the frame of reference of the particle.

It would of course be possible to represnt the tranectory with more than two stright-line segments, in which case the avdot terms would have to be carried. In light cone probglems in the first frame of reference, and always in the second frame of reference, the ax2 terms are in the same order as the avdot terms. The avdot terms integrate to acceleration terms in the time dt, so not much should be expected of solutoins that do not include the avedot terms.

Even though the Newtonian accelertaion and velocity terms are both curvature terms, they can be separated in prolonged observations, and there is sometimes a need to separate them.

It would of course be possible to divide the path into three or more segments, and in more general solutios the acceleration terms should be carried even with only two segments.

No matter how small the interval is, the midpoint of the line element is not at the middle, and the error is of the first order. The behavior is unphysical. If a long thin wire with slowly varying properties is stretched, the elongation will not be uniformly distributed along the wire, but it will be uniform in any aritrarilly short section.

The behavior is unacceptable for physical rather than mathematical reasons. The clock in the frame of reference of a distant and detached observer runs at a constant rate.

There are two ways of viewing this problem. In our frame of reference, there is only one way of viewing it, because we have to use a clock that is at rest in our frame of reference as a time standard. We cannot use a clock in a differnt frame of reference as a standard. The distinction between the two perspectives is not important for the first infitesimal step, but it does become imporetant is subseqent steps. More generally, only first order equations can be reduced to first order in one step.

If the signal arives at the time dtf then it must have been transmitted approximatly at the time . It become necessary to start over with the better estimate.

This calculaion illustrates that only first ordee equations can be reduced to first order in one step, even though the first order of the infitesimal is free of error. This principle is not unique to the four dimensional space [].

The equaion is similar to the equation for the Thomas precession [], which occurs in the other frame of refernce. In this case the othere frame of refeence is our frame of referene. We can never know which frame of reference we are in. A distant and detached obserer would not perceive any accelertion, so a second transform is required to take it out. A third transform would be required to take out the avdot terms if the vx4 terms were carried.

.. The vector has the same magnitude as it would have in 3+1 space, but it is rotated. The rotations of the Lorentz group[,] represent the 4-space angular relationships in a more general manner.

The calculations have not used the chain rule for differentiation, but is should be possilbe to obtain the solution in a more concise form with it.

The time inteval indicated by dt is correct, but the elapsed time shown by the other terms depends on when the observation is performed. The behavior is unacceptble in the first frame of refernce, and it is difficult to obtain a solution that an observer in the second frame of reference would not object to. tf0 is a free parameter. A common meeting ground is obtinable by choosing it so that the time in the first frame of refence is 0

These relationships could be expressed more elegantly with the chain rule for differentiaron, but other methods are sometimes easier to visualize. The chain rule is required whenever the variables are not independent. The time and space coordintes are not independent variables in light cone solutions, or in more general solutions in the second frame of reference. Light cone solutions are special cases of more general solutions.

The chain rule for differentiation is required when the variables are not independent, and the retarded potentials exist only for the purpose of being differentiated. The terms of a Taylor expansion are mutually orthogonal, but the terms of the Newton series cease to be orgthogal in light cone solutions, or in more general solutions in the second frame of reference. Light cone solutions are special cases of more general solutions.

For light cone solutins only, the retarded postion vector can be expanded in a Taylor series.

This series expansion does not represent the trajectory of an inertial particle, nor does the Newton series. The terms of a Taylor series are mutually orthogoanal. The chain rule for differentiaon is not needed when the terms are orthogoanal, but it is necessary to insure that the terms in the equations actually are orthogonal.

The Newtonian acceleration and velocity terms are not independent variables in light cone solutions, making it difficult to separate them, as both terms cause the perceived trajectory of a particle to be curved. The terms are nevertheless separable.

In being quadradic in time, the curvature is in the same order as acceleration, so the solution for the perceived curvature would not be complete without the acceleration terms. But acceleration is obtained by differentiating velocity. The acceleraton soluton is not obtainable without knowing the velocity solution first. (It would be possible to integrate acceleration to obtain the velocity, but those equations would be of a different form.)

The particle is still on the light cone and the potentials propogate at c, but they were not emitted by the particle when the LW equations assume that they were.

.. is where the observer thinks the particle is. .. is where it acutally is. A field of observers would know where the particle actually is. How could one observer know? Actually, there is a way for one obsever to know. That is to observer the partcile for a long time and then integrate the trajectory. The trajectory cannot be integrated unless the derivatives of the function are computabhle from it.

?? The light cone equation is traditionally solved in our frame of reference, but it can also be solved in the frame of reference of the particle. It is possible that the way the other observer perceives us is not the same as the way we perceive ourselves.

****

The solution will be shown in a more compact form in a later version of this paper.

The constant of integrataion was not determined in this calculaion, so the solution is not necessarily unique.

The thing that matters is not the constant of integration itself. The thing that matters is that the function and its derivatves are not consistent. Discovering a function that diffrentates correctly is an indirect way of refining the constant of integation.

Fortuously, dt has dropped out of the solution. Since the solution does not depend on the value of dt, it remains valid when dt is zero.

The solution will be shown in a more compact form in the next version of this paper.

In representing the integral of the fields, potential solutions are arbitrary to within a constant of integration. Retardaion equations express a realatinship between one observer and one particle. The constant of integration has no special significance for other particles or other observers. The thing that matters is that the function and its derivatives should be self-consistent. Discovering a function that differentates correctly is one way of refining the constant of integration. There is probably more than one method of discovery.

With continued use and familiarity, the potentials can seem to become more real than the fields, which is fine if it assists the comprehension of the equations, but the model can develop fractures if carried too far. Worse than that, it can cause an explorer to become lost in space and time, seeking a solution that does not exist.

It is sometimes necessary to know if an accelerometer attached to the particle would register an acceleration, and the Newonian parametrization is needed for that purpose. The accelerometr dial can be read from afar, so it is the same for observers in all frames of refrence.

The acceleration terms also vary as t^2, so they are in the same order as the angular velocity. In periodic solutions, it would not be consistent to obtain the soluton for the angular velocity without including the acceleation terms. Non-periodic solutions are included in the more general solution as special cases.

The retarded accelertion may or may not be zero in solutions containnin d2Adt2 terms. Angular velocity and acceleration are difficult to separate. The need to separate them is sometimes artificial, but it can be done.

Two consecutive infinitesimal reparamterizations would be required in order to carry the $\dot{\boldsymbol{a}}$ terms. In the second frame of refernce, the a^2 terms are in the same order as the $\dot{\boldsymbol{a}}$ terms. The a^2 terms will become important in intense field solutions.

****

The Lorentz transform is for one point. It is not known to represent a smooth and continuous differentiable function without additonal processing.

When the interval is small, the first difference behaves in the same way as the first derivative. The second and third differences are closely related to the second and third derivatives, but they are different.

Discrete difference equations are widely used in digital engineering applications, but they are not covered in calculus textbooks. Some basic characteriscs are shown at ...

In being a velocity transform, the second difference is appropriate for computing acceleration with the Lorentz transform. The first derivative of velocity would be satisfactory if the Lorentz transform was an accelertion transform, but it is not. The solution can no doubt be obtained with the second derivative, and it should be done, but the second difference will be used here.

****

The curvature in the infinitesimal is more than just a visual effect. It will influence the interactions of the particle with other particles.

That does not imply that curvature can be reduced to first order in two steps. A parabola can be in closer contact to a circle than a straight line tangent to it, but a cubic equation is even better.

The angular velociy of a particle depends on where the observer is. Its value does not have a meaning for other observers unless it is transformed. The particle velocity cannot exceed c, but its angular velocity is unbounded. Obtaining accurate soltions becomes progressively more difficult as the size of the system becomes smaller.

When drv is arbitrarily small, the acceleration of each particle passing through the line element vainshes in the infitesimal. The velocity of all the particles in the infitesimal line element can be assumed to be the same, so the solution of the n particles within the line eleamnt at any given instant is simply n times the solution for one particle.

However, the length of the line element, and therefore the amount of charge within it, depends on its angular velccity and the acceleration of the particles. The time dt at the field point maps into a Δts interval in the current loop, and there are $(\delta t_s)^2$ terms in the equations, even when the particle is not accelerated. An accelerated particle acquies the velocity $\boldsymbol{a}\delta ts/2$ at the midpoint of the line element, then the velociy change integrates to a position change during the second half of the traverse.

As shown in the SOM, the location of the particle at time dtf1+dtf2 also depends on the avdot and avddot terms.

This solution is only valid when the line element and the observer are in the same frame of reference. The potentials have to be transformed in other cases. The 4-potential transforms in the same way as the coordinates.

This model exists in 3+1 space, and it contains a loophole. A moving clock does not run at the same rate as an at-rest clock. The time shown by a clock, as read from afar, depends on its history.

These relationships exist in the first frame of reference. It should not matter what frame of reference the problem is worked in, but transforming incomplete solutions to the second frame of reference would not be helpful.

The calculaions of the Thomas precession [] assume that the trajectory is Newtonian. The tranformed Newton equations are the Newton equations in disguise.

The particle is not where it was thought to be. But where is it actually? The angular velocity of a particle depends on where the observer is, so there is no general answer. It is the transform that matters.

The tensor of the zeroth rank is a scalar The zeroth order retardation equation for charge is the Coulomb solution. It is often possible to obtain useful solutions with the lowest order potential equations, but they contain an error of the first order. That is because the magnetic field is of order v^1. The error resulting from the neglect of the magnetic field becomes arbitrarily small as the transverse velocity becomes arbitrarilly small, but it does not vanish quadradically, meaning that it does not vanish at all. The theorems of Eucludian calculus apply, but the chain rule for differentiation is required when the variables are not independent. No matter how low the velocity of the particle is, the zeroth order potential equations contain an error of the first order.

As shown in the SOM, the $\dot{\boldsymbol{a}}$ terms affect the location of the particle at time dt_1+dt_2. One of the velocity terms at time $dt1$ is $d\boldsymbol{r} = -3\dot{\boldsymbol{a}}r_0v_0^2/c^3\ dt_1$, but that does not imply that curvature can be reduced to first order in two steps.

appears in the solution. There are no conflicts between these solutons and the theorms of Eucludian calculus, as the chain rule for differentian is required when the variables are not independent. It it is capable of conferting first derivatiess into derivatives of any order. The terms of the Newton series not not idependet variables in light cone solutions, where the time and space coordintes are strongly coupled.

The following calculations assume that the observer and the current loop are in the same frame of reference. The calculaions would necessarily be more elaborate in other cases.

When $d\boldsymbol{r}$ is arbitrarily small the particle accelereation vanishes in the limit. The velocity of the particles in that region can be assumed to be constant throughout the interval, then the LW equaions can be applied to the amount of charge that exists in that infinitesimal line element.

The space and time coordinates are strongly coupled in light cone solutions. One of the consequences of the coupling is that acceleration and velocity not independent variables in global solutions. The chain rule for differentiaon is required when the variables are not independent, and the retarded potentials exist only for the purpose of being differentiated. The chain rule is not used in the following calculatons, but that is only because it is not needed if the terms are orthogonalized first.

In the first frame of reference, we are one step behind – sometimes two steps.

The behavior of the first derivative and the first difference are the same when the interval is small. The second difference is closely related to the second derivative, but it is not the same. In not being an accelertion transform, velocity has to be reduced to second differences when applying the Lorentz transform. The second differences sometimes work for the Newton equaions, but not always.

Eq – shows where the first observere thinks the particle is. Eq – showss wher the other observer thinks it is. So which solution is the right one? From the perspective of an observer in a third frame of reference, neither of them. It is the transform that matters.

The t in these equations is actually delta t, with the actual time being t+delta t. In the interest of breity delta t is abbrevviated to t, however the solutions are only valid for short time intevals.

Using more than two steps has no effect at all on the soltion, showing the the locatin of the particle at time tx2 can be reduced to first order in two steps. The same is true of the Newton equaions, but the equations are not the same.

The chain rule for differentiaion is capable of converting first derivaries into derivtives higer order. In that case, the theorms of Eucludian calculus do not necessaryly apply until *after* the chain rule has been used.

Translating the coordinate sysem in space by the amount rv would not affect the final solution. The coordinates could also be translated in time. We can never know where we are in space and time, so coordinate-dependent solutions are not of physical significacne. Furthermore, coordinate systems are not portable. Portable coordinate systems lead to egocentric theories.

It is nevertheless true retardaion equations are intrinsically egocentric. There is a practical need for egocentric solutions, but those solutions are special cases of more general solutions.

There would be 1/2 av tx2 terms in the solution if the tx2 terms were carried. However, the Lorentz transform is a velocity transform and the veocity at that time is 1/2 j0 tx2. The solutin would be incomplete without the j0 terms. The Newton series is always one step behind.

avdot terms apper in the soluton after the trajectory is differnteiate in the first frame of reference. ... The solution is for a jerked particle.

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Accurate solutions become progressivly more difficult as the systems become smaller. This solution is not the last term of the series.

t0 is a constant for differenting in time. The t0x2 terms could be carried, then an accelertion transform would be required for diferentiating the solution at time t0x2. The Lorentz transform is not an acceleration transform. The location of the particle in the second frame of reference depends on both its location and velocity in the first system. Newtonian concepts do not apply. More generally, the chain rule for differentain is required when the variables are not independen. Acceleration and velocity are not independent variables in 4-space.

The vector from the particle to the field point is rotated in the fourth frame of reference. The pivoit point for the rotation is the same as the origin of the coordinate system. In this soluton the origin is at the particle, so the coordinates of the tip of the vecor are different in the fourth system. The field point would be the pivot point if the orgin is selected to be there. A vector is not affected by a translation of the coordinate system.

There is, of course, no requirement that the problem be worked in the first frame of reference, although it is as good as any.

There are several similarities between this soluton and the equations of the Thomas precession [,]. That is not surprising, as our frame of reference is not special, and it is not differnt. Indeed, it should not matter which frame of reference is used for working the problem. However, the Thomas calculions represent the transformation of the Newton equations to the second and third frames of refernce. The transformed Newton equations are still the Newton equations.

That amounts to projecting the solution into 3+1 space, then proceeding as though it were a real space. The interpretation would not be acceptable for field equations, but these are potential equations. Physical arguments should not be applied to potential equations until they have been suitbly differentiated.

The solution represents a projection of the retarded potentials into a flat 3+1 space. The equations are applied in the same way the LW equations are applied. 3+1 space is not a real space, but physical arguments should not be applied to potential equaions until they are suitably differentiated, because they do not represent locally measurable relationships. Local in a cosmological context covers a lot of territory.

There are three known ways of differentiating the solutions.

The E field transforms into a magnetic field. It is not yet known what the gravitational field transforms into.

After transfoming the Newton equations to the second frame of reference, they are still the Newton equations, but from a different perspective. It does not matter which frame of reference the problem is worked in, but the equations do have to be right in some frame of reference before they are projected elsewhere.

The expansion factor of the cosmos is not zero, so it is doubtful that the velocity terms are real either. The expansion factor does not affect the measured speed of light when the test particles are stationary []. It is not yet known if it can be neglected high order local equations, but, for now, it will be neglected.

Since the time in the equation is not the time shown by an accelerated clock, the terms in the equation are not real in a physical sense. The equation is in 3+1 space. 3+1 space is an abstract mathematical space. However, it is the space where the theorms of Eucludian calculus were discovered. The chain rule for differentiation is one of those theorms. It is capable of converting first derivatves into higer order derivativs, in which case other theorms for the first derivative do not apply until aftrer the chain rule is used.

The expansion factor is not zero inside a mass shell, but it does not affect the locally measured speed of light []. The expansion factor of the cosmos is not zero, but its space and time aspects are linearly dependent in low order laboratory measurements, so it should be all right to neglect it at first. The expansion factor is assumed to be zero in the following calculations.

The Liénard–Wiechert[9] (LW) retardation equations are for the contravariant tensor of the first rank, a vector. History skipped the gravitatioal solution for the first rank tensor. The Newtonian gravitational field is the 3-space gradient of a scalar. The equations do not contain the speed of light. The LW equations do. That is not because electrical equations are essentially different. It is because history skipped a step that is of too low an order to be of much interest. The first ordere electrical solutions are more interesting because an E field transforms into a magnetic field. The first order gravitaional field transforms into itself.

The following calculations do not assume that the location of the particle at the simultaneous point is knowable. The considerations would be different for a field of observers, but that is not the way of the retarded potentials.

The a v cross terms vanish when av and vv are parallel or anti-parallel. There are various other similarities to the calculation of the Thomas precession []. However, there is no point in transforming to the second frame of reference until the coorditates are known in the first system. The transformed Newton equations are the Newton equations from a different prespective.

The basis of these relationships is that the time and space coordintes are strongly coupled in light cone solutions. The equaions do not behave in the same way as they would in an orthogonoal coordiante system. The relationships would be representable in an orthogoanl system, but the chain rule for differentiation is required when the variables are not independent.

A second observer at a greater distance receives the signal when the velocity of the decelleating particle was higher, with the result being that the potential does not decay with distance in a simultaneous system. The behavior might or might not be unphysical, depending on how the derivatives behave. The LW equations have this characteriscic. Even less intuively, the avdot terms can actually increase with distance in a simultaneous system. However, the paticle velocity would quickly exceed c with this artifical trajectory, so the consequencs of the odd behavior are bounded.

The fields computed from the retarded potentials are for sensors that are at rest in the frame of reference of an unaccelerated observer. The advanced potentials are needed when a sensor is moving relative to a stationary charge. The advanced potentials in our frame of reference are the retarded potentials for an observer in the other frame of reference. The advanced potentials do not matter for us, but they do matter for everyone else.

The tensor irreducibility theorm[6] represents the differential 3-space angular relationships. The 4-space equations are different, but a space rotation is still a space rotation – in some frame of refernce. A space rotation does not affect the invariant quantity $\boldsymbol{r}\cdot\boldsymbol{r} - c^2t^2$.

The expansion factor also does not affect the invariant quantity[8]. The expansion factor of the cosmos is not zero, but it will be neglected in the following calclations. However, it remains to be determined if the expansion factor can be neglected in laboratory measurments of sufficently high order.

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When tfx2 is arbitrarily small, it can be replaced by the sum dtf1+dtf2. Only two steps are needed to reduce the equaion to first order unless the vxn terms are carried.

This solution differs from the LW solution in the ordere vx3. With the LW equations, the vector potential is parallel to the retarded velocity vector. This solution contans an rv dot vv term, meaning the it depends on the angular velocity of the particle.

Provided that the vx4 terms are neglected, approximating the trajectory with more than two straight-line segments does not affect the solution. The calculatons for three line segments are shown in the sOM. The solution is

The differnces from the LW solution are small, so they would be difficult to detect in the laboraty in direct comparisons. Solutions with symmetries alien to the LW equations would be easier to experimentally isolate.

The Liénard–Wiechert[9] (LW) retardation equations are contained in the representations of the contravariant tensor of the first rank, a vector. In deriving the equatons, the coodinates are transformed to the velocity of the particle at the retarded intersection. In that frame of refernce the pontential solution for a charged particle moving tangentaily to the trajectory is the Coulomb solution,

$$\begin{aligned}\psi &= q/(4\pi\epsilon_0 R')\\ \boldsymbol{A} &= 0.\end{aligned}$$

The potentails are then transformed back to the first frame of reference. In being representable as a 4-vector, the potentials transform in the same way as the coordinates. The solutions are in the Lorentz gauge, satisfying the condition $\boldsymbol{\nabla}\cdot\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t = 0$ (SI units). The potentials are then differentiated to obtain the fields with the equations

$$\begin{aligned}\boldsymbol{E} &= -\partial\boldsymbol{A}/\partial t - \boldsymbol{\nabla}\psi\\ \boldsymbol{B} &= \boldsymbol{\nabla}\times\boldsymbol{A}.\end{aligned}$$

Ill behaved solutions are mathematically possible[5], but in real problems the solutions are always solutions to the Maxwell equations.

The fields computed from the retarded potentials are for sensors that are at rest in the frame of reference of an unaccelerated observer. The advanced potentials are needed when a sensor is moving relative to a stationary charge. The advanced potentials in our frame of reference are the retarded potentials for an observer in the other frame of reference. The advanced potentials do not matter for us, but they do matter for everyone else.

The invariance of the speed of light is sometimes interpreted as meaning that our frame of reference is the only one that matters. The advanced potentials do not matter for us, but they do matter for everyone else.

Invariance is a necessary but not sufficient condition. Invariant solutions are not of physical significance if they contain a coordinate dependency. Coordinate systems are not portable. The Lorentz transform is easily misinterpreted in this respect.

The meaning of the equation is best found by relacing tf with dt. Like the zeroth order potential equations, no matter how small dt is, the solution contains an error of the first order. The chain rule for differention is required when the variables are not independent. The Newtonian vv and av are not independent variables when the speed of light mattrs. There is no point in transforming to another frame of reference until we know where the particle is in ours.

In the most general solutions, the av and avdot terms are also independent variables with the Newton equations.

With further development, the right way of telling time should be testable in the laboratory.

XXXII. THE METHOD OF RETARDATION

This behavior is not contrary to the LW equaions. It is indluded in them. For a given transverse velocity, the magnetic field goes to infinity as the radius goes to zero.

For unaccel erated particles, this behavior is not in any way contrary to the solutions of the LW equations. It is rather that this is the way the LW equations work. The angular velocity terms do not appear in the solutions until the potentials are differeentiated to obtain the fields.

The following calculations do not require that the location of the particle at the simultaneous point be known before working the problem.

There are additional terms in the equations when the retarded accelertion is not zero.

From the perspective of an observer at the field point, the trajectory does seem to be curved, and it does seem to depend on the angular velocty of the particle, even though the actual trajectory is straight line unaccelerated motion.

For unaccelerated clocks, the two ways of telling time are identical.

The various observers cannot agree on where the particle is at time dt. Depending on the perspective, the solution is coordinate-dependent or observer-dependent.

These calculations are closely related to the calculations of the Thomas precession []. Those calculations are interpreted as showing that the coordinates are spinning in the frame of reference of the particle. Their actual meaning is that if we do not know where the partticle is in our frame of refrence then we do not know where the particle is in any frame of referene.

This way of telling time has a meaning in our frame of reference, however the time shown by a field of synchronzed clocks has no special significance for observers in other frames of reference.

The angular velocity terms cause the trajector of the particle to appear to be curved, even though the partile is moving in straight line unaccelerted motion.

When working the retardation problem, the particle is on the light cone for an instant at the location $\boldsymbol{R}$ and the time $t_s = -R/c$. The time at the field point is $t_f = 0$. The particle then continues it journey to the simultaineous point at the location $\boldsymbol{R}_0$. The time at the field point is then R_0/c.

An observer watching a moving particle does not glimpse it at two closely spaced instants. It is continuously visible. It is permanently on the light cone.

The coordinates of the particle are

$$\begin{aligned} \boldsymbol{r}_s &= \boldsymbol{r}_0 + \boldsymbol{v} t_s \\ t_s &= t_s. \end{aligned}$$

The coordinates of the field point are

$$\begin{aligned} \boldsymbol{r}_f &= 0 \\ t_f &= t_f. \end{aligned}$$

The trajectory appears to be curved. The curvature has the form $t_f(v/c)^2$ times the angular velocity of the particle. The angular velocity terms vanish when t_f is zero, however the retarded potentils exist only for the purpose of being differentiated.

The converse relationship exists for an observer in the frame of reference of an unaccelerated particle. Due to the continuously changing propgation delay to the field point, the trajectory of the field point is perceived as being curved. The curvature is not included in the LW equations.

The curvatue is more than just a visual effect. It will influence the interaction of a particle with other particles.

From the prespective of the observer, a trajectory that is perceived as being curved cannot be integrated in one step.

The curvature is more than just a visual effect. It will influence the interaction of the particle with other particles. Even though the trajectory is perceived as being curved, an accelerometer attached to the particle would not register an acceleration unless the retarded acceleration is not zero.

The trajectory of the particle could include acceleration terms. The retarded potentials for light cone events would not be expected to depend on the history of the particle, although the history of the particle is required for computing the location where it is on the light cone. (It can in some cases be necessary to integrate the history of the fields to obtain a potential solution, but the following calculations are not of that form.)

The equation is a polynomial in t_{s0}. It has two roots. Choosing the root for which the signal propagates from the particle to the field point,

When working in series form, the light cone equation can also be solved by the method of successive approximation.

These calculations are not in the second frame of reference, but our frame of reference is not special, and it is not different. Furthermore, it is not possible to transform to the second frame of reference until we know where the particle is in our frame of refernce. The invariance of the speed of light is a necessary but not sufficient condition. Invariant equations are not of physical significance if they contain a coordinate dependency. Coordinate systems are not portable.

The retarded velocities computed by different observers on the surface of the sphere are not for the same particle. Alternatively, in being coordinate-dependent, the soluton is not of physical significance.

That is better, but the tensor of each rank is irreducible []. We are always one step behind – sometimes two steps. The tensor of the first rank is not sufficient.

In so far as is known, the accelertion of the particle does not affect the retarded potentials at any given instant, but the accelertion is nevertheless required for computing the location of the particle at the retarded time. It is plausible that the acceleration does matter, but it is reasonable to defer the question until contradictions arise.

XXXIII. THE TWO WAYS OF READING CLOCKS

The basis of this relationship is that the location of a partile in the second frame of refernece depends on both it location and its velocity in the first frame of reference. In the first frame of reference, we start out one step behind, and can never catch up.

The tensor of each rank is irreducible []. We are always one step behind – sometimes two steps.

All of the observers on the surface of the sphere are not performing calculations for the same particle. ??Like the zeroth order potential equatios, the LW equations contain an error of the first order. It is of order vx2 instead of the vx1 of the zeroth order equatios, but it is first order in dt. The tensor of each rank is irreducible []. We are always one step behind – sometimes two steps.

There are two ways of computing the retarded velocity. In eq –, the particle moves along a marked course. The location of the particle is the nearest grid intersection and the time is the time shown by the nearest stationary clock in a field of clocks. The retarded time is the same for all stationary observers at different angular locations. Eq – implies that this method of calculation is only usable for observers at infinity.

The time at the particle can also be obtained by reading the number displayed by a moving clock. With this method, the time read is delayed by the light time from the clock to the observer. The time read is different for other stationary obsercvers with diferent angular relatinships relative to the velocity vector, but if the retarded location of the particle is known, the delayed time read by the observer can be corrected for the propogatoin delay. After applying the correction, both ways of telling time are the same in 3+1 space, as is illustrated by the following calculations. The coordinates are orthogonal in 3+1 space.

With this parameterization, the LW equations [] become

The identitis .. and .. can be applied as appropriate. These equations are in all ways equivalent to the solution with Newtonian parameterization. The labor required to obtain the solutions to practical problems is about the same either way.

These relationshis can be extended to the retarded acceleration. If that is done, rvddto is generally not zero for straigh line unaccelerated motion, so the occurence of avddot terms in a solution does not necessarly mean that the rerarded acceleration is not zero.

Similar relationships exist for the advanced intersection. The advanced potentials tend to be neglected, but neglecting them assumes that our frame of reference is the only one that matters. While it is true that our frame of referene is the only one we can ever measure anything in, we are nevertheless free to send messengers elsewhere. Egocentric theories have an irresistable appeal, but they do make simplifying assumptions.

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The constant has a meaning similar to that of pi, except that it can only exist when the particles are in motion. Comparing the computed value of constants with the values measured in the laboratory provides a means of evaluating the depth of our understanding.

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The calculaions in the SOM do not use tensor notation, but tensor relaionships are sometimes representable by carrying vectors in component form in a Cartesian coordinate system. The essential relationship is that the gradient of a vector is a tensor of the second rank.

****

If a particle is approaching an obsever and decellerating, the retarded velcity for a second observer further away from the paricle is higher. Consesuenly the potaentials do not decay with distance. The LW equations have this characteristic. Depending on the global behavour of the derivatives, it is not necessarily unphysical.

Terms that do not decay with distance, when combined with the 1/rx2 terms of Eq –, decay as 1/r. 1/r terms are radiative. However, the near field 1/rx2 terms in quasistatic solutions are experimentally more accessible.

For quantities that are of order v^1, such as the magnetic field and the momentum of a particle, v^3 terms represent a $(v/c)^2$ relativistic correction.

In the solutions for rotating electrical equipment, the quantity of interest is not the velocity of the conduction electrons, but rather the sum of their velocity and the rim velocity. The velocity of the conduction electrons is not necessarily in the same direction as the rim velocity. The protons must of course be included in the calculation.

We canot tell which partile is which. The Lorentz transform is not helpful in obtaining the soluton for an inertial particle.

The basis of these difficulties is that coordinate systems are not portable. Choosing one that follows as we move about in space and time has a certain appeal, but it leads to contradictions if there is, or could be, another observer in the problem.

The solution depends on where the origin of the coordianate system is. It is coordinate-dependent. coodinate-dependent solutions are not usable, for we can never know where we are in space and time.

Similarly, if we choose a coordinate system that follows us as we move about in space and time then the soluton is coordinate-dependent if there is, or could be, another observer in the problem. All things measurable are relative, but it is not us that they are relative to.

Potential equations reperesent the view from infinity. That is a view we can never know, so it is not our perspective.

For the purpose of reconstructing the trajectory of the particle with subsequent processing of the delayed measurements, both observers have to be able to agree on where the particle was.

A correction must be applied to account for the propogation delay from the souce to the field point when using the measurements of Rv and its derivatives to reconstruct a Newtonian trajectory.

The basis of this discrepancy is that the perceived values of Rv and its derivatives are delayed by the light time from the source to the field point. If the objective is to reconstruct a Newtonian trajectory from the perceived values then a correction is required to account for the delay.

... is zero, showing that the particle is on the light cone, but it is not the same particle as in Eqs ().

Three steps would be required if the v^4 terms are carried or if the solution needs to be differentiated three times.

The inconsistency vanishes when vv and rv are parallel or anti-parallel at the simulteneous point. It is only when there is a transverse velocity component that the problem occurs.

In 3+1 space, we are free to choose a coordinate system that follows us as we move about in space and time. We still are with this solution, but only by a tiny amount. This solution would not work if there was a third navigator at the location ... .

$\dot{\boldsymbol{R}}$ requires a relativistic correction for inertial particles.

While it is possible to determine where the particle was, distant observers cannot know where it is.

Actually, we do not know where the particle was, but in working the retardation problem it might be necessary to assume that the location is already known. The assumption will require further evaluation. If the assumption is valid, then the $\dot{\boldsymbol{R}}$ in Eq () should be replaced by $\dot{\boldsymbol{R}}'$.

Another way of working the problem would be to obtain the location by integrating $\dot{\boldsymbol{R}}'$. In 3+1 space, the solution could be obtained by integrating $\dot{\boldsymbol{R}}$.

The time ticks along the trajectory of the particle will be evenly spaced if t_s is taken to be the independent variable in the light cone equation. But then the time ticks are not uniformly distributed for an observer at the field point. There is a conflict.

The invariance of the speed of light is a necessary but not sufficent condition. Invariant solutions are not necessarily of physical significance. Laboratory evaluation is required to be sure of which invariant solutions are the right ones. However, when there is a conflict between a function an its derivatives, it means that the solution should be obtained by integration.

A distant assistant reporting the times that the moving clock is nearly coincident with a field of at-rest clocks would not agree with this equation. The assistant is usually not available, although it could be done. In any case, it is the transform that matters.

The virtual acceleration occurs because the solution is coordinte-dependent. Coordintate dependent solutions are not of physical significance, because we have no way of knowing which coordinte system we should use.

drv has dropped out of the solution, showing that it does not matter where the navigator is. However, this solution would not work if the navigator was at the location ... .

In 3+1 space, we are free to choose a coordinate system that follows us as we move about in space and time. In 4-space, such solutions are coordinte-dependent if there is, or could be, another observer in the problem.

Suppose that the perceived velocity of a particle, as evaluated by a distant observer, is $\gamma\boldsymbol{v}$, with ... . With this corretion, the radius vector from the observer to the particle appears to be spinning, relative to where it would be in 3+1 space. Similarly, the radius vector appears to be spinning in the calculaitons of the Thomas precession[7]. The angular relationships of the four dimensional space are represented more generally by the rotations of the Lorentz group []. The 3-space angular relationships are represented by the tensor irreducibility theorm []. There are additonal terms in 4-space, but a space rotation is still space rotation.

Equations of physical significance also have to work in the same way when applied in different places.

The Newton equations and the LW equaions are for one navigator. Both equaions have a respectable level of accuracy and they are not wrong, but they are incomplete.

XXXIV. THE THIRD TERM OF THE RETARDATION SERIES

The first term of the retardaion series for charged particles is the Coulomb solution. It is the basis of all the other terms, except that it requires adjustments in a cosmological context. The second term of the series is the LW solution. Each term of the series stands alone.

When vv and rv are perpendicular there is no first order constraint on the particle location. The light cone equation is slippery.

The second observer requires this choice, and the first observer did not know where the particle was in the fisrst place.

To first order, the transverse location of the particle is unconstrained. Equations of physical significance have to work in the same way when applied in different places. A second nearby observer is helpful in obtaining a better estimate of where the particle was. Distant observers cannot know where it is, but it is possible to obtain useful estimates of where it was.

It may be possible to obtain a more complete solution by considering an observer at ..., or in other ways. There is always more than one way of working a problem. The equations in the rotations of the Lorentz group [] are probably relevant. The calculaions of the Thomas precession [] are closely related to those equations. However, the calculaions of the Thomas precession assume that the location of the particle is computable with the Newton equations, which leads to contradictions.

drv has to drop out if the equation is to be usable without knowing where the origin of the coordinate system should be.

Two observers will not be able to agree on where the particle was unless drv drops out. The solution cannot be differentiated unless two nearby observers can agreee on where the particle was. Actually, the equation could be differentiated, but the solution would not mean anything unless both points are for the same particle. The first derivative is not sufficient unless the equation is of first order, but it is a necessary starting point.

The two observers cannot agreee on where the particle was unless drv drops out. The first derivative does not mean anything unless both points are for the same particle.

drv has to drop out. If it does not drop out, the two points needed for computing the first derivative are not for the same particle.

... The particle is on the light cone for the first observer, but its location depends on where the second observer is. Neither observer knows where the particle was.

Both observers now agree on where the particle was. The two points required for computing the first derivatvie are for the same particle.

The curvature term vanishes when tf=0. The function is not consistent with its derivatives. When there is such a conflict, it means that the solution should be obtained by integration.

XXXV. INERTIAL TRAJECTORIES

Distant observers cannot know what is happening at the simulteneous point while it is happening. It is necesary to either extrapolate to that point with light cone events or wait a while to find out what happened there.

This inconsistency does not occur unless there is a velocity component in the transverse direction, which requires at least two directions in space. More generally, the gradient of a vector is a tensor of the second rank. However, $(d\boldsymbol{r}\cdot\boldsymbol{\nabla})\boldsymbol{R}$ is a vector.

The angular velocty term in Eq – can be retained by integrating in time by the amount r0/c, which cancels the 1/r0 term. The integral does not look like an angular velocty term, and it is not, however the retarded potentials exist only for the purpose of being differentiated.

Except in special circumstances, the gradient of a vector is a tensor of the second rank, which is probably the appropriate way of perfoming the integration. There appears to be a shortcut way of obtaining the solution.

... The light cone constraint is slippery in the infinitesimal.

In the first frame of reference, there are powers of velocity in all orders at time dtf, but they are a consequence of representing 1/(1+v/c) in series form. The implied accuacy of the series expansion, expecially in the transverse direction, is illusionary. It is not possible to quickly determine where the particle was with light cone equations.

$\Delta_{01} = \Delta_{12} = \Delta^2\boldsymbol{R} =$

The magnitude of $\boldsymbol{\Delta R}$ needs to be considered in relation to the magnitude of R.

$$\frac{\Delta(\boldsymbol{\Delta R})}{R^2} = \tag{14}$$

When $\boldsymbol{R}$ and $\boldsymbol{v}$ are perpendicular, the equation has the form of the angular velocity of the particle, multiplied by a $(v/c)^2$ relativistic correction.

From the perspective of the second observer, the particle is not where the first observer thought it was. The second observer is probably wrong too.

The equations would also be interpretable as meaning that the first observer was right and the second observer is wrong. The retarded potentials exist only for the purpose of being differentiated. The differnce between the solutions is more important than the value of either.

The two observers cannot agree on where the particle was. The retarded potentials exist only for the purpose of being differntiated. The difference between the two perspectives is more important than the correctness of either.

The equations could be developed further by taking Eq () as the assumed starting point instead of Eq ().

The particle is on the light cone, but it is not the same particle.

Something is amiss.

The full transform in Eq () will also halt the particle in a single step. Are we sure the solution is for the right particle?

The propogation delay from the particle to the field point, when combined with the constantly changing angular relationships, results in a virtual accelertion term that is parallel to the retarded velocity. The virtual acceleration is perceived, but it is not real. A relativistic correction for the angular velocity of the particle is needed.

XXXVI. ONE PARTICLE, MANY OBSERVERS

The LW equations can be derived from the Lorentz transform, suggesting that the Lorentz transform is not for an inertial particle.

When computing the retarded potentials for two partilces, the retarded velocity of each particle must be computed, but that is only the first part of the problem. The contributtion from each particle must be delayed by the light time to the field point. The two signals cannot be added simultaneously. When the two partices are closely spaced, and there are only two particles in the problem, it is only the differential delay that needs to be considered when adding the two signals. There are similar consderations for one particle at two times.

The impied curvature of the trajectory is not real. It is an observational artifact. A relativistic correction for the angular velocity of the particle is needed.

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The derivatives of the function are not consistent with the function. That normally means that the function must be obtained by integration. But since the retarded potentials exist only for the purpose of being differented, it does not appear necessary to actually perform the integration. However, because the integral of the derivative is arbitrary to within a constant of integration, the potential solution obtained with the shortcut method is not necessarily unique. It is only the derivatives of the potentials that are measurable, so a potential equation does not have to be unique. Gauge transforms exploit this characterisc of potential equations. It is necessary to assume that we know where the particle was when working the retardaion problem, but it is difficult to be sure that we do know where it was. An observer at infinity would know, but that is not our perspective.

If Rdot were computed in three orthogoal directions and interpreted as the components of a vector, the result would be the same as the LW vector equation in Eq (). That would be the right way to differentiate a vector in 3+1 space.

It has to be possible to apply retardation equations without knowing what time it is. They have to work in the same way for any value of tf0.

The only thing that this calculation shows is that the derivtives of a function are consistent with the function, and we knew that already.

This would be the solution for the retarded potentials in 3+1 space. The problem is not in 3+1 space.

This solution is for the first time derivative. The soltion for the second derivatve will be more elaborate unless the relationships are degenerate.

There are two ways of viewing this solution. Since the particle is not accelerated, it is a visual artifact and is not real. On the other hand, it is the delayed signal that a charged particle will respond to.

The expansion factor within a mass shell stretches distances and times by the same amount, so the measured speed of light is not affected[8]. It is not yet known if the cosmological expansion factor affects laboratory electrical measurements, but it should be possible to find out if it does by including it in the retardaion equations. The location of a particle at the simultaneous point contains a hidden degree of freedom in this reqpect.

Unsurprsingly, as shown in the SOM, the Lorentz transoform is obtainable by integrating the infitesimal transform. However, the integral is arbitrary to within a constant of integration.

There can be one observer and many particles, or one particle and many observers. When there is one particle and many observers, each observer is not free to choose their own coordinate system. It is not meaningful to ask which perspective is the right one. It is the transform that matters. It is more difficult to transform from one observer to another observer than to transform from the particle to one observer.

From the perspective of an observer in the frame of refence of an unaccelerated particle, a clock in our frame of reference would appear to be accelerated in the direction of the retarded velocity, so a clock in our frame of refernce would not be a good choice for a time standard.

In Eq (), it can be seen by inspection that R=r0 when tf=0. The equations assume that the solution is known before working the problem. The formulation is overconstrained, leading to an unphysical solution.

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Potential equations exist only for the purpose of being differentiated. In not representing locally measurable relationships, local physical arguments should not be applied to them.

In Eq (), the derivatives of the function are not consistent with the function. It is sufficient to obtain a function that differentiates correctly. Always in concept, and sometimes in practice, potential equations can be gauge transformed, so there is no requirement that the function be unique. There is also no requirement that it have local physical significance. Potential equations allow us to see ourselves as others see us, to know what we cannot see.

The solution is the LW solution when tf=0. However, the solution is for the wrong particle at time tf.

Since the coordintes appear to be spinning, the transform must be performed in two steps, first to take out the translaional velocity, then to take out the spin of the particle. A point charge cannot actually spin, but it does seem to be spinning.

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Other choices would be equally valid, but choosing other locations does not accomplish anything, because curvature is not representable in 3+1 space. However, 4-space can be projected into 3+1 space with potential equations.

The method of retardation is so different from the methods of field equations that it is often difficult to see why there should even be a connection, but there is. Neither field equations nor retardation equations form a complete representation. Field equations constrain the fields without specifying the physical processes responsible for them. Retardation equations do specify the physical processes, but they are inept in other ways.

It remains to be determined if there are electrical terms in the solution for a hammered mass, and conversely.

Refer to Ref. 11 and `www.s-4.com/som2/som.htm` for some useful background material and equations.

It is unlikely that the second derivtive can be obtained by differentiating this solution in 3+1 space, but obtaining the second derivtive that way should be a good approximation when the velocity is low. Conversely, it would be easy to carry more powers of velocity in the solution for the first derivative, but their validity will need further investigation.

drv and dt would be orthogonal if both were defined at the field point, in which case the drv*dt cross term would vanish. In this soluion dt is at the field poilnt and drs is at the particle. The two terms are not orthogonal.

The spin of the particle must now be taken out by transforming from the tip of the vector to the tail of the vector.

The rotation causes the velocity vector in the second frame of reference to not be parallel to the vector in the first frame of reference. The rotation is not inclued in the derivation of the LW [] equations. It is also not included in the calculatinns of the Thomas precession []. It is not included in any of the calculations of the special theory of relativity [].

It is possible to interpret this solution in a different way – that simultaineouls events in the first frame of refernce are not simultaneous in the second system.

The solution can be interpreted in two ways. With one interpretaion, events that are simultaneous in the first frame of reference are not simultaneous in the second system. With the other interpretaion, the velocity vector in the second system is not parallel to the vector in the first system. Light cone solutions can be helpful in resolving the ambiguity.

.. A second transform is required to take out the virtual spin of the particle. The equations would be muddled if the angular and translational velocity terms were not separated.

The angular velocity of the particle is v=w r0. When vv and rv are perpendicular, the equation simplifies to delta $r = w(v/c)^2$ [?at time t=0].

The traditional interpretaion of this and similar relationships is that simultaneous events in the first frame of reference are not simultaneous in the second system [].

It follows from the calculations of the Thomas precession [] that two consectutive Lorentz transforms are not representable with a single Lorentz transform. Two transforms are equivaltt to a singl transform followed by a space rotation. It is the velocity of the tip of the radius vector that is relevant in the second frame of refence. Since is is not the same as the velocity of the tail of the vector, two consecutive transforms are required for transforming the potentials back to the field point.

Potential equations can usually be viewed as reprenting the perspective of an observer at infinity. The perspective can be mathematiclly convenient, but it is one that we cannot know.

This solution can be interpreted in two ways. In one way, events that are simulteneous in the first frame of reference are not simulteneouls in the second sysem. With the other interpretaion, the radius vector is spinning in the second system. The second interpretaon is used in the following calculaions. However, there is not necessarily anything wrong with the first perspective, provided that is consequences are developed. It is the transform that matters, not which perspective is the right one.

As shown in the SOM, this calculaion can be obtained by solving the light cone equation twice, first for time tf0 then for time tf0+tf1, and the solution is the same as this one.

This equation applies equally well to doppler shifted acoustic signals. although they do not transform in the same way as electromagnetic signals.

The constant of integration would evidently depend on where the observer is, relative to the particle, so it may be difficult to determine it in a general way.

In 3+1 space, this would be the retarded velocity of the particle at time tf, and the retarded acceleration would be zero. For light cone solutions, and more generally in the second frame of refernce, the time of an event depends on both the location and the velocity of the particle, so the acceleration cannot be computed until the velocity becomes known. That requires one more step. Similarly, an extra step would be needed for the $\dot{\boldsymbol{a}}$ terms. In 3+1 space, we always one step behind. There is nothing wrong with being one step behind, provided that we do not try to lead.

The solution indicates that a relativistic correction for the anglar velocity of the particle is needed. Eucludian calculus does not neglect this form of curvature, because the chain rule for differentiation is required whenever the variables are not independent. The investigation will be continued.

The trajectory is perceived as being curved. The trajectory is a straight line, but the time intevals along the line are not properly spaced.

Since the trajectory is perceived as being curved, a more accuate soltion can be obtained by representing the trajectory with two segments. The two segments are parallel, but their lengths have to be computed.

The LW equations are not form invariant for translations in time. This relationship causes the first time derivative of the LW equations to be wrong, even though the equations are correct at time tf=0. The derivatives of the function are not consistent with the function. If the time at the field point were the only availble time that would mean that the function would have to be obtained by integration. For the purpose of deriving the retardaion equations, the time at the source is also available, although it is not availbe for field equations.

The mapping of the time at the source to the time at the field point is not a linear function of the time. The midpoint of a time interval at the source does not map into the the midpoint of the corresponding interval at the field point, no matter how small the interval is. It is easy to misapply the theorms of Eucludian calculus when the midpoint of an infinitesimal line element is not at the middle. However, the chain rule for differention always required when the variables are not independent. The chain rule is not needed when the time at the source is taken as the independent variable.

The LW equatios work correctly at time tf=0 and time tf=+r0/c. The eqations have a different form at other times. It is not possible to apply the equations without knowing what time it is. We cannot know what time it is.

Our frame of reference is not special for anyone except us. Coordinate systems are not portable if there is, or could be anothere observer in the problem. One observer evaluating a system at two widely spaced times counts as two observers. It is nevertheless possilbe to obtain equations where the coodinate system seems to be portable, within limits. That is equivalent to stateing that the equations can be differentiate specified number of times. The count for the LW equations is zero.

The gradient of a vector is not a vector, but (dr vdot del) Rv is, and the equation can be applied recursively. It is possible to obtain vector equaios that behave like vector equaions, within limits.

The LW equations are also form invariant for translations in time, provided that they are not differentiated.

The solution shows that the LW equations cannot be differentiated without knowing what time it is. We cannot know what time it is. Coordinate dependencies can exist in either space or time.

The propogation delay from the particle to the field point casuses the angular velocity of a particle to become coupled to the perceicved translational velocity. This relationship is included in the LW equations. The retarded velocity is still v0.

The basis of this relationship is that the midpoint of a time interval at the source does not map into the midpoint of the corresponding time interval at the field point. The durations of the two intervals are also different.

An observer in the frame of refernce of an unaccelerted clock would insist that the clock runs at a constant rate, as would an observer at the field point. Dynamically adjusting the clock rates is not an option. Distorting the mapping by adjusting the midpoint of one or the other is not possible either, because it would affect the propogation velocity.

However, the time shown by the other clock was not known in the first place, so now we have the opportunity to synchronize it, to the extent that it is possible with the retarded potentials. By displacing all of the events associtated with the other clock in time, the two midpoints can be brought into alighment.

The midpoint is now at the middle, and the false acceleration term has vanished. Or is it that a curvature term has been added? That depends on the perspective. It could be that the charged particle is at rest in our frame of reference and the other observer is moving, so both perspectives are needed.

It can seen in Eq – that the closest approach occuas at time tf1=0. Something is amiss.

The retarded potentials exist only for the purpose of being differentiated, so it is not necessarily true that we have to know where the particle actually was if the derivatives are consistent. But when applying retardation equations to specific configurations, it is necessary to assume that we know where it was. However, potential equations are subject to gauge transformations, so the assumed potentil solution is not necessarily unique. In any case, potential equations do not represent locally measurable relationships, so local physical arguments should not be applied to them.

This adjustment seems contradictory, but perhaps not. In 3+1 space, the time of an event depends only on its location. For light cone solutions, and more generally in the second frame of referece, the time of an event depends on both the location and the velocity of the particle. It is difficult to be sure that all of the events in Eq () are for the same particle.

The perceived transverse velocity is parallel to the retarded velocity, but there is a realtivitic correction for it when the particle is nearby.

Either Eq – is not for an inertial partcile, or all of the events are not for the same particle. It is difficult to be sure which particle is the right one when the angular and translational velocites are coupled.

There are no longer any angular velocity terms in the solution. The angular and translational velocites have been decoupled.

There are still angluar velocity terms in the solution, but they are no longer coupled to the translational velocity. That is how it would be if there were no propogation delay.

The constantly changing angular relatioships, when combined with the propogation delay, cause the other clock to not be perceived as running at a constant rate, even though it does. The rate for an unaccelerted clock is not subject to dynamic adjustments, but it can be that the time shown by the clock is not what it was thought to be.

The perceived transverse velocity is not the same as the retarded velocity when the particle is nearby. It is the same when the particle is at a great distance.

The nonlinear mapping, when combined with the propogation delay, causes the rate of the other clock to seem to be variable. An unaccelerated clock will run at a constant rate, so the perceived variation in the rate can be used to derive a constant offest to the times that were incorrectly assumed to be the time shown by the other clock in Eq ().

The other clock is now perceived as running at a constant rate.

However, taking the Thomas precession out of the retardaion equations puts it in to the solutions at the field point. From the perpective of an observer in the frame of refernce of the particle, the LW equations contiain terms that do not belong there. From the persective of the observer at the field point, the LW equations are missing terms.

The nonlinear transfer function, when combined with the propagation delay, causes the clock to appear to run at a non-uniform rate. An unaccelerated clock does run at a constant rate, but it is difficult to determine the time shown by the clock when there is a transverse velocity component. Since the clock is known to run at a constant rate, it is possible to synchronize it by choosing a time offset from the times assumed in Eq () that causes the rate to appear to be constant. The times assumed in writing Eqs() seem reasonable, but they lead to subtle contradictions.

From the perspective of an observer at the particle, the correction removes a visual abbertion. From the perspective of an obervrer at the field point, it adds a minor correction to the much larger curvature terms that are already present in the LW equations. The LW equations are not in the same order as the Newton equations. The static Coulomb solution is in the same order as the Newton equations.

??The problem could be worked in the first frame of reference, and the equations would be interesting, but it turns out that it is computationally easier to synchronize the other clock by solving the light cone equation in the second frame of reference.

It has to be possible to apply retardation equations without knowing what time it is, because we do not actually know the time. Coordinate dependencies can exist in either space or time.

The magnitude of $\Delta \boldsymbol{R}$ needs to be considered in relation to the magnitude of R. The second difference is

For the purpose of applying the retardation equations to specific configurations, a suitable reference point can be obtained by requireing the the time of closest approach in Eq () coincide with the time of closest approach in Eq (), even though we cannot know where the particle actually is at the simultaneous point until after waiting a while.

Our frame of reference is not special or different, so it should also be possible to work solve all of the equations in the other frame of reference.

There is a very old belief that we know the time in our frame of reference, and it is true that our frame of reference is as good as any.

If our frame of reference were the only one that matters then the effet of the correction would be to add a minor correction to the much larger curvature terms that already exist in the LW equations.

Potential equations are arbitrary to within a constant of integration. It is the nature of retardaion equations that the constant of integration depends on where the observer is, relative to the source, so it does not look like a constant. That makes the chain rule for differentianon necessary, because the variables are not independent. Potential equations that do not require the chain rule are more convenient, but they do not repersent locally measurable relationsips, so local physical arguments should not be applied to them.

It should be possible to work the problem in either frame of reference, but the calculations are simpler if the light cone equation is solved in the second frame of reference.

Retardation equations have to work in the same way when applied at different times, because we have no way of knowing when the time tf=0 should be. The time tf=0 can be chosen arbitrarily, but once selected, it cannot be readjusted.

Eq () assumes that we can know what is happening at the simultaneous point while it is happening. That is not possible. There is no physical basis for asssuming that the location of the particle at time $t_f = 0$ in Eq () is knowable. There is a particle at that point in space and time, but it is difficult to be sure that it is the right particle.

the the particle is at the location $\boldsymbol{r}_0$ when the time at the field point is r_0/c. There is a particle at that point in space and time, and it is on the light cone, but it is difficlut to be sure that it is the same particle as when the time is $-r_0/(2c)$. It might be possible to determine if the two particles are the same if the transfer function were linear, but it is not linear. It does become linear if the location of the partice on its trajectory is displaced in time.

The retardaion equations now look the same at three different times. The quadradic terms have been taken out. The equations will be more difficult to linearize when the velocity is high enough that the t_{so}^2 have to be carried.

The perceived velocity contains vv/R vsq terms, which represent a relatiisic correction to the angular velocity.

This is the LW solution when the coordinates are first-known at time tf=-r0/c. It has the same form as when they are first-known at time tf=0.

This is the LW solution for the time tf=-r0/(2 c). We have no way of knowing when the time tf=-r0/(2 c) is.

This relationship causes the derivatives of the function to contradict the function. That can mean either that the derivatives are being computed the wrong way or that the function is wrong. The two perspectives are not necessarily mutually exclusive, but the equations do need to be self consistent.

Form invariance has about the same meaning as the requirement that the equatons should not contain a coordinate dependency. Coordinate dependencies can exist in either space or time.

The time at the source is

Differentiating with respect to tf

The clock rate will be constant if the first derivative of ts is 0.

Orthogonalizing the angular and translationl velocites has the effect of taking out the Thomas precession. However, taking it out of the light cone equation has the effect of putting in to the solutions of the retarded potentials, where it appears in clearly recognizabe form as a rotion of the retarded vector potential. The Thomas precession is of order ax1 vx1. The vector potental is of order vx1, so the rotaion is of order ax1 vx2 when the particle is accelerated.

The potential equations do not contain acceleration terms, but they are the integral of the fields, so the acceleration terms do not appear until the solutions are differentiatd.

This transform takes out the quadradic terms, but there are other corrections needed if the vx4 terms are to be carried.

No matter how short the time intervals are, the midpoint does not map into the middle. That would complicate the theorms of Eucludian calculus if the chain rule for differentation is not applied first. The chain rule is not needed if the equations are orthogonalized. It is doubtful that complete orthogonalizaion is possilbe, but the problem can be developed as a series expansion.

It is for this reason that the LW equations do not work the same way at the midpoint of the interal as they do at either end. The midpoint of an interval at the particle does not map into the midpoint of the corresponding interval at the field point. The transfer function is not linear in time. The location of the particle is linear in the time at the source, so the time at the source should be taken as the independent variable for inertial particles.

The time at the field point then does not progress at a uniform rate. There are circumstances when we should care about what time it is at the field point, but approximate solutions for the retarded potentials can be obtained without knowing what time it is there. That is because the retarded potentials exist only for the purpose of being differentiated. The only requirment is that the eqations be linear for a short time at the field point.

There is no upper limit for the anglular velocity of the partile, so this approxiation will become unusable if the particle is too close. It is possible for the interval at the particle to be far longer than the interval at the field point, in which case the model is not accurate.

The observer at the field point cannot know where the midpoint of a line is, causing the equations to be nonlinear in a way that Eucluid neglected. In 3-space, straight lines do not have tic marks at regularly spaced intervals along their length.

The retarded potentials exist only for the purpose of being differentiated, so we do not need to know where the midpoint of a long line is. There are circumstances when we should care about what time it is in our frame of reference, but approximate solutions for the retarded potentials can be obtained without knowing the time.

The tic marks are almost equally spaced along short lines, but the observer at the field point probably does not understand what short lines are. In Min– space [], the distance between light cone events is zero.

The other observer already knows the time, so it is easier to work the problem from the other perspective. The retarded potentials exist only for the purpose of being differentiated, so the actual time in our frame of refernce does not matter. It is only the derivtives that matter.

From the perspective of an observer at the particle, this soluton removes a false visual abberation. From the perspecive of an observer at the field point, it adds curvature term. It is the transform that matters.

The dt interval would have to be subdivided into three intervals when the vx4 terms are needed.

The other observer already knows the time, so it is easier to work the problem in the second frame of reference. The retarded potentials exist only for the purpose of being differentiated, so the time in our frame of refernce is a secondary consideration if the derivatives are correct. In being the integral of the fields, potential solutions are arbitrary to within a constant of integration. The potential solution obtained this way is therefore not necessarly unique, and there is no requirement that it be unique.

As was first realized by Einstein [], and developed from a different perspective with the equations of the Lorentz group [], the Lorentz transform contains hidden degrees of freedom. We cannot tell the difference between angular and translational velocity in observations of short duration, but we need to know the difference in global solutions, as we could not otherwise tell if we are orbiting a particle, or if the particle is orbiting us. It could be either way.

This equation is the appropriate equation when we do not care what time it is, although it there are circumstances when we should care. But in obtaining retardation solutions for one observer and many particles, it is only necessary to know the solution for a short time. However, the solution will not be valid for a long time unless more terms are carried in the equations. The solution has to be accurate for longer times if it is to be differentiated more times. The LW solutin is accurate enough to compute the first derivatves. The Maxwell equations are in terms of the second derivatives.

This solution satisfies the equation $\boldsymbol{r}' \cdot \boldsymbol{r}' = \boldsymbol{r} \cdot \boldsymbol{r}$. That is possible, because a space rotation does not affect the magnitude of a vector. Rotations are mathematically simpler in the complex number domain, but they are harder to understand for most of us. There is an extensive and relevant literature on the subject []. No attempt has been make to find equivalent equations in the literature, but they almost certainly exist, and in a more elegant form.

In the first frame of reference, the vector from the field point to the particle rotates as the particle moves. The observer in the frame of refence of the particle perceives the vector as rotating in the other direction. For one choice of the coordinates, right handed rotation becomes left handed rotation. For unobvious reasons, the other observer does not think the particle is where we think it is. The potential solution is first-known in the frame of reference of the particle, so the other perspective is the appropriate one.

Which of us is right? Since we do not know our own velocity, an arbitrer at infinity might be able to resolve the question. Potential equations can usually be viewed as representing the perspective of an observer at infinity. The 4-potenital transforms in the same way as the coordinates, it is implied that the coordinates can be viewed as being a potential representaion.

Potential equations do not represent locally measurable relationships, so local physical arguments should not be applied to them.

If the coordinates are not viewed as being a potential representaion, then the chain rule for differention is always required when the variables are not independent. The time and space coordinates are not independent variables in light cone solutions or in the second frame of reference. It is not sufficent that light cone solutions specify the function. They must also specify the derivatives of the function, which can require integrating the derivatives in order to obtain mathematical self consistency.

There is an extensive literature on invariant equations []. There is a good possibility that a solution equivalent to this one exists somewhere in it, perhaps in an abstract form that is difficult to recognize.

The chain rule for differentiaon is required when the variables are not independent, so there should be a way of working this problem with the chain rule. The chain rule is not needed if the relationships are orthogonalized. In this case, orthogonalization appears to have the same meaning as linerizing the equations by using the time at the source as the independent variable in the light cone equation.

It should also be possible to linarize the tajecory of the particle with the chain rule for differention . If the chain rule is used, the valuc of a vaector at a nearby point is Rv + (dRv vdot del) Rv. The thing that matters is that the time shown by a clock in our frame of reference is not the right time base for the kinematic relatinships of the particle. In containang velocity, eq – makes it look like there is a connection, but the equation is misleading. The equation represents a sequence of static light cone events having no connection to the trajectory of an inertial particle. The eqation looks like it is in 4-space, but it is in 3-space. The equally spaced tick marks along straight lines are not visible in 3-space, but they are there.

The mapping is linear when the motion is radial, and a simple correction can be applied. The mapping is not linear when there is a transverse velocity component and the partilce is nearby. The time shown by a clock in our frame of reference is not a satisfactory time standard for computeint the trajectory of an inertial partcile unless the angular velocity is zero. Corrections could be applied for the nonlinearity, but they are not simple, and they become more elaborate when the particle is closer.

It is for this reason that the solution to the LW equaions in Eq - looks different. The midpoint of the three soloutions is not at the middle in the sense that it canot be obtained by linarly interpolating the other two solutions. The midpoint would be much closer to the middle if a short time span for the three solutions were used, but the solution would have to be integrated, and the large discrepancy would re-appear after integration. There is no known way of reducing the four dimensional space to first order, although it can be linearized in various stages.

The propogation delay, when combined with a transverse velocity component, causes the transfer function to be nonlinear in time, making it difficult to use the clock at the field point as the time standard, although it could be done.

Each of the events in Eqs () is for a particle on the light cone, but there is no physical basis for assuming that all of the events are for the same particle. The function is technically correct, but the derivativs of the function contradict it. That means that either the function has to be obtained by integration or a differet function computed. The time and space coordintaes are not independent variables in light cone solutions, so the chain rule for differention would be required if the function is to be obtained by integrating the derivatives.

It would be possible to use the clock in our frame of refernce by apply a corrction for the nonliear transfer function. The location of the partcile is already a linear function of the time at the source, so it is easier to use the time at the source as the time standard.

It is for this reason that the LW solution in Eq – at the midpoint of two times cannot be linerly interpolated from the other two solutons. The transfer function is not linear in time when there is a transverse velocity component. Since the particle veocity cannot exceed c, the nonlinaritey is only important for observations of short duration when the particle is nearby. But even when it is not nearby, it is not where it was though to be.

The retarded potentials exist only for the purpose of being differentiated, so the time in our frame of reference is not very important. We do not even need to know the time in our frame of reference to compute the derivatives of the retarded potenials. The location of the particle is a linear function of the time at the source, so it is easier to use the other clock as the time standard than it would be to apply a nonlinear correction for the time in our frame of reference.

A short time interval in our frame of reference maps linearly into a short interval in the frame of refernce of the particle. The retarded potentials exist only for the purpose of being differentiated, so it is not necessary to know what time it is in our frame of reference. There is therefore no need for nonlinear corrections to the time shown by our clock.

The location of the particle is already a linear function of the time at the source, so it is easier to use the other clock as the time standard. We do not need to know the time in our frame of refernce to compute the retarded potentials, so it is then no longer necessary to apply a nonlinear correction for the time shown by our clock.

There is no upper limit to the angular velocity of a paricle when it is nearby, so there are probably limitations to this model, with the neglect of the acceleration terms being one of them. It is doubtful that large powers of velocity can be carried in these calculations without additional considerations. There may be additional considerations for the v^3 terms.

The following calculations neglect the acceleration terms. They will be considered in a later version of this paper, but velocity terms are of lower order.

The value of R0 in our frame of reference has no meaning for the other observer.

Taking out the false acceleration term has the effect of applying a relativistic correction to the curvature terms that already exist in the LW solutions.

The propogation delay causes a false acceleratin term to appear in the light cone solution when there is a transverse velocity component. This difficulty does not occur when the motion is radial.

The Newtonian parameters are obtained by differentiatig with respect to the time at the tip of the radius vector. The perceived quanatiaties are obtained by differentiang with respect to the time at the tail of the vector.

The particle is perceived as moving along a straight line, but it does not appear to move at a uniform rate along the line when it is nearby and there is a transverse velociy component. The trajectory seems to be curved in a way that has no meaning in 3-space. Straight lines in 3-space do not have equally spaced tick marks along their lengths. 4-space can be projected into 3-space, but the tick marks are not a part of 3-space geometry. The curvature causes the angular and translational velocites to become coupled when the particle is nearby. The two kinds of velocity are easily distinguishable in global solutions, but it takes a while to determine which is which when the observations are delayed by the light time accross the system. The perceived velocity depends on both the distance to the particle and the angle of the trajectory. The angular and translational velocites are coupled. It is not possible to quickly determine which is which unless the retarded velocity is known by independent means.

The propagaion delaty results in a virtual acceleration parallel to the retarded velocity. It is not possilbe to quickly distinguish between the virtual acceleration and real acceleration unless the retarded velocity is known by independent means. (An accelerometer in free-fall does not register an acceleration, so the meaning of real acceleration is negotiable.)

Translational and angular velocites are easily distinguishable in global solutions, and in any solution when the propagaion time accross the system is zero. When the observations are delayed, it is not possible to quickly determine which is which unless the retarded velocity is known by independent means.

The Lorentz transform has the same problem, and for the same reason. The midpoint is not at the middle. The particle appears to be accelerated in a direction parallel to transform velocity. For this solution, there is no connection between the transform velocity and the kinematic behavior of the particle. The false acceleration terms need to be taken out if the other observer is to concur with what we already know – that the particle is at rest in our frame of reference.

There is no ambiguity in the reading shown by an accelerometer dial attached to the particle. The dial can be read from any frame of reference. The other obsrver should read the same dial.

It has been established that the Lorentz transorm is not the right transform for inertial particles[8]. There is an extensive literature on invariant equations []. The equations tend to be abstract and difficult to interpret. It is possible that a satisfactoy transform has been discovered but has not been recongnized.

The calculations of the Thomas precession[] do not distinguish between the virtual acceleration terms and the acceleration measured by an accelerometer attached to the particle.

The midpoint is at the middle when the motion of the distant observer is radial, but the special case is not of much interest. The solution is otherwise for a ficticous particle. On the other hand, our frame of reference is the only one we can ever measure anything in. The best that can be done is to understand the idosyncyry of our perspective.

Eqs – represent a family of light cone events. The events are not all for the same particle. The midpoint of a time interval at the field pont does not map into the midpoint of the corresponding time interval at the particle. The nonlinearity of the transfer function is shown in Eq () and in Fig. 1.

The coupling is a consequence of the non-orthogonality of the time and space coordinates. They are not independent variables in light cone solutions, so the chain rule for differention is required. The tenosor of each rank is irreduciblre [], so it is not possible to fully orthogoanlize the coordintes with the chain rule, but it can be done in stages. The zeroth rank tensor is a scalar, and the Coulomb solution is the retardation equation in that order. The solutons can be differentited, but the derivatives are degenerate. The LW equatons are for the tensor of the first rank, a vector. The solutions can be differentiaed, but the derivatives are degenerate, becasue the gradient of a vector is a tensor of the second rank. The contravariant tensor of the second rank represeents the first derivatives. Its decomposition products are a scalar, a vector, and a quadupole []. The scalar is the Lorentz condition, ... The scalar is always zero in the solutions of the LW equations. The vector is the E field, ... According to the LW equations, the magnetic field is a transformed E field, so it does not have an independent role. It is possible that the magnetic field has a more general interpretaion.

There is no known need for the quadurpole in 4-potential equations, but it is there unless the Lorentz condition is zero. The analogs of the elongation and shear terms contribute to the quadupole. They are not mathemarically separable [], but they do seem to have distintuishable intuitive meanings, and their meanings would be more clear in the decomposition products of the tensor of the third rank, which are a scalar, three vectors, two quadupoles, and an octopole. As will be develped later, the scalar is $\boldsymbol{\nabla}\cdot\boldsymbol{E}$. The scalar is always zero in the solutions of the LW equations. There are too many zeros in those solutions.

Since the time and space coordinates are not independent variables in light cone solutions, the chain rule for differentaion is required. The location of a particle at time dt_f is

$$\boldsymbol{R} = \boldsymbol{R}_0 + \frac{\partial \boldsymbol{R}}{\partial t_f} dt_f + \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{R}_0} \cdot \frac{\partial \boldsymbol{R}_0}{\partial t_f} dt_f$$

The second term in this equation can be dropped when the velocity is low. $\partial \boldsymbol{R}/\partial \boldsymbol{R}_0$ stands for a tensor of the second rank. The equation reduces to the form used for the solutions of the LW equations when the tensor has no signifiant off-diagonal terms, which is the appropriate form at low velocites. The equations could be applied recursively if the potential solutoin needs to be diffrented twice, which is frequently the case, but it will be better to proceed one step at a time. The second derivatives of the first step can still be computed after the solution is obtained, but they will be incomplete.

It is likely that a more appealing representaion exists. Obtaining more terms in the series should be helpful in inferring its form.

3+1 space exists only in the mind, but it is a convenient space.

The Rvdot in Eq – would be the correct value in an orthogonal coordinate system, and the coordinaes are indeed approximately orthogonal at low velocites.

The angular and translational velocities have been decoupled. Whether or not they need decoupling depends on the perspective.

It is for this reason that the solution for the midpoint of the time interval in Eq – cannot be linearly interpolated from the other two solutions. The three solutions are not connected by a linear equation. It is not a linear eqation becuse the solution is not for an inertial particle. The location of an unaccelerated particle is linear in time.

The coordinates are not orthogaonal in light cone solutions, but one small step in space, along with one small step in time, is the same as one small step in space-time. The space-time cross term vanishes in the infinitesimal of the first frame of reference. It does not vanish in the second frame of reference. The second frame of refernce could be our frame of reference next time, so it also vanishes in the second system when the coordinates are first-known there.

The equations are linear for small steps in our frame of refernce. There is a space-time cross term in the second frame of reference. However, the equations are also linear for one small step in space-time in the second frame of reference if the coordinates are first-known in the second frame of reference. The retarded potentials are first-known in the other frame of refernce. It is the perspective of the other observer that matters.

The second step works in the same way as the first step, but it is not possible to take the second step in either frame of referene without the chain rule for differentiaon.

Now that the coordinates are first-known at time dt, the second infitesimal step would work the same way as the first step. The second step is not needed when the vx4 terms are small enough that they can be neglected, but the solutions will contain multipole terms that are not present in the solutions for the first step.

A clock in our frame of reference is not a satisfactory time base for separating the angular and translational velocities of light cone events. On the other hand, our clock is the only one that we have.

In our frame of reference, the radius vector to the particle rotates as the particle moves. From the perspective of the other observer, it rotates at a different rate than we think it does. There are irreconcilable differences. The retarded potentials are first-known in the other frame of refernce, so it is the perspective of the other observer that matters for them. That would not be the appropriate perspective for most calculations.

The acceleration terms can be neglected when only the first derivatives are needed. The Maxwell equations are in terms of the second derivatives, so there will be some inconsistencies.

The midpoint of a short time interval is closer to the middle when the interval is short, but it is never exactly at the middle. That can mean that the chain rule for differntaion is needed.

The meaning of these calculations is that the chain rule for differention is required when the variables are not independent. The chain rule was not used in the calculations, but there is usually more than one way of obtaining a solution.

t_s has dropped out of the solution. We do not need to know the time in either frame of rererence to compute the derivatives of the retarded potentials. They are special in this respect. However, in being the integral of the fields, they are arbitrary to within a constant of integration. This solution is therefore not necessarily unique. We do not know where the particle was yet, but when applying retardation equations to specific configurations, it is necessary to assume that we already know where it was.

Rudot dot ru is 1 when the motion is radial. Substitutng Rudot ru = 1 into eq –

The equatons are the same when the motion is radial. The diffrence between eqs – and – occurs because the gradient of a vector is a tensor of the second rank.

Equations have to free of first order error before they can be integrated, as the error will otherwise grow without bounds when the equation is integrated. Only first order equations can be reduced to first order in one step, but it a necesary step in the progression. The order of the four dimensional space is not yet known.

tf0 does not drop out of the solutin, so there is an error that exists in the first order of the infinitesial. The error will grow without bounds when the equations are integrated, but it will be small in observaions of short duration. tf0 can be set to zero in those solutins.

These relationships could be experessed more elegantly with the chain rule for differention. The chain rule is always required when the variables are not independent, although it does not necessarily need to appear explicitly in the equations. The essential consideation is that, after extrapolation to a nearby point, a vector is still a vector, but the two vectors are connected by a tensor of the second rank. Similarly, three closely spaced vectors are connected by a tensor of the third rank []. The tensor of the third rank is irreducible in the sense that the solutions will contain multipole terms that are not representable with the first derivative.

As is more clear in the form shown in Eqs (), the solution mostly represents the first derivative, even though it does not look like a first derivative. We do not yet know where the particle is, but it is necessary to assume that we already know where it is when applying retardaion equations to specific configurations.

While the transfer functionn from the time at the source to the time at the field point is not linear, any short section of it is approximately linear. Indeed, even quite a long path is linear enough for the LW equations, and there is nothing wrong with that when the solutions are accurate enough. A shorter section is not perfectly linear, and the problem becomes harder when the particle is closer. No matter how short the path is, the midpoint is not perfectly at middle. There is no known way of reducing the four dimensional space to first order, although it can be orthogonalized in stages.

While the mapping is not linear, a short inteval at the source maps into a short interval at the field point, and the mapping is approximately linear for short sections. We do not need to know the time in either frame of refernce to compute the derivatives of the retarded potentials. But because we do not yet know the time shown by either clock, the equations of the retarded potentials are inept in many ways. They are just one perspective of a much larger problem.

The light cone equation has to be solved twice so that both the function and its first derivatives will be defined.

dru dot ru0 is *plusminus*1 for radial motion. dru drops out altogether in that case.

The retarded potentals exist only for the purpose of being differentiated, so the terms multipled by tf0 are in the same order as the solution for the fields. It is assumed in the following calculations that the error introduced by setting them to zero is small. They are set to zero, but it is indicated that the solution for the second derivaties is needed.

The light cone equation must now be solved a second time so that both the function and its derivatives will be defined. The retarded potentials exists only for the purpose of being differentiated, so the derivatives are more important than the function.

After differentiaoon to obtain the fields, the tf0 terms in the solution are in the same order as the fields, so they should be carried, but they cannot be carried when applying the equaions to specific problems. It is indicated that the second derivatives are needed. Since the LW equations have the same problem, it is likely that the error from setting the tf0 terms to zero is small.

This should be a linear equation. The quadradic terms make it seem that the particle is accelerated when it is not. In some sense, the quadradic terms are real, but they do not belong in the equations for the retarded potentials.

we have never known where the particle was. We have always assumed that the particle is either approaching or receding from us. That is not true. The transverse velocity matters, expecially in tiny systems.

That is computaionally convenient, but it is also a weakness, because there are many calculations where we do need to know what time its.

The first derivative at a displaced point is normally one of the contributors to the second derivative. But in this case, since the equaions must be form invariant, it must not matter when the frirst derivative is computed, so it is only what it looks like – a first derivative.

The error at the midpoint can be neglected when the v^3 terms of the vector equation are not needed. There is nothing wrong whe the LW equations when they are applied within their range of validity.

We are always one step behind in 3+1 space. A calculaion that would normally be for the second derivtive is only for the first.

The LW equations have the same problem, so there is a precident for neglecting the residual, even though it does not vanish properly. First order terms do not vanish at all, but they can be small enough to neglect.

Unsurprisingly, the LT has the same problem. That is beause the LW equations can be deriived with the LT. The basis of the problem is that we cannot tell the differenc bewewen angular and translaions velocitits in obsertions of short duration if the signal is delayed by the light timie accorss the system. There are two velocities in a problem where there would be only one if the speed of ligth did not matter.

The basis of this relationships is that the midpoint of time interval in the second frame of reference does not map into the midpoint of the corresponding interval in the first system. That represents a form of curvature that has no meaning in 3-space. The midpoint is closer to the middle in short time intervals, but is is never perfectly at the middle. The decrepancey is small, but of first order, no matter how short the interval is. The behavior is different than in 3-space, where curvature vanishises in the limit. However, the 3-space curvature equation is singular for straight lines.

The infitesimal transform can halt a particle in a single step, no matter how high the velocity of the particle is. The equations should not work this way. The infitesimal transform is only valid at low velocites.

There is not necessarily anything wrong with being one step behind, but the consderations are not the same as for the leader. The equations have to look like second derivatives to compute the first derivative. They have to look like third derivatives to compute the second derivative.

To order vxx, this solution reduces to the Lorentz transform when the motion is radial. The midpoint is at the middle of the time interval in both frames of refence for radial motion.

The trajectory seems to be cureved in a way that has no meaning in 3-space. The trajectory of an isolated an unaccelerated particle is not actually curved. The solution is not for an inertial particle. As has long been known [], the Lorentz transform is not for inertial particles.

The basis of this abberation is that the midpoint of a time interval in the first frame of refernce does not map into the midpoint of the coorespoinding interval in the second frame of reference when there is a transverse velocity component. The abbrration is real in some sense, as an observer would not perceive a partile as moving at a uniform velocity when there is a transverse velocity component and the particle is nearby. While it is possible to view the abberation as being real, that is not the perspective of the retarded potentials.

As has been known from the beginning, the Lorentz transform in not for inertial particles [].

In many ways, retardation equations have an inverse meaning to field equations. They do have one thing in common. The solution should be for an inertial particle.

No matter how high the velocity of the particle is, the infinitsemal transform can halt it in a single step. The solution looks like it is exact, but of course it is not.

For observations of short duration, the composite of an angular and translational velocity can be viewed as being the sum of two translational velocites, although we are not yet sure of which is which.

There is no upper bound for angular velocity, so the problem becomes harder when the particle is closer.

The solution reduces to the Lorentz transform when the motion is radial. The midpoint is at the middle in those solutions.

Like the Lorentz transform, this solution holds the quantiy r dot r - cx2 cx2 invariant. There is an extensive literature on invariant coordiante transforms []. It is unlikely an invariant transform exists that was not discovered long ago, although the literature does tend to be abstract and difficult to interpret.

The velocity of the tip of the vector from the other observer to the field point is v0 and its acceleration is zero. It is from the perspective of an observer at the tail of the vector that the motion of the particle is not perceived as being uniform along the straight-line trajectory. These two perspectives are not distinguishable when there is no propation delay, but 4-space equations without a propagaion delay are not actually in 4-space.

Thus, the Lorentz transform is not for an inertial particle, and for the same reason that the light cone solution is not. The midpoint in one frame of reference does not map into the midpoint of the other frame of refernce. It has been known from the beginning that the Lorentz transform is not for an inertial particle [].

It is not sufficient that equations hold the speed of light invariant. They must also not depend on the choice of a coordinate system. Since the transfer function is not linear in time, projecting the time shown by a clock in our frame of refernece onto the second frame of reference causes the solutions to become coordinate-dependent if there is a transverse velocity component. The midpont remains at the middle if the motion is radial.

The curvature vanishes when only the v^1 terms are carried. The infinitesimal Lorentz transform represents the v^1 terms, and it can be applied recursively.

The solution is the LW solution when tf=0. However, the solution depends on what time it is, and we have no way of knowing what time should be used. Retardaion equation have to work without knowing the time.

There is an extensive literature on invariant coordiante transforms [], and it is likely that there is a satisfactory transform for obtaining the equations of the retardaion potentials, but it is not the Lorentz transform if the solution is to be for inertial particles.

There are no longer any tvx1 terms in the solution. That would be acceptable if only the first derivatives were needed. The Maxwell equations are in terms of the second derivatives.

The Lorentz tansform is not for inertial particles, and for the same reason that the light cone equation is not. The midpoint is not at the middle when there is a transverse velocity component.

We have no way of knowing where on this curve an isolated light cone event is. Retardaion equations have to work in the same way at any point on the curve.

The Lorentz transform has the same problem, and for the same reason. The midpoint of a straight line is not at the middle. .. The transfer function is linear for radial motion, but that subset of the solutions is of limited interest.

Due to the nonlinar transfer function from the source to the field point, the equations of the retarded potentials have to be usable without knowing which region of the nonlinear curve the solution is for. The rigth region would be identifable in global solutions, but the global solutin is not yet known. The retarded potentials exist only for the purpse of being differentiated, so it is sufficient the the derivatives be correct for any region of the curve. Conversely, in being the integral of the fields, potential equations are arbitrary to within a constant of integation. The potetial solution obtained below is therefore not necessarily unique. The solution obtained does not specify where the particle actually is, wheras an invariant coordinate transform should be able to specify its location, at least in differental form.

It can be inferred from Fig 3 that the second derivative become smaller, relative to the first derivative, when the time t=0 is further in the future. That makes it possible to predict when the time when the particle will be at its closest approach. There are obviously other considerations for accelerated particles, but the velocity terms come first.

It would be easy to carry more poweres of velocity in this calculaion, but it might be that the third derivative should not be neglected, so more accurate soltions will not be considered at this time.

This solution is for one space dimension and one time dimension. With this simplificalion, the trajecrory of particle is always conincident with the field point at some point in the future. That is not true when there are two space dimensions, so the solution for two space dimensions will need further analysis.

Fig .. The geometry of light cone events. This figure represents the light cone events from the perspective of an observer at the particle. The construction is not usable at the field point because time does not progress at a uniform rate at the field point in this construction.

In the construction as shown, Rvdot is the same of all of the events, but R is longer at earlier times. The Rvdot/R term in Eq – is therefore not an invariant equation. There is an extensive literature on invariant equaions [], and it may well be that a satisfactory cooreinate transform exists, but it is not the Lorentz transform.

The particle emits a signal at time t=-r0/c. The light ray arrives at the field point at the time tf=0. The particle arrives at the tip of the vector r0 at the same time that the signal arrives at the field point. When there is only one space coordiate, the particle is conincident with the field point at some point on the trajectory. That is rarely the case when there is more than one space coordiante.

The figure assumes that the location of the partile at time t=0 is already known. There is no simple way of knowing where the particle actually is then, but when applying retardation equations to specific configurations it is necessary to asume that the location is already known. However, in being the integral of the fields, potential equations are arbitrary to within a constant of inegation. The soluton obtained is therefore not necessarily unique, nor is it necessarily true that we know where he particle actually was.

? tf0 and tf1 have dropped out of the solution, so it does not matter what time it is.

Despite the different angles and lentgths of R for the events in the figure, the equation for the retarded poententials has to look the same for all of them. It has to be an invariant eqquition. The LW equations are invariant, but only at low velocites.

These calculations can be extended to three or more virtual particles.

discussion

At time $(\Delta t)^2$, the real $at^2/2$ terms are in the same order as the v^2 terms, so they should be carried in periodic solutions for small systems. The virtual $\dot{\boldsymbol{a}}$ terms are small but in the same order as the real acceleration terms, so they should be taken out when good accuracy is needed. The Newton series is not an orthogonal series when the speed of light matters. Individual terms of the series, considered in isolation, do not have a meaning.

In many ways, retardation equations have an inverse meaning to field equations. A false term from one perspective can be a neglected term from a different perspective. The perspectives do have one thing in common. The solution should be for an inertial particle.

Since the trajectory is perceived as being curved, it is necessary to approximate it as a series of stratigh-line segments. The segments are all parallel to each other, but their lengths vary. A low velocities one segment is sufficiently good approximation, and the solution in that order is the LW solution.

There are quadradic terms in Eq –. No matter how small dt is in our frame of reference, it is not small enough to satisfy the other observer. Dividing dt into two steps is enough to represent the lowest order curvature terms.

it is also not zero when the retarded Newtoian acceleration is not zero. When obtianing solutions for the retarded potentials, we do not need to know which case is which. There are other circumstances where we do need to know the difference. Retardation equatins are just one perspective of a much larger problem.

Since the time and space coordiantes are not independent varialbes in light cone solutions, it should be possilbe to subdivide the infitesimal with the chain rule for differentiaion, as it is capable of converting terms that look like first derivatives into second derivatives. There is always more than one way of working a problem.

The transfer funcion is linear for radial motion, but the special case is not of much interest.

These calculations are based on the premise that it is not possible to transform to the second frame of reference until the coordintes are known in the first system. There is a possibity that the same relationships can be interpreted as meaning that the Lorentz transform is not the right transform for inertial pariciles. There is an extensive literature in invariant coordinte transforms []. In any case, it is the transformations amongst the perspectives that matters, no which one is the right one.

The segments are all parallel to each other, but their lengthts vary in a way that has no meaning in a pure 3-space.

While the transfer function is linear in time, the linearity is misleading. A radially approaching source will eventually coincide with the field point. Assuming a near miss, the higher doppler shifted frequency of an approaching transmitter instantly switches to the lower but still constant doppler frequency for a receding transmitter. The LW equations are form invariant for either portion of the trajectory, but they are not capable of bridging the discontinuity between the two segments.

There is no discontinuity when the motion is not radial, but then there is a transverse component to the velocity.

The retarded potentials depend on both the velocity and the location of the particle, and two closely spaced points on the trajectory are sufficint to defefine them. However, the retarded potential exist only for the purpose of being differntiated, and three points on the trajetory are required to represent the first time derivative of the potential solution. It will be computationally convenient if the differentions can be performed without having to consider the fact that the midpoint of the three points on the trajectory is not at the middle. The differentaions could be performed anyway, but the chain rule for differention would be required. Potential solutions that do not require the chain rule for computing the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields will be easer to apply.

More than two straigh-line segments would be requird if the second derivatives are needed. The Maxwell equations are in terms of the second derivatives, but it is sometimes best to proceed one step at a time.

At low velocities, it is only the radial velocity that is important. The doppler shift depends mostly on $\dot{R}$.

As shown in Fig –, at high velocities, the differnce between a scalar and a vector becomes important. $\dot{R}$ is no longer a good indicator of the doppler frequency. $\dot{\boldsymbol{R}}$ is now required in the equations. The assymetry between negative and positive times is due to the propagation delay from the source to the field point. The quantity that matters is where the transmitter was, not where it is. The assymetry between negative and positive times is responsible for the transverse doppler effect[20]. The assymetry is of order v^3, but the doppler shift is of order v^1, so the transverse correction is of order v^2. (The figures are based on exact calculatins. The v^3 calulations shown elsewhere are not accurate enough.)

These figures would apply equally well to doppler shifted monochromatic acoustic signals. Acoustic signals are not in 3-space. They are in 4-space.

The LW eqations are form invariant for either portion of a radial trajectory, but they do not provide a means of connecting the two portions.

The basis of this inconsistency is that the LW equations are not the right equations for inertial particles unless the motion is radial.

The shift in the midpoint is small when the three points are closely space, but it is in the same order as the retarded Newtonian accelertion of the particle.

The time and space coordinates are not independent variables in light cone solutios, and the chain rule for differentiation is generally required when the variable are not independent. The chain rule could be used to apply a correction for the midpoint shift.

When the velocity is low, the scalar $\dot{R}$ is a good indicator of the doppler shift, even when there is a small transverse velocity.

The LW equations are form invariant for either portion of the trajectory. There is a discontinuity between the two segments that no differentiable equation can bridge.

The equaions obtained this way represent a projection of 4-space into 3+1 space. While 4-space can be projected into 3+1 space, the reverse projection is incomplete. 3+1 space is not a real space, so the equations are unphysical.

It will be computaionally convenient to obtain approximate potential equations that do not require the chain rule for computing the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields.

The doppler shift is too small to be easily visible in the figure, but if pulses are transmitted at regular intervals by the source, they will by doppler shifted to a slightly higher frequency when the particle is approaching on the left side of the figure, then shifted to a lower frequency when it is receding on the right side. The LW equations are a good approximation in this regime. They do in fact represent the transverse velocity terms. The transverse terms are responsible for the magnetic field, but they are only accurate to the extent that the rate of change of a doppler shifted frequency can be represented with the rate of change of three scalar $\dot{R}$ terms in three orthogonal directions, considered separately. In 4-space, the rate of change of a vector is not necessarily representable with vector equations.

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While the terms of the series in in Rv are mutually orthogonal in the first frame of reference, they are not orthogonal in the frame of reference of the particle. The lack of orthogonality results in an unfamiliar coupling between the orders. The dt interval has to be subdivided into two steps if the potential solutions are to be differentiated once. It would have to be subdivied into three intervals if the second derivative is needed. When working in 3+1 space, we are always one step behind.

In more elaborate problems, where there are both real and virtual accelearation terms, the virtual terms usually mean that we do not yet know where the particle was. This perspective would be appropriate for orbit detrmination. Orbit determination will be more difficult when the retarded virtual jerk is significant.

There are other problems where the trajectory of the particle is artifically constrained, such as in rotating electrical equipment. When the location of the particle is already known by independent means, the virtual acceleration terms need to be taken out. In more extreme circumstances, the retarded virtual jerk must also be taken out.

The light cone equation can be solved with t_s being the independent variable. If that is done the pulses would be transmitted at regular intervals but received at irregular intervals. The LW equations can be derived either way. The retarded potentials depend on both the velocity and the position of the particle, so their first derivative is the second derivtive of position. It should be possible to work the problem either way, but the irregularly spaced pulses at the field poilnt would need consideration in computing the second derivative. In 3+1 space, we are always one step behind. What looks like a first derivative is actually a second derivative.

Being one step behind is not a serious condition, but the methods are not the same as for the leader.

No matter how how short the inteval is, the midpoint at the field point does not map into the midpoint of the interval at the particle. The midpoint shift is small when the interval is small, but it is in the same order as the retarded Newtonian acceleration.

The retarded potentials are not physical quantities. It is only their derivatives that are of interest. That makes it possilbe to append a virtual velocity term to the retardaion equatins, which, after differentiation, takes out the virtual acceleration terms.

An approach better aligned with physical concepts would be to use the chain rule for differention to correct for the midpoint shift. The calculations have not been carried through, but it should be possilbe to work the problem that way.

It is possible to solve the light cone equation with t_s

being the independent variable. The pulses would then be transmitted at regular interavals, but they would be irregularly spaced at the receiver, and the same problem would be present from a different perspective.

This behavior seems contrary to the theorms of Eucldian calculcus, but it is not, because the chain rule for differention is one of those theorms. The chain rule is required when the variables are not independent, and they are not independent in light cone solutions. The chain rule is capable of converting terms that look like first derivatives into derivatives of any order.

The time shown by a clock at the field point is obviously not a good choice for a time standard when evaluating the kinematic behavior of the particle, but the observer has no other option unless the retarded velocity of the source is known by independent means.

The $\dot{\boldsymbol{a}}$ terms are degenerate with the Newton equations. The $\dot{\boldsymbol{a}}$ terms are not zero in the solutions, but it is not possible to carry them when integrating numerically. There is no known way of reducing the four dimensional space to first order, so it follows that the $\dot{\boldsymbol{a}}$ terms, whether virtual or real, are not degenerate in 4-space.
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Since the potentials are the integral of the fields, a more accurate solution can be obtained by integrating before differentiating. The simplest possible form of integration consistes of approximateing a curve with two straight-line segments. While the midpoint shift is not representable with the first derivative in the first frame of reference, it does influence the integral. That makes it possible to subdivide the infinitesimal. The subdivision is only possible with potential equations, as the subdivisions would otherwise be linearly dependent. The midpoint is always at the middle when the relationhips are linearly dependent.

The actual meaning of the infinitesimal is that the relationships are of first order. Infinitesimal quantities can be of substantial magnitude, yet not be capable of subdivision because the relationships are linearly dependent.

The following calculations do not explicitly use the chain rule for differentiation, but recursive applications of the chain rule is the actual mathematical basis for the calculaltions. The chain rule is capable of converting terms that look like first derivatives into derivatives of any order.

The midpoint shift results in a virtual retarded Newtonian accelertaion term that does not represent the behavior of an inertial particle.

The retarded potentials are not physical quantities. They exist only for the purpose of being diferentiated, so the potential equations should be compensated for the midpoint shift so that they will differentiate correctly.

In the frame of reference of the field point, the retarded position vector represents three copies of the scalar $\dot{R}$ in three orthogional directions. The same is true for the advanced position vector in the frame of refence of the particle. However, the retarded and advanced position vectors are not connected by a vector equation.

The retarded potentials are first-known in the frame of reference of the particle. It is the perspecive of the other observer that is relevant for their derivation. From the perspective of the other observer, the advanced potententials at our location would exist without our presence.

The differences between the two doppler curves are subtle and not important at low velocites. At low velocites, the connection between the advance and retarded position vectors can be aproximated with vector equations, and the LW equations are a satisfactory approximation in that regime. Indeed, the velocity of conducton electrons in wire is so low that there is no possibility of detecting the difference in those configurations. The behavior of electron beams near the wires is a different problem.

All of the data shown in the figures is based on the exact root of a polynomial. The series calculations shown elsewhere are not accurate enough for the figures.

The midpoint shift introduces a false acceleration term into the solutions for the E and B fields. Four points on the trajectory would be required to compute the second derivatives of the potentials. The Maxwell equations are in terms of second derivarives.

The midpoint shift does not affect the retaded velocity of the particle. After diffeentaion to obtain the the fields, vecocity terms become acceleration terms, then the midpoint shift introduces false Newtonian acceleration terms.

The basis of the inconsistency is that the LW equaions are not for an inertial particle. They assume that an unaccelerated particle is half way to the destination in half the travel time. For light cone solutions, that is only true for radial motion.

It is for this reason that the solutin in Eq cannot be obtained by linearly interpolating the other two solutins. The three solutions are not connected by a linear equation, but the LW equations assume that the connection is linear time.

By dropping the quadradic terms, the time intervals become of first order in both frames of reference.

The LW equations are only exact in 1+1 space. When the transverse velocity is low, 3+1 space can be approximated with the three scalars ..., but there are cross terms in the derivatives that require the chain rule for differentiaon when the velocity is high.

It would obviously be necessary to take more steps if more powers of velocity were carried.

The velocity of a particle can be decomposed into a component v ru parallel to the line of sight and a component dv in a different direction.

The solution shows that the time and space coordinates of the doppler equation are difficult to separate.

The solution shows that the perceived acceleration of a particle depends on both its distance and the angle of the trajectory.

The basis of the inconsistency is that the LW equations assume that the particle reaches half way to the destination in half the travel time. Isolated and unaccelerated inertial particles do behave that way, but light cone solutions do not when there is a tansverse velocity component.

Since the trajectory is perceicved as being curved, it needs to be represented by a series of straight-line segments. Two segments are better than one segment.

This solution assumes that the location of the particle is already known. That is the appropritate perspective when the trajectory is artifically constrained or otherwise independently obtained, as in the soltuions for rotating electrical equipment. It would not be the appropriate perspective for orbital calculations.

As with the Lorentz transform, there are cross terms in the light cone solution for the sum of two velocites. Vectors do not add as vectors. In 1+1 space, the sum of two velocites is still one velocity. The cross terms of 2+1 space are not representable in 1+1 space. There are other cross terms in the solutions.

The solution for a low transverse velcity is shown in Eq –. There is some doppler shifth in the figure, but it is barely visible. Even though the LW equations belong in 1+1 space, they are a usable approximation in this regime.

A time interval at the field point maps linearly into the time interval at the particle when the motion is radial. Equivalently, a doppler shifted frequency is constant for radial motion.

None of the solutions are affected by translating the coordinates in either time or space.

The angular velocity and the perceived Newtonian acceleration are coupled in light cone solutions. It is not possible to quickly determine which is which unless the retarded velocity is known by independent means.

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There are essential differences for the light cone equaton in 2+1 space. There are related differences for the doppler frequency. The doppler shift is constant for radial motion, but it is constantly changing when there is a transverse velocity.

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The cross terms between radial and transverse velocites are visible in the 3+1 space derivatives. Light cone solutions in 2+1 space are not representable with two copies of the 1+1 space equations.

Solutons to the light cone equation contan ... cross terms in the first frame of reference. The terms are similar in form and meaning to the Thomas pdrecession []. The solution shows that light cone solutions in 2+1 space are not representable with two copies of the 1+1 space equations. The rotations of the Lorentz group [] represent the 4-space angular relationshops in a more general way.

The calculations of the Thomas precession[7] do not have to be for light cone events, but they can be, in which case transforming the wrong equation to the second frame of reference confounds the problem. The $av \times \boldsymbol{v}$ terms of the Thomas precession exist in the first frame of reference when the speed of light matters. That is as it should be unless our frame of reference is special. It does seem special, and it actually is special in some ways, because it is the only one we can ever measure anything in. However, the retarded potentials are first-known in the other frame of reference.

At low velocites, the vector sum of the rate of change of the three scalars $\dot{R}_x$, $\dot{R}_y$, and $\dot{R}_z$,condsidered one at a time, is a good indicator of the rate of change of a doppler shifted signal. The cross terms can be neglected at low velocities. The chain rule for differentiation would be required at higher velocites. The chain rule is not needed when there is only one space coordinate.

Due to the presence of cross terms in the derivatives of the light cone equation, the chain rule for differentiaon would be required at higher velocites.

At low velocites, the doppler shift can be accurately computed with the three scalars The relativistic doppler equation[20] is a good approximation in this regime. For the same reason, the LW equations are also a good approximation in this regime.

The midpoint shift does not affect the retarded velocity. However, after differentiating the potential solution to obtain the E and B fields, velcoity terms become acceleration terms, and three points on the trajectory are required to represent them. The midpoint shift introduces false acceleration terms into solutions for the retarded fields.

The retarded potentials are not physical quantites. They exist only for the purpose of being differentiated. The solutions should show the retarded Newtonin acceleration to be zero when it is already known to be zero. The false accelereation terms can be canceled by appending virtual acceleration terms of the opposite sign.

The correct interpretatin depends on the circumstances. When the conditions are mild the distrance in the equation stands for itself, and equations in that ordee are the LW equations. When the solution is more demanding, the qantity in the solution is int Rdot dt. Under still more exterme coneditions the solution shoud be ineterpreted as int (int Rddot dt) dt.

The constant of integration has to be determined from physical principles. The retarded Newtonan acceleration should be zero in solutions where it is known to be zero.

When the solution is a double integral, the first constant of integration is not a constant after the second integration.

While angular and translational velocities do not need separate consideation with vector equations in the first frame of reference, the distinction is important for the other observer.

This solution is for the first derivative. The Maxwell equations are in terms of the second derivatives, but it will be best to proceed one step at a time.

One of the considerations is that the first constant of integration in a double integral is not a constant in the solution. There appears to be a connection to the rotations of the Lorentz group ... . Those equations may provide a more reliable basis for the next step.

It is not normally necessary to distinguish between angular and translational velocities in the first frame of reference. Indeed, there is really no point in it when the speed of light does not matter, but the distinction can be importaint in more general solutions.

It is not necessary to distinguish beween angular and translational velocites in 3+1 space. There is only one velocity associated with a particle. But because the two kinds of velocity are coupled in light cone solutins, it is necessary to formulate the problem in such a way that is is possible to determine which is which in order to separate them. The chain rule for differentaion is not necessary when the variables are independent. For short times, angular and translational velocites can be represeted as two translational velocites. The pivot point for the angular term cannot be at the field point. One point is not a satisfactory angular reference.

The solutin in this form looks like it is exact, and its compactness can be convenient, but it is no more accurate than Eq ().

In the calculations for the Thomas precession in Ref – contain an error. The calculations assumed that velocities add as vectors in the light cone equation. That is not true. There are cross terms the the soluiton. The error is corrected in the following calculaions.

There is a close connection between this solution and the calculations of the Thomas precession []. The Thomas calculations project 3+1 space into 4-space. This calculation projects 4-space into 3+1 space. The equations are essentially the same either way, but the meanings are different.

Fig. 4 shows the same data as Fig. 3 with the same scales but half the velocity. The false acceleration varies as $(v/c)^3$, so it is only signifiant at very high velocites. On the other hand, since the doppler shift is of order v^1, it requires a v^2 correction to the doppler shift.

?At very low velocities the false acceleration term is insignificant but at a maximum at the point of closest appoach. The maximum moves to progressively earlier times at higher velocites.

Figs. 5 and 6 show the data from other prespectives.

The Newtonian retarded av and avdot terms are in the same order as the calculaions of this section. The velocity terms are of lower order, and it will be best to proceed one step at a time.

This equation applies to the solution at the tip of the radius vector $\boldsymbol{R}$, as interpreted from delayed observations. An observer at the other end of the vector would see the world differently.

The cross terms between the radial and transverse velocites depend so strongly on the velocity of the particle that they can usually be neglected.

The false acceleration terms are in the same order as the Newtonian $\dot{\boldsymbol{a}}$ terms, indicatating that they could be cancelled by appending false $\dot{\boldsymbol{a}}$ terms. However, the false terms should not be in the solution in the first place.

There is an extensive literature on invariant coordiante transforms []. It is likely that this solution is obtainable in other ways.

The terms of the Taylor series look the same as those of the Newton series, but the Taylor theorm has a more general meaning.

This solution is in 4-space. The Newton equations are in 3+1 space. 4-space can be projected into 3+1 space, but the reverse projection is incomplete.

Direct calculaton shows that the vxn terms should not be carried without also carrying the avddot terms.

The particle is on the light cone, but it is not the same particle as in Eq –.

No matter how small Δt_f is, it is not small enough to cause the midpoint to be at the middle, as there are still terms quadradic in Δt_s in the light cone solution.

A smaller interval could be selected by dropping the quadradic terms, but then the solution would missing terms at time dt_f, because the dt_s interval is not linear in dt_f. Light cone solutions cannot be reduced to first order in the same way that the equations of 3 space are.

Straight lines in 3-space do not have tick marks at regularly spaced intervals, making it impossible to determine when they have been streteched. In 4-space, we know when they have been stretched.

It becomes necessary to subdivide the infinitesimal dt_f interval. The corresponding segments of the trajectory are all parallel if the retarded Newtonian acceleration is zero, but their lengths vary. In effect, it is necessary to integrate before differentiating. The midpoint of the curve represented by several straight-line segments is still not at the middle, but the equations are otherwise of first order, and the first order relationships are recovered after the potential solutions are differentiated.

At low velocites, it is not necessary to subdivide dt_f in order to obtain satisfactory accuracy, and the equations of that order are the LW equations.

To the extent that the curvature of the trajectory can be approximated by two straight-line segments during the dt_f interval, the particle is now on the light cone at time dt_f.

The discontinuity goes away when the motion is not radial, as the particle will always miss the field point when there is a transverse component to the velocity.

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The solution is of first order in dt_s, but it contains terms quadradic in Δt_f. From the perspective of the other observer, the equations are not of first order, and there would be no point in subdividing dt_s, as the equations would be linearly dependent. The retarded potentials are first-known in the other frame of referene, so the perspective of the other observer is important.

The equation can be reduced to first order by dropping the terms quadradic in dt_s. But from the perspective of the other observer, the solution would be missing terms, because the quadradic terms belong in the solution, no matter how small dt_s is. The equations can be reduced to first order from both perspectives by integrating over the dt_f interval.

In performing the integration, the curved trajectory can approximated with just a few straight-line segements.

When the equations are of first order from both perspectives, the midpoint is at the middle of each segment. One segment is sufficient when the velocity is low enough. The equations of that order are the LW equations. No segments at all are needed at the still lower velocites where the Coulomb soltuion is adquate.

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The potentials are the integral of the fields. The integral is already known, so there is no point in differentiating and then integrating again. The considerations would be different for orbital calculations.

In orbital calculations, the derivatives are integrated to obtain the loction of the particle. Retardaion equations assume that the location is already known, then the derivatives are computed. There are many case where the location can be independently obtained, such as in the solutions for rotating electrical equipment.

Potential equations are arbitrary to within a constant of intgration, so the solution obtained this way is not necessrily unique. It is the nature of retardaion equations that the constant of integration depends on where the particle is, relative to the observer, and we are not sure yet of where it is. Alternatively, potential equations can sometimes be gauge transformed. Local physical arguments should not be applied to potential equations, because they do not represent locally measurable relatinships. They exist only for the purpose of being differentiated.

An isolated and unaccelerated inertial particle will reach half way to the destination in half of the travel time. The LW equations assume that light cone equations behave the same way. That is not true unless the motion is radial.

The solution represents the value of Rv from the perspective of an observer who cannot tell the difference between angular and translational velocity. The observer would know the difference if the retarded veloctity were known by independent means, but that is frequently not the case.

The cross terms causes the Taylor theorm to behave differerently in 4-space than it does in 3-space.

The Newton series, ... looks like the terms of a Taylor expansion, but the problem is more fundamental than that. The problem is that the Taylor theorem exists in 3-space. Due to the presence of cross terms, it is necessary to carry more terms in 4-space than would be necessary in 3-space. Vector equations do not have cross terms, so the calculations cannot be performed with vector equations.

This behavior is a fundamental property of the four dimensinal space. Perceiving it is an everyday experience. It occurs when the doppler shifted tone of a passing vehicle is constantly changing. The equations for it are not simple.

The midpoint shift is small when the interval is small, but it is in the same order as the retarded Newtonian acceleration. The particle is perceived as being accelerted in the same direction as the retarded velocity.

The midpoint shift does not affect the retarded velocity. But after the potential soltuion is differentiate to obtain the E and B fields, velocity terms become acceleration terms. Three points on the trajectory are required to represent accelertion terms. The midpoint shift introduces false accelerations terms into the solutions for the retarded E and B fields.

In being of order $(v/c)^3$, the midpoint shift depends so strongly on velocity that the LW equations remain usable even at moderately high velocites.

That does not mean that the midpoint shift does not belong in the equations. It is rather that its meaning is ambiguous.

Since Eqs (2) can be derived with the Lorentz transform[9], all of these calculations could be performed equally well with the Lorentz transform. However, the light cone equation should be included in the calculations if the solution is to be for light cone events. (The calculations in Ref.[11] contain an oversight in this respect.)

The retarded Newtonian acceleration terms are in the same order as the calculaions of this section, so they should be included. In being of lower order, the veclcity terms are more important. In the interest of simplicity, the acceleration terms are neglected in this section, but they are included in the calculation of the next section.

For observations of short duration, the signature of a retarded Newtonian that is parallel to the retarded velocity would be the same. The light cone equation does not specify the behavior of inertial particles. The interpretaion of the solutions is ambiguous on short time scales.

The first term of the retardaion series is the Coulomb solution. It is usable in quasi-static solutions when the magnetic field is weak enough that it can be neglected. The second term is the LW solution. It is usable when the symmetric terms are small enough that they can be neglectd. The Lorentz condition, $\boldsymbol{\nabla}\cdot\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, is a symmetric vector equation, but is is always zero in the LW solutions.

This behavior is one of the signatures of a four dimensional space. The same effect occurs when the doppler shifted tone of a passing high speed vehicle is perceived as constantly changing. The v^3 term depends so strongly on velocity that it is usually not noticeable, althought it is occasionally obvious, as the acoustic location of a low flying high speed aircraft is not the same as the visual location. Similarly, the minimum of the $\ddot{R}_y$ term in Fig. ... occurs before the closest approach. At low velocites the minimum would occur nearly at the closest apprach.

Since we have no way of knowing where we are in space and time, each observer is free to choose a coordinate system that is centered upon themselves. But when there are two observers in the same problem, both observers must use the same coordinate system. One observer evaluating a system at two widely separated times counts as two observers.

The particle is on the light cone at time tf, but there is no solution for the time tf+dtf. Tbe calculation assumes that the angular velocity does not matter. The assumption leads to a contradiction when the particle is nearby.

The difference between two closely spaced vectors is a vector, but the vector is not computable unless a global vector solution is already known.

This result can be interpreted in two ways. It could be taken to mean that the the Lorentz transform is not sufficiently general, and more genearal invariant coordinate transforms are known. With the other intrpretaion, there is no point in transforming to the other frame of reference until the coordintaes are known in the first system.

It is for this reason that Eq () is inconsistent. The LW equations are not for an inertial particle.

The particle seems to be on the light cone at time t+dt, however the calculation is only accurate to first order. Only first order equations can be reduced to first order in one step. The second derivative is required for computing the first derivative at a displaced point.

The solution is also invariant when Rv and vv are perpendicular. However, the light cone condition does not provide a first order constraint on the magnitude of R at that point on the trajectory. The transverse location of the particle at time $t_s = -r_0/c$ is not accurately constrained.

When $\hat{\boldsymbol{v}} and \hat{\boldsymbol{r}}$ are perpendicular, the point of zero doppler shift does not occur at the point of closet approach. It occurs slightly earlier. The wavelenght stretch is of order dR^2, but $d\boldsymbol{R}$ is $v_0 t_s$, so it is of order $(vt)^3$.

The transverse doppler term[20] has no first order effect on the light cone equation, but it causes the solutions for the LW equations to be for a particle that is not on an inertial trajectory. There will be additional transverse doppler terms when the retarded Newtonian acceleration is parallel to the velocity vector. They will not have a first order effect on the solution, but transverse doppler is not of first order either.

The particle appears to be accelerated. An accelerometer attached to it would not register an acceleration, and the accelerometer dial can be read from any frame of reference. The LW equations are not for an inertial particle.

The first derivative at time t+dt is always different than the first derivative at time t unless the second derivative is zero.

Only first order equations can be reduced to first order in one step. Higher order equations can be reduced to a system of first order equations[2], but not in one step.

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The particle seems to be on the light cone at time t+dt, however the equation is only accurate to first order. Only first order equations can be reduced to first order in one step. The second derivative is required for computing the first derivative at time t. The Maxell equations are in terms of the second derivtives of the potentials.

The retarded potentials exist only for the purpose of being differentiated. It is not sufficient that the particle be on the light cone at time t. It must also be on the light cone at time $t + dt$, but it is not.

$\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{v}}$ is ± 1 for radial motion and the inconsistency vanishes. It also vanishes when $\hat{\boldsymbol{R}}$ and $\hat{\boldsymbol{v}}$ are perpendicular, or when the v^3 terms are dropped. At low velocites, the particle is on the light cone for any angle.

The solutin in Eq could be obtained for the time tf, then the substitution tf=tf+dt made, and the particle would seem to be on the light cone at time tf+dt. To first order, it would in fact be on the light cone, but only first order equations can be reduced to first order in one step. For more general equations, computing the first derivatve at a displaced point requires the second derivative. The second derivative has to be free of second order error. The first order of the infitessmal is free of error, yet differential equations of all orders exist.

The solution becomes invariant if the t_1^2 terms are dropped, suggesting that an invariant solution could be obtained by integration. The solution would require a constant of integration.

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Coordinates do not have an absolute significance. It is only coordinate differences that are relevant in either frame of refernce. The origin of the coordinate sysem in our frame of reference has no special significance for an observer in the other system, and conversely.

The perceived acceleration of the particle depends on both its distance and the angle of the trajectory. The angular and translational velocites are coupled and difficult to separate unless the retarded velocity is known by independent means.

As shown by the dashed line in the top panel of Fig.–, the midpoint shift depends so strongly on velocity that it can be neglected even at moderately high velcites. At low velocities, the magnitude of $\ddot{R}$ is at a maximum at the point of closest approach. The maximum occurs earlier at high velocites.

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The retarded velocity at time $t+dt$ should be $\boldsymbol{v}$, but it is not. The LW equations are not for an inertial particle, which is why Eq – is inconsistent.

There is no mathematical contradiction in this calculation, because only first order equations can be reduced to first order in one step. The equation has to be reduced to first order if it is to be integrated.

The solution seems to be for the wrong particle, and it may be. On the other hand, since we cannot know what is happening at the simultaneous point while it is happening, it is difficult to be sure of which particle is the right one.

Even though dts/dtf containts quadradic terms, choosing a smaller value for dtf does not accomplish anything, because the solution for a smaller value of dtf is linearly dependent. But from the perspective of the other observer, no matter how small dtf is, it is not small enough to reduce the problem to first order.

The third derivative would be required to compute the first derivative at a twice displaced point.

Eq () becomes invariant if the t_f^2 terms are dropped.

The $-r0/c$ term can still be arbitrarily large, but the particle will only be on the light cone for a short time.

The midpoint shift only occurs when the clock at the field point is used as the time standard. In Fig. ..., the light cone equation was solved with t_s as the independeent variable. From the perspective of the other observer, the midpoint shift occurs at the field point.

Equations of physical significance must not depend on the orientaion of the coordiante system. The principle is much older than the theory of relativity.

The acceleration terms should be carried in equations that are quadradic in time, but they will be neglected in these calculaions.

The acceleration terms are in the same order as the transverse doppler terms, so they should be carried. Due to the lenght of the expressions, only the velocity terms are considered here. Similarly, the $\dot{\boldsymbol{a}}$ terms are in the same order as the v^4 terms, but they can be neglected when the conditions are mild.

The $-r_0/c$ term can still be aribtrarily large, but the particle will only be on the light cone for a short time – just long enough to compute the second derivatives.

The light cone equation does not provide a first order contraint on the transverse location of the particle.

The equation reprsents the transverse doppler effect. When .. is zero, the point on the trajector for which the doppler shift is zero does not occur at the particle's closest approach. It occurs slightly sooner.

Differentiating Eq – with respect to tf0

This solutin is wrong. The transverse doppler when ... is zero is half of that value.

From the perspective of the other observer, this is the right equation for the transverse doppler. Sometimes we need to see ourselves as others see us. It is too easy to write the equations as though we were at the center of the universe, while denying that we are doing what we are doing.

From the perspective of the other observer, this is the equation for the transverse doppler. It is probably our equation too.

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The time and space coordinates are not independent variables in light cone solutions, making the chain rule for differentiaion necessary. The equations do not work in the same way as they do for one independent variable.

This relationship indicates that the chain rule for differention is required.

dR=paretial rv/dt dt + (drv dot del) rv drv

del rv is a tensor of the second rank. rv is the Newtonian location of the particle. Rv is the perceived location.

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There are no relativistic corrections when tf is 0 and ru dot vu s also zero. The solution is the same as the solution for a stationary charge at that location.

The solution does contain transverse terms at time dt, but they are lost when dt is set to 0 to obtain the retardation equations. There are no assurances that they can be recovered by differnting in the first frame of reference.

There is a connection bewteen the doppler shift of a moving monochromatic source and the retarded potentials. Flux is emitted by a charged particle at a constant rate. When the time interval at the field point is shorter than the interval at the source, the flux becomes concentrated into a shorter time, enhancing the field strength. (A source of power would be required for a stationary particle to emit anything. This is just a model, one that needs refinement.)

The retarded potentials are first-known in the frame of refernce of the particle, so the time interval in that system is a more plausible choice for computing the flux rate.

dtf can be the period of a monochromatic signal. There is no doppler shift at the point of closest approach.

Since the speed of light is the same in all frames of refrence, it would be easy to conclude that we can always consider events in relation to ourselves. The retarded potentials would exist if we were not here.

When we are at one fixed point, angular relationships in relation to ourselves have no meaning. On the other hand, there is a very old question as to whether an event without an observer has a meaning.

The doppler shift seems to be zero at the point of closest approach.

This is the right equation for transverse doppler.

It may seem that we have always known where the prticle is when k is zero, but it is difficult to be sure, because the time at the field point is then +r0/c. The location of the particle at that point on the trajectory could be determined by an assistent near there. The assistant could observe the time that the particle is adjacent to a nearby clock in a field of synchronized clocks. However, the observer at the field point could not know the outcome of the observation until a later time. We cannot know what is happening at the simultaineous point while it is happening.

The solutions conain fluctuating virtual charge. Preliminary indications are the the global sum of the virtural charge is zero.

The light cone equation does not impose a first order constraint on the location of the partcle at time tf=+r0/c. A more accurate solution can be obtained by extrapolating to that point from a region of the trajecory where the constraints are of first order.

k has dropped out of the solution, so it does not matter when the coordinates are first-known.

There is no transverse doppler term if the location of the partice is the same as the one assumed in the derivation of the LW equations. Transverse doppler is of order v^3, but doppler is of order v^1, so it reprsents a $(v/c)^2$ correction.

When integrating numerically in n steps, it is not sufficent that the error vanish in the limit. The term must either vanish as $1/n^2$ or be carried. After integration, terms that vanish as $1/n$ are in the same order as the solution.

dt_f could be the period of a doppler shifted monochromatic signal. This is not the right equation for radial doppler.

... There is no transverse doppler at the point of closest approach.

For unaccelerated particles, $\dot{R}$ is zero when $\boldsymbol{R}$ and $\boldsymbol{v}$ are perpendicular. The light cone equation does not impose a first order constraint on $\dot{R}$ at that point on the trajectory. It is difficut to be sure of exactly where the particle is then. That does not matter for the retarded potentials, as they are free of first order error. However, they exist only for the purpose of being differentiated twice.

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The particle is on the light cone, but it is not the same particle that the LW equations are for.

We cannot know what is happening at the simultaneous point while it is happening. We can find out what happened there, but not until after waiting a while. Determining exactly where the partile was then is not easy, and it cannot be done quickly.

In Eq (), dts/dtf is 1 at the point of closest approach. There is no transverse doppler.

An assistant near the source could record the time that the particle passes the nearest clock in a field of synchronized clocks, but the assistant is usually not available. A distant observer must infer the number that the assistant would read by extrapolating to that point with several delayed observational data points.

The solutions are the same as those of the relativistic doppler equation.

There are two ways of viewing this solution. In one way, the Lorentz transform in not the right transform for the problem, and other invaraint coordinate transforms are known. In the other way, there is no point in transforming to the other frame of reference until the coodintes are known in our frame of reference.

The particle is on the light cone, but it is not the same particle that the LW equations are for. The LW equations are not for an inertial particle.

This solution represents a projection of 4-space into 3+1 space. The location of the particle appears to be where it is thought to be in a conventinal Newtonian light cone calclation. This statement represents an interpretation, one that may need further evaluation. The considerations for computing the orbit of a free particle would be different.

A more accurate solution can be obtained by extrapolating to the poorly constrained portion of the trajectory from the solution for other times where the light cone constraint is of first order.

The particle is on the light cone. To first order, it is also on the light cone in Eq (), but this solution is more accurate. There is no transverse doppler term[17,20] in Eq ().

dt_f can be the period of a doppler shifted monochromatic signal. There is no transverse doppler at the point of closest approach.

The sign of $\hat{\boldsymbol{r}}$ can be inverted if the usage is consistent. There are no relativistic corrections when $t_f = 0$. Translating the coordinates has no effect.

The light cone constraint does not impoese a first order constraint on the loccation of the particle when $t_f = 0$. The calcultion assumes that we already know where the particle is, but we actually do not know exactly where it is yet. There are some fairly obvious ways of discovering a good answer, but it would be better to understand why it is the right answer.

There is a particle at the origni of the coordinate system when tf=r/c, and it is on the light cone, but it is difficult to sure that it is the right particle.

The observes cannot both assume that they already know where the particle is. The retarded potentials are first-known in the other frame of refernce. The opinion of the other observer is more relevant.

There can be many particles and many other observrs. It is not clear that the location of the particle has an absolute meaning, but its location can be bounded.

In a simultaneous system, the location of the particle is

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When extrapolating around a circle, the first infitesimal step is free of first order error. If the equations are then reparameterized by coordinates first-known after the first step, the the second infitesimal step works in the same way as the first step.

There are necessarily other ways of working the problem. Some of the equations of the Lorentz group[6,10] are probably relevant to the retardation problem. Retardation equations should not contain time as a parameter, because we do not know what time it is.

In being the integral of the fields, potential solutions are arbitrary to within a constant of integration. The retarded potentials exist only for the purpose of beingf differentiatd, so the constant is probably superflous, but it could be important from other perspectives. In any case, in being subject to gauge transformations, potential equations are not necessarily unique.

The solution would have been the same as the LW equations if the integration had been performed in one step. There does not appear to be anything wrong with the LW equations when they are of sufficient accuacy.

We have no way of knowing what time it is. Retardation equations have to work in the same way at time t+dt as they do at time t.

When a paticle is moving parallel to the vector pointing from one observer to another observer, either observer should be able to solve the light cone equation for the other oberver. The solution should not depend on how far apart the two observers are unless a cosmological term is inclded.

The expansion factor within a mass shell is not locally detectable[8], but it is responsible for the gravitaional redshift for distant observers. The expansion factor will be assumed to be zero in these premininary calculations.

The acceleration terms will need development, but velocity terms do come first.

This terms results in fluctuating virtual charge. Preliminary indications are that a positive virtual charge in one region is canceled by a negative virtual charge in another region.

This calculaion secifies the magnitude of this term. However, in so far as the Proca equations are concerned, T is a free prameter. T will be carried as a free parameter until its magnitude can be better constrained. This behavior suggests that T has an obscure connection to a constant of integration. A constant for the first integration is not a constant after the second intetration. In being the integral of the fields, potential equations are arbitrary to within a constant of integration. There can be more than one constant of integration in multiple integrals. Alternatively, potential equations are subject to gauge transformations.

T has no first order effect on the solution wheh t_f is zero, but it does affect the solution at other times. The calcualons in Ref were obtaine by integgrating the first time derivative the time $t_f = 0$, while this solution is for any time. This solution is more general.

Most solutions are unaffected by choosing a time other than $t_f = 0$, which can be misleading if there are other observers.

Since the relationships of space and time are not independent, it is indicated that the textbook calculations of the Thomas precession are missing terms representing the intgration to a different place but at the same time. A tensor of the second rank is required to extraplate a vector to a nearby point unless a global vector solution is already known. If a global vector solution is already known then there is no point in extrapolating a vector. Vector equations contain an enormous loophole in this respect.

Translating the coordintes has no effet on Δt_f. A choice other than $t_f = 0$ would immediately drop out of the equations, there would be no point in choosing a different time.

The scalar and vector solutions are not symmetrical. The gradient of a vector is a tensor of the second rank. The gradient of a scalar is a vector. The equations should be symmetrical if they do not need to be differentiated, but not otherwise. Furthrmore, the second derivative of the scalar equation is a tensor of the second rank. This solution is incomplete. It must be so, for the tensor of each rank is irreducible[6].

It cannot be a true free parameter, but it is likely that further investigation will be required to accurately determine its value.

There is no point in extrapolating a vector when a global vector solution is already known, which is why there are potential equations. However the retarded potentials do not represent locally measurable relationships, so local physical arguments should not be applied to them. The are just one of many perspectives.

The equations behave as though the clock at the source were running slow. This equation is similar to the transverse doppler equation.

The other observer could attach an accelerometer to the particle. The other observer's opinion of what the accelraton of the partice is is more reliable than ours – unless the other observer is in free fall.

Probably so, but that should not be assumed until it is shown.

The transvers doppler equation suggests that T is 1. However the doppler equation does not include acceleration terms, so the degrees of freedom of the problem may need further investigation. T cannot be a free parameter, but it will be carried as one until a more convincing argument for its correct value is obtained.

The calculations of the Thomas precession do not include the virtual acceleration term. Invariant coordinate transforms that are more general than the Lorentz transform are known, and they may be relevant for the retardaion problem.

The T in the equation is an undetermined coefficient.

The solution is the same as for a charge that is at rest at the same location. There are no transverse doppler terms.

The solution is the same as for a charged particle at rest at the same location. There are no transvers doppler terms[20]. There are no transverse terms becaue the doppler equation repreents the derivatives of the function, wheras the LW equations represnt the function itself. That would be all right if the function did not need to be diffeerntited. Once the transverse terms are lost, they cannot be recovered by differentiating in the first frame of reference. In being of order v^3, the transverse terms can be negleced even at moderately high velocites.

The relativistic doppler equaton does not include accelertion terms. The equations of the general theory of relative do[8].

This soltion does not imply that we already know where the particle is at time t=0. The retarded potentials exist only for the purpose of being differentited. When applyig retardaion equations, it is necessary to assume that the location of the partile is already known. The considerations for computing the orbit of a free particle would be different.

The acceleration terms should be included in the following calculations. They will be neglected in these preliminary results. In being of lower order, the velocity terms do come first.

The following material is presented for those who wish to persue independent investigations. More complete calculations will be shown here later.

There is always more than one way of working a problem. The ways of transformation amongst the perspectives is the solution.

The solution contains a false accelertion term, with its basis being that the acceleation and velocity are not orthogonal.

The solution for the retarded location of an accelerated particle will need further investigation.

The point of zero doppler shift does not occur at the point of closest approach. The relativistic doppler equation is probably not a reliable guide for inferring the form of the acceleration terms for the retardaion equations, and conversely.

For that observer, a particle moving in th x-y plane is timeless at first. At first, the velocty of the charge has no effect on d phi/dt. The soluton might as well be for a charge that is at rest at the origin. But for the observers in th x and y diretions, d phi/dt is of first order. In some ways, d phi/dt behaves more like a vector than a scalar. It is implied that, in some ways, te static vector solutin in Ref. ... behaves like a tensor of the second rank.

The obserer in te +z direction could be us. We need two assistants in the +x and +y diretions if the solution is to be differentialbe with respect to x or y.

It seems that we did not know where the parcicle was in the first place. If the three observes make three independent calculaions for d phi/dt then they are for three differnt particles. That could be confusing if the three obervers believed that the solutins were all for the same particle.

We need an assistnat in the +y direction. But if the particle is movng paralleel to the x axis then the assistant has the same delima thar we have. d phi/dt is zero, but the Maxwell equaions are in terms of the second derivaives. For the observer at +y, if d phi/dt is zero the d^2phi/dt^2 is also zero.

The soluton is timeless at first. But for a third obgserver in the +x diretion, the motion is radial and the equaions are first order. Between the three of us, we can know more than any one alone.

However, the coordinates have to be communicated at the speed of light from one obsrver to the other, which imposes observational restrictions. Observational restrictions and computational restrictions do not have the same meaning.

These relationships can be misleading, beause it seems that one obserer shold be able to figure out precisely where the partticle is by simply waiting a while. Direct calclaion shows that the equations contain linear dependencies. The linear dependencies are easily misinterpreted as meaning that the particle is where we thought it was. The conclusion is logically circular when there is only one observer.

Both solutions are based on the Lorentz transform, but the doppler equation represents the derivatives, while the LW equatins represent the undifferentiated function.

Angular relationships in relation to ourselves have no meaning unless there are two other points. There are two other points when the location of the particle is represented in relation to the tip of the vector to the point of closest approach. $vv\times\hat{\boldsymbol{R}}$ is zero at the point of closest approach.

The particle is also on the light cone for us, but we are no longer alone. It is better that way, because angles in relationship to ouselves have no meaning.

By proceeding in this way, it is possilbe to work the retardaion probem for someone who believes that velocities are relative to them. It turns out that their assumption is correct.

Our eqtions are different because the partcicle is usually not on a collision course with us.

With the Newton equations, once the velocity terms become known, the acceleration terms are also known. These are not the Newton equations, but the solution for the velcity terms should be meaningful when the conditions are sufficiently mild.

The motion is radial for the third observer, and there is some point in the history of the particle when it was coincident with the observer. That provides a reference point for determining where the particle is at other times. The third observer knows where the particle is.

Unless the partice is on a collision course, there is always some point on the trajectory when Rv and vv are perpendcular. The particle is then at the point of closest approach. One observer can assume that the location is already known. The calculations will reaffirm the assumption in great detail, but they are for the wrong particle.

The vector points to an observer who knows the radial distance to the particle.

The other two observers could assume that they already know when the particle is at the point of closest apprach, and the calulations would reaffirm their assumption in great detail, but they would be for the wrong particle. Only one of the three observers knows accurately when the partile is at that point. Eq – is for the wrong partcle unless ru dot vu is +/- 1.

There are no terms in the solution that are identifiable with the transvers doppler equatio. The soluton is the same as it would be for a partcle at rest at that location.

The four dimensional space is adept at convincing us that we know more than we do. The behavior is not malicious. It prevents the equations assiciated with a tensor of a given rank from contradicting themselves.

The other two observers can assume that they know where the particle is, and the calculations will reafirm their assumptions in exquisite detail. However the solutions are for the wrong particle. The third observer is the only one who knows where the particle is. Eqs () are for the wrong particle unless vu dot ru is +/- 1. R is a scalar. From the perspective of three observers, it behaves more like a vector.

There are no transvers doppler terms for the other observer. The retardaion equaions are simpler for the other observer than they are for us.

We thought we knew where the particle was, but the other observer's calculations are simpler and more reliable than ours.

The retardaion equaions for the other observe are the LW equations, but the are not our equations.

These are the retardion equaions for the other observer. They are the LW equations. The other observer does not have to include the transverse doppler terms, so the equations are simpler than they are for us. These are our equations.

The partilce is on the light cone for us too, but the other observrs calculaion's are more accurate than ours. At the point of closest approach for us, R is at a quadradic minimum. The light cone equation does not impose a first order constraint on its location. At the point of closest apparoach for us, the particle is conincident with the other observer. The other observer knows where the particle is.

These are the retardation equatons for the other observer. They are the LW equations. They are simpler than our retardation equatons, because the other observer does not have to include the transverse doppler terms.

There are no terms that are identifiable with the transverse terms of the relativisc doppler equation[20]. The scalar solution is the same as for a particle at rest at the same location.

It would not be difficult to carry more powers of velocity in this solution, but it is doubful that very many powers are meaningful in periodic solutions without including the acceleration terms.

That may be because the calcullations in the reference dropped a term, but it is indicated that the Thomas equations are too egocentric. It is not possilbe to use the first derivative to detemne we are orbiting a partcile or if the particle is orbiting us. It could be either way, but the acceleration terms are not the same.

The distant observer is us.

According to the relativistic doppler equation[20], the doppler shift is not zero at the point of closest approach. There are no terms in Eq – that are identifiable with a transverse doppler term. The scalar solution is the same as for a particle at rest at that location.

In being at a quadradic minimum, the solution for R at time dt does not impose any new constraints. The constraints at time t+dt1+dt2 are adequate for computing the second time derivative.

We are one step behind, so the equations will require more terms than the Newton equations would.

Two points on the trajectory are enough for computng the Newtonian velocity. Three points are requird for the retarded velocity. We are always one step behind.

The acceleration terms will be needed in most practical calculatins, but the velocity terms do come first.

The dipole terms are the same as those of the LW equations. The LW solutions do not have other multipoles. In being the terms of an orthogonal serees, each of the vectors of the Newton series is in and independent direction.

This solution could be represented with the spherical vector harmonics[2].

The LW equations assume that the particle is at the origin when tf0=0. Each observer on the surface of the sphere is free to make the assumption, and there are no contradictions within it, but two widely spaced observers cannot make the same assumption at the same time.

There would be transverse terms in Eq () if the particle is not where the LW equations assume that it is. The light cone equation does not impose a first order constraint on the location of the particle when rv dot vv is zero. It is difficlut to be sure of exactly where the particle is then. There is a partiele at that point in space and time, and it is on the light cone, but it is difficult to be sure that it is the right particle. The right particle could be at the same place at a different time. We can tell the differnce, because there transverse terms in the solution if the LW equations are for the wrong particle.

One of the terms has been multiplied by T to tag the Thomas terms in all solutions. They would otherwise be difficut to recognize. The solutions are solutions to the Proca equations for any value of T. The LW solutions are obtained by setting T to 0. T cannot be a free parameter, but it is possible that these calculations are too limited to constrain its value. It will be carried as an undetermined coefficient until its value can be evaluated.

The acceleration terms will be required for the second infinitesimal step. In periodic solutions, it would not be meaningful to carry very many powers of velocity without also carrying the acceleration terms.

It would be better to subdivide the dt1 interval, but the avdot terms should be carried if that is done. The ax2 terms are in the same order as the avdot terms.

The Newton series is not orthogonal for light cone events. The ax2 terms are in the same order as the avdot terms. The ax3 terms are in the same order as the avddot terms.

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There is a connection between the a terms in the solution and the acceleration of the particle, but the connection is nebulous, because Newtonian accelerateion is not computable with two points on the trajectory.

The chain rule for differention would be required in the following calculaions if the time at the field point is taken as being the independent variable.

When integrating velocity in several steps, the acceleration and other terms are in the same order as the solution, so they cannot be negelected in practial problems, but the geometry of the velocity terms is a prerequisite for those calculatins.

It may be possible to view the constant as existing in the infinitesimal of 3-space. The question borders on the philosophical, so it will be deferred.

In being of very low order, the solution for an unaccelerated partcile will not be of much practical intrest, but it is the second term of the series. The first term is the Coulomb solution.

The second infitesimal step works in the same way as the first step if the coordinates are first-known at time ts0.

The second intitesimal step works in the same way as the first step if the coordintes are first-known at time dtf0. The chain rule for differentaion would otherwise be required for the second step.

The integral represents an infinity of infinitesimal steps. The second infinitesmail step follows the first step.

The acceleration terms will be required for the second infinitesimal step. In periodic solutions, there is no point in carrying very many velocity terms without the acceleration terms.

If two observers are in free fall, one behind the other, then each will perceive the other as being accelerated. There are relativistic corretions to the problem, but the gradient of Newtonian acceleration is a tensor of the second rank. It is easier to difference two vectors, but the tensor is still there.

The gradient of static Newtonian is a tensor of the second rank. It is easier to obtain the solutin for a nearby point by differencing two vectors, but that only works when a global vector solution is already known.

The diffrence between two closely spaced vectors is itself a vector, but that does not imply that the solution is always obtainable with vector equations.

From the perspective of an obsrver at the field point, the two points on the trajectory defined by the points rv and rv+drv define an angle. The two points defined by the terms rv and rv+vv dt also define an angle, but the calculation assumes that the time and space coordintes of light cone events are independent variables.

To first order, the acceleration of the particle does not affect its location at time t+dt. But computing the coordinates at time t+dt and rv + drv requires two steps unless the coordinates are orthogonal. At time t+dt, the light cone coordinates do depend on the accelertion of the particle.

The LW solution might be the right equation for the wrong particle. How would we know? Particles do not have name tags.

On the other hand, if the trajectory of the particle is known by independent means, then we do know which particle the solution is for, and the retardaion equations can be used to compute the derivatives.

The particle velocity in this series will eventually exceed *c*, so there are limitations to its applicability.

The LW solution could be the right equation for the wrong particle.

There will be additional terms in the solution when the retarded Newtonian accelertion is not zero. The solutons shown in the SOM are of too low an order to be of much practictial interest, but the velocity terms do come first.

The doppler frequency changes as the partcle moves, but the doppler equation has to work in the same way at any time, because we do not know what time it is. It is not sufficent that equations of physical signifcance be invariant. They must also not depend on the choice for a coordinate system. Coordinate depencies can exist in either space or time. This solution depends on when the time ts=0 (or tf=0) is chosen to be, so it is not of physical signifiance.

However, the tsx2 terms can be negtlected for observations of short duration, so the solution is a useful approximation in that regime, even though the v0x2 terms of the doppler equation are missing when ts0 is set to zero. The velocity of conduction electons in stationary copper wire is so low that the angular reationships do not change significanly for substantual times. Indeed, relativisstic corrections are undetetcable in those configurations, even at currents high enough to melt the wire. There is nothing wrong with using true vector equations when they provide satisfactory accuracy. The LW equations are a legitimate term of the retardaion series. The Coulomb soluton is the first term of the series, which is of satisfacotry accuracy in quasi-statice solutions when the magnetic field is weak enough that it can be neglected.

The series expansion for 1/(1+...) has powers of velocity in all orders. The terms have to be carried for reasons of mathemacial self consistency, but the additial terms do not improve the accuracy of Eq –.

There will probably be another constant of integratio in the solutions if the particle is accelerated, but it should be all right to negelect the accelertion terms in observations of short duration.

The particle velocity in this series expansion will eventually exceed *c*, so the equation is only valid for a short time. It is usable for a longer time if the particle velocity and acceleration are low.

If observations are made at two different points on the trajectory of a partcile then enough derivatives must be known that the trajector can be extrapolated to the other point in order to insure that both data sets are for the same particle.

We cannot be sure of how long it has been since the moment of creation, so equations of physical significance have to work without knowing what time it is.

That does not matter for the retardation equaions, as they are free of first order error. However, they exist only for the purpose of being differentiated twice.

Retardation equations have to work in the same way when applied at different locations and at different times. Unless the cosmos is evolving, it should not be necessary to know what time it is.

All things measurable are relative. It cannot be us that they are relative to, because we do not know where the particle was.

An assistant near the source could record the time that the particle passes the nearest clock in a field of synchronized clocks, but the particle would be elsewhere after the data point is transmitted at the speed of light to the field point. It is easy to assume that we know more than is knowable.

The particle not on the light cone when tf and ts are both zero. We cannot see where it is then. We do not know where it is yet.

This resut is not a contradiction, because we did not know where the particle was in the first place. An assistant near the source could record the time that the particle passed the nearest clock in a field of synchronied clocks, but that would not allow us to find out where it was any sooner unless superluminal communication is possible.

The location of the particle at time r0/c depends on when the observations are made. The light cone constraint is slippery. It is difficut to be sure which particle the solution is for.

The light cone constraint is slippery when rv dot vv is zero. There would be transverse terms in the solution if the particle is not exactly at the origin at the time tf-r0/c. Due to the lack of a first order constraint, it is possible for the particle to be at a slightly different location while still being on the light cone.

It is less obvious, but the quasi-static solutions were also lost, and they tend to be more accessible for laboratory investigations.

XXXVII. A COORDINATE DEPENDENCY

The solution is the LW solution at time tf0=0, but not for any other time. It should not be necessary to know what time it is in order to apply retardation equations.

This relationsbhip can be misleading, beause the derivation of the LW equations is unaffected by translating the coordintes in time. However, once a coordinate system is selected, it cannot be readjusted as needed.

The $1/r^2$ terms can be neglected at great distances, indicating that the solution is missng angular terms.

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Due to the difference between the behavior of approaching and receding particles, retardation equations do not exhibit time reversal symmetry. One of the consequences of the asymmtery is that the time of zero doppler shift occurs slightly before the point of closest approach.

making it possible to compute when the particle is at the point of closest approach. The time of zero doppler shift occurs slightly before the point of closest approach. consequently, the LW solution is the right equation for the wrong particle.

There are no relavistic coorrections when ru dot vu is zero. There are no terms that are identifiable with the transverse doppler effect.

It is just as likey that the particle is moving in the other direction. That solution can be obtained by substituting -vv for vv.

The particle is on the light cone, but it is not the same particle that the LW equations are for.

There are no relativistic corrections when $\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{v}}$ is zero. There are no terms in the solution that are identifiable with the transverse doppler effect[20].

These solutions will need further development when the $\boldsymbol{a}$ and $\dot{\boldsymbol{a}}$ terms are not small enough that they can be neglected. However, the velocity terms do come first.

Photons cannot travel backwards in time. It is possible to obtain retardation solutions using only forward time progression.

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The solution is the LW solution when tf=0, but not for any other time. The solution can be for any time if the coordiates are translated in time, but it is still only valid for one specific time, meaning that it cannot be differentiated in time.

In general, the gradient of a vector is a tensor of the second rank. It represents both symmetric and antismmetric terms[6]. The LW solutions do not have any symmetric terms, nor do true vector equations.

The equation is not form invariant for translations in time unless T is 1. Form invariance has about the same meaning as the requirement that equations of physical significance should not contain a coordinate dependency.

In practical applications, the missing $(v/c)^2$ relativistic corrections are so small that they can be neglected even at moderately high velocites.

In the frame of reference of the particle, the velocity is initially zero. Acclereration is the first term of the series in that frame frame of reference. Velocity is the first term from the perspective of a distant observer.

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For local equations, it does not matter whether the sun is orbiting us or we are orbiting the sun. It does matter in global solutions. The speed of light is locally invariant, but light rays travel slower in the gravitational field of a distant object. Potential equatins represent a global perspective.

There will be additional terms in the solutin for the second infinitesimal step. It follows from the tensor irreducibility theorm[6] that angular relationships cannot be fully reduced to first order.

We are always free to choose a coordinte system that is centered upon ourselves. However, unless the sun is orbiting us, the coordinate system is not portable.

The solutions for accelerated particles will need further development. However, from the perspective of a distant observer, acceleration is not the first term of the series.

There will be other terms in the solution when the particle is accelerated. The velocity terms are of lower order, so it should be all right to neglect the relatistic corrections for the accelertion terms when the orbital radius is large.

Eqs – and – are not necessarily for the same particle. We cannot be sure which particle is the right particle until a global solution is obtained. More derivatives would constrain the trajectory more accurately and make it easier to identify the right particle.

This solution looks like the LW equations, but it is not, because rv0 is not where the particle was.

In Eq –, when tf0=0 the particle is not at the origin. The LW solution appears to be the right equation for the wrong particle.

Retardation equations have to work in the same way when applied in different places and at different times. The solution become form-invariant if tf0 is set to ... .

Form invariance has about the same meaning as the requirement that equations of physical significnce should not depend on the choice for a coordinate system.

There are no assurances that the solution for accelerated particles can be obtained by differentiating this solution in the first frame of reference. But since the velocity terms are of lower order than the accelertion terms, it should be all right to neglect the reltivistice corrections for the accelertion terms when the orbital radius is large.

In Eq –, the events at the source and field point are simultaneous when tf0=0. In Eq –, they are not simultaneous unless T is zero. The solutions of the Lorentz transform generally do not exhibit absolute simultaneity[2,7].

We cannot know what is happening at the simultaneous point while it is happening. We can find out what happened there, but not until after waiting a while. There is no obvious reason for assuming that the location of the particle is knowable before working the problem. The LW solution appears to be the right equation for the wrong particle.

In Ref. –, the first time derivative of the retarded potentials was computed in a manner that directly parallels the derivation of the Thomas precession[7]. The first time derivative was then integrated in time for the dipole and current loop antennas. The method of calculation results in the loss of the static solutions. It is less obvious, but the quasi-static solutions were also lost. To order r0x3, the solutions to Eq – are idential to those solutions, but they include the static terms.

The solutions for non-degenerate accelertion terms might nevertheless be more interesting in the laboratory.

The light cone constraint is slippery. There is a small family of particles that is on the light cone at the time tf=r0/c. It is not yet clear which particle is the right particle.

Using the relativistic doppler equation[20] as a guide, a plausible form for the retarded location of the particle is

vv is the Newtonian velocity, but the measured velocity of the particle cannot exceed c with this equation. In this context, the Newtonion velocity is a mathematical abstraction.

Two of the terms have been multiplied by T to tag the terms that represent the Thomas precession. T should be set to 1 in the final solution. The solution reduces to the LW solution when T is 0. It would not be difficult to carry more poweres of velocity in the solution, but not many powers would be meaningful in periodic solutions whithout including the acceleration terms.

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The first infitesimal step

Curvature in the infinitesimal is not representable in our frame of reference. The retarded potentials are first-known in the other frame of reference. Ours is not the one that matters. The retarded potentials could have originated long ago at a distant place. They would exist if we were not even here.

But also, the source for the future retarded potentials for distant observer could be a charge that is at rest in our frame of reference. The advanced potentials would be needed for those solutions.

In representing dts/dtf, the doppler equation is in differential form. The retarded potentials represent an integral. Even though both eqations are obtained from the Lorentz transform, it is conceivable that the potential solution would have to be differntiated in a suitable manner and then be integrated to obtain a commensurate solution. A constant of integration could affect the potential solution a the point on the trajectorey where ru dot vu is zero, even though the constant has no role in the diffenential solution.

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Integration in the infitnitesimal degnerates to multiplication, which is basis of the Newton equation dv=av dt. From the perspective of the other observer, dtf is not an infitesimal quantity. The additional velocity of an accelerated particle acquired at the time dtf/2 contributes to the light cone solution at the time dtf. The equation is not Newtonian. The solution would have to be obtained by integration.

As discssd in §–, angular velocity and acceleration are not instantly separable with delayed observational data.

Repeating the calculaton with more steps shows that two steps are sufficient to reduce velocity to first order unless the vx4 terms are carried. The first infitesimal step is much smaller than it seems to be in our frame of reference. But then, our frame of reference is not really special or different. It only seems that way.

The avdot terms integrate to acceleration terms, which inegrate to velocity terms, but it should be all right to neglect the avdot terms when the orbital radius is large.

dtf has not dropped out, showing that the LW equations do not work in the same way at time t+dt as they do at time t. We cannot tell the difference between the two times.

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The LW equations do not look the same at time t+dt as they do at time t. They are not form invariant for translations in time. Form invariance has about the same meaning as the requirement that the equations should not contain a coordinate dependency.

The gradient of a vector is a tensor of the second rank, which represents both symmetric and antisymmetric terms. The LW solutions do not have any symmetric terms. Retardation equations that are form invariant just long enough to compute the first derivative would be useful.

There will be additional terms in the soltion if the acceleration terms are carried. However, the velocity terms do come first.

The Proca equations would allow other values for T if more refined calculations show that it is not 1.

The Proca equations reduce to the Maxwell equations when T is 0. The additional vx3 terms of the Proca equaions are so small in relation to the first order terms that they can be neglected even at moderatly high velocities. The LW equations appear to be the correctly determed retardation equaions for the tensor of the first rank, a vector. There are no symmetric terms in true vector equatons. The Lorentz condition is nevertheless a symmetric vector equation.

Since the gradient of a vector is a tensor of the second rank, it is not mathematiclly possible to differentiate vector equations unless the symmetric terms are small enough that they can be neglected.

As discussed in §– and –, we cannot tell whether the tagged terms in this solution are for an accelerated particle or for a constant veocity particle with a transverse velocity component until a global soution is obtained. It is a conventional interpretataion that potential equations exist only for the purpose of computational convenience. They do not represent locally measurable physical quantites.

The solution reduces to the LW solution when av and vv are prallel or antiparallel. The solution represents the integral of the Thomas precession in potential form. The solution will be much more elaborate if the $\dot{\boldsymbol{a}}$ terms are carried. The a^2 terms are in the same order as the $\dot{\boldsymbol{a}}$ terms.

From the pespectivew of the other observer, delta Rv is an infitesimal quantity.

The mathematical meaning of the infitisimal is that the equations are linearly dependent if it is subdivided. delta tf behaves like an infitisaml quantity in our frame of reference, so it can be written as dt. Eqs – become

We are always one step behind, so the av terms of the solution will need further development. Beyond that, the ax2 terms are usually in the same order as the avdot terms when the equations are not Newtonian.

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ts1 has dropped out, indicating that the solution is valid for any value of ts1, including the time dts1=0.

We have no way of knowing what time it is. The equations have to work in the same way for a range of times if they are to be differentited in time.

Acceleration has dropped out of the solution, but that does not necessarily mean that the particle is not acceleerated. Three points on the trajectory are required to datermine if it is accelerated. The solution is for two points. The equatioan is ambiguous until a global solution is obtained.

The equation could be extended to three points, but we would still be one step behind.

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Due to the nonlinerity of the light cone equation, the solution for the sum of two velocites is generally not in the same direction as the vector sum of the velocites. The additional velocity that an accelerated partillce acquires at the time dt counts as a second velocity. Since the av0 dt and vv0 terms do not add as vectors, the light cone solution cannot be obtained by the methods of vector analysis.

The Lorentz transform has the same characteristic, regardless of whether or not the events are on the light cone. The more general solutions are usually better suited to kinematic calculations.

The calculaions of the Thomas precesion assume that the trajector is Newtonian at time dts1+dts2. It is not. The velocity is initially zero in the frame of reference of the particle. The veclocity at time dt is av dt, and there are no relativistc corrections that are first order in v. There are corrections at time dt1+dt2. They are small, but they accumulate as the integration proceeds.

The acceleration and velocity terms can be orthogonalized by solving the light cone separately for each of them. The equations still contain quadradic terms, but truncated polynmials are nevertheless linear equations.

The accelreration terms have dopped out, but thy will reappear after the solution is differntiated to obtain the fields.

In being the terms of a Taylor series, the terms of the Newton series are mutually orthogonal. rv is a vecor, av dt is a vector, and 1/2 jv deltx2 is still a vector. The cross terms in light cone solutins cause the orthogonality to be lost. Since velocities do not add as vectors, the methods of vector analysis cannot be used to obtain light cone solutions. The terms of the Newton series have to be processed sequentially. Each term in the series is a vector, but only when it is considered in isolation.

The lack of orthogonalty can be viewed another way. The chain rule for diffrentaion is required when the varialbes are not indepedent, and the time and space coordinates are strongly coupled in light cone solutions. The chain rule is not needed if the terms are orthogonalized first.

The tagged velocity terms can be velocity terms, or they can be integrated acceleration terms. A longer observation would be required to determine which is the case.

Fortuously, ts0 has dropped out of the solution. It is essential that retardation equations work this way. The equations would not be usable if we had to know what time it is in order apply them. Other equations may be valid, but they are not retardation equations.

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ts0 has dropped out of the soluton. It is no longer necessary to know when ts0=0 was. Retardaion equation have to work this way, because we have no way of knowing when the time ts0=0 occurred.

We do not know yet if the vx1 terms reparesnt velocity or the integral of acceleration.

av has dropped out of the solution. It does matter if the particle is accelerated, but we cannot quickly determine if it is.

Potential solutions do not represent locally measuable relationships. They repreent a global perespective. A global perspective cannot be acquired quicky by one oberver. If it is obtained by a field of observers then the propagion delay in commuicatimg the data points from one observer to another is part of the problem.

... This is a vector equaion, but it is one that is alien to the Newton equations.

According to the multivariate Taylor theorm, the vx3 v1 terms are in the same order as the vx4 terms, so they have been dropped.

The perspective of field equations is different. The chain rule for differentiaon is required when the variables are not independent.

dt has dropped out of the solution. Retardaion equations have to work this way, because we have no way of knowing when the time t=0 should be.

The LW equations also behave properly in this respect. However, the gradient of a vector is a tensor of the second rank. The LW solutons do not have any symmetric terms. Other than that, there is nothing wrong with them. Vector calculations will always reaffirm that the LW equations are the right equations, and they are.

Both particles are on the light cone, but it is not possible to dermine which particle is the right particle with the first derivatvie.

The quanty ... in this solution is the same as in Eq –, but the solution is not for the same particle. The vector is rotated. A rotation does not affect the magnitude of a vector.

We do not know yet if we are orbiting the particle or if the particle is orbiting us. We would know in a global solution.

The potentials are the integral of the fields. They represent a global perspective. They do not represent locally measurable relationships.

The speed of light is invariant, yet photons travel slower through the gravitaional field of a distant galaxy. Local and global perspectives are not interchangable.

Since velocites do not add as vectors, the methods of vector analysis are not necessarily reliable for light cone calculations.

They are small, but they accumulate when integrating in time. They can be avoided by transforming to a fourth frame of refence with the velocty av dts. The paticle is initally at rest at time dt in that system, then another acceleration term can be appended.

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In the first frame of reference, the dtf interval is an infitesimal quantity. An infitesimal quantity can be subdivided, but it does not accomplish anything, because the equations are linearly dependent.

whan a particle is apporaching us at high velocity, the elapsed time at the particle in a light cone solution can be arbitrarily greater than the elapsed time at the field point. There is ample time for the acceleration of the particle to influence the first derivative. From the perspective of the other observer, dtf is still free of first order error for velocity terms, but not for acceleration terms.

There is more than one way of working a problem. The chain rule for differentiation is required when the variables are not independent. The time and space coordinates are strongly coupled in light cone solutions.

We cannot determine whether the terms in the solution represent velocity or the integral of acceleration until a global solution is obtained. The terms are distinguishable when the time $t = 0$ is known. With retardaion equations, we have no way of knowing when the time $t = 0$ should be.

The terms in this solution can represent either velocity or the integral of acceleration. When the time t=0 is known, the diffrence is distinguishable. Retardaion equations do not contain time, so it is not possilbe to determinee which terms are accelertion terms until a global solution is obtained.

It would be dimensionally possible for retardaion equains to contain av or avdot terms in some order. The solutons for accelerated particles will need further investigation.

the other observer would be inappropriate.

The LW equations apper to be the properly determined retardaion equations for the tensor of the first rank, a vector. They are usually the only retardation equations needed. However, the tensor of each rank is irreducible citegelfand.

The gradiant of a vector is a tensor of the second rank, which includes both symmetric and antisymmtric terms. The solutions to the LW equations do not have any symmetric terms.

The light cone equation does not not impose a first order constraint on the transverse location of a particle, so it is not very important for us. It is important for a different observer. The other observer could be us next time.

dr has dropped out, indication that the solution is valid for both us and a nearby observer.

The difference between two nearby vectors is itself a vector. But in general, the two vectors are connected by a tensor of the second rank, which includes both symmetric and antisymmetric terms. The Lorentz condition, ... , is a symmetric vector equation.

There is a radial component for other observers. The light cone equation is not as slippery for them, so they can perform a more accurate calculation than we can. Since we did not care about the precise transverse location of the particle in the first place, it is all right for us to use the more precise value provided by other observers.

dr has dropped out of the solution. If it is the right equation for the other observer, then it is also our equation, because it would remain valid when dr=0.

For a short time, the orbit can be approximated by a straight line.

The approximations used in the following calcultions restrict the solutions to the extrior region of a long magnetic solenoid or charged sphere. The interior solutions need further investigation. There is a possibilty that the LW equations are the right equations for the interior region.

This solution is only valid for a short time.

For a short time, the velocity of the barycenter, relative to ourselves, is

As the solution shows, our velocity does not matter – unless we wait a while and check again.

It is possible to interpret the location of the other observer as being at the barycenter. If so, we cannot tell whether we are orbiting a local barycenter or a distant particle is orbiting its barycenter. We know how to observe the difference between the two cases, so it matters. Including acceleration terms in the calculation should help in resolving the ambiguity.

Since the equation works for all observers except us, it should work for us too.

The opinion of another observer would be helpful in refining the transverse location of the particle.

All observers except one think they know where the particle is.

All of the other observers provide the same solution, so it must be our solution too.

The other observes are all in agreement. Since we were not sure of the transverse loction in the first place, it is appropriate to use their calculation.

When the time t=0 is known, velocity is the integral of acceleration. It is easy to detrmine which is which. Retardation equations are timeless. We cannot tell which terms are acceleration terms until a global solution is obtained. Then we will know if the particle is orbiting us, or if we are orbiting the particle. Until then, we are not sure. We know how to observe the differnce, so it does matter, but potential equations do not represent locally measurable relationships.

The speed of light is invariant, yet photons travel slower through the gravitational field of a distant galaxy. Local and global perspectives are not interchangable.

In some sense, the LW equations appear to be the properly determined retardation equations for accelerated charged particles. However, as discussed in §–, four points on a trajectory are required to compute the retarded acceleration.

Since the solution does not depend on where the other observer is, it should work for us too.

We know how to observe the difference, so it does matter, but potential equations do not represent locally measurable relationships. There are limitations to how much they can reveal of a system. Including acceleration terms in the calculation may help in resolving the ambituity.

The retarded potentals depend on both the radial velocity and angular relationship. There are no angular relationships for one observer and one partile. There are angular relationships for the same partile at two times, but connecting the events requires assumptions. When ru dot vv is zero, the light cone equation does not impose a first order constraint on the transverse location of the particle. Applying the ligth cone equation to the same particle at two closely spaced times therefor does not adequately constrain the angular relationships.

The solution does not contain acceleration terms, so we cannot tell whether we are orbiting the particle or the particle is orbiting us. We would know the difference in a global solution.

The angular relationships are better constrained if they are represented relative to the particle's barycenter rather than relative to us. There is still some slipperyness, especially in the near field solutions.

r0 is the radius of curvature, which is not necessarily constant throughout the orbit.

This calculation is similar to the calculations of the Thomnas precesssion[7]. The difference is that the angles in the solution are relative to the barycenter, while in the Thomas solution they are relative to us. However, as shown in Ref –, differentinat the Thomas equations in time, then integrating in time, provides the same solutions as the solutions to these equaions. There is evidently nothong wrong with reprasint angles relative to ourselves, but the solutions are harder to obtain. But since the solutions are the same either way, the precessional terms of these solutons clearly represent the Thomas precession from a different perspectie.

Differentiating the Thomas equaions twice in time and then integrating twice should provide more accurate solutions, so this inconsistency is not necessarily fundamental. It could also be due to an oversight in the calculainss in the reference. In any case, it is the transformations amongst the perspectives that matters, not which perspective is the right one, so the inconsistency needs further investigaion. It also needs to be kept in mind that there is no known way of integrating around a circle with one linear equation unless it contains π.

The angle from the barycenter to the location of the particle at times t and d+dt is larger than the angle from the observer to the particle. The θ^2 terms drop out of the solution for a distant observer, but not for the barycenter angle.

The θ^2 terms do affect the time derivatives of the radiative terms if the equations of the Thomas precession[7] are used in the calculations. There is a possiblity that the second time derivatvie of the Thomas precession, after being intgegrated twice, will be in better agreement with the dipole solutons obtained here, but the calculains have not been carried through.

There does not appear to be anything wrong with the LW equations when the angles are small enough that the θ^2 terms and their derivatives are not needed, which is always the case for stationary current carrying wires. There might be exceptions for semiconductor fabrication methods.

The magnetic field of a static current loop is of order r0x2, so it is not representable with the solutons of this order. Other terms are representable, and more complete solutions are shown in the SOM.

When integratig around a unit circle in 3-space, the first infitesimal step is

Eq – contains r0 and dts rather than x and dx, but its form is similar to a 3-space rotation. 3-space rotations are not representable with linear equations when they are parameterized by x and y. They are linear when θ is the independent variable, which is sugestive of the form for the second infinitesimal step.

The distant observer did not know where the particle was in the first place, so there are not contraditions in that respect.

The solution is interpretable as meaning that this is not the right way to derive the LW equaions.

The origin of the coordinte system is at the barycenter, but a differnt choice would not affect the solution.

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The light cone equations does not impose a first order constraint on the transverse location of a partile whe ru dot vv is zero. It is not possilbe to determine precisely where the particle was. The radius vector could be rotated to an infitesimally different angle and it would not affect a first order solution.

From the persective of an observer at the center of a small orbit, the angle to two points on the trajectory is larger than the angle to the same two points from the persepctive of a distant observer. The thetax2 terms are important, while they drop out of the solution for a distant observer. That would not matter if the thetax2 terms were not radiative, but they are.

There is still some slipperyness in low order constraining reltionships for an observer at the barycenter.

The Thomas precession looks different from the perspective of an observer at the barycenter, but it is the same solution from a different perspective.

The solution reduces to the LW equations if the ax2 and j0 terms are not carried. They appear to be the correctly detrmined retardaion equations for accelerated particles, except that the distant observer is always one step behind.

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But since dtf is not an independent variable from the perspective of the observer at the barycenter, the quadradic terms can also be captured by subdividing it and then integrating over the interval. From the perspective of the other observer, dtf behaves like a macroscipic quantity in light cone solutions, no matter how small it is.

While the observr at the barycenter is in a preferred location, it should not be necessary to know where the barycenter is to apply retardaion equations.

There are mathematical similarites between 3-space rotaions and light cone solutions, so this constant might also exist in 3-space. The constant may be known, but it is difficult to reliably identify it with this method of culaion. there is also the possibity that 4-space constants do not correspnd exactly to 3-space constants.

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An infintesmal rotaion of the radius vector maps into a shorter section of the trajectory for an observer at the barycenter, so the other observer knows more accuratly where the particle was, although there is still some slipperyness in the equations.

The actual location of the particle is not computable this way. In applying retardaion equations, the location of the particle is assumed to be already known, then the derivatives are computed. On the other hand, the derivatives are integrated to obtain the trajectory of a particle. The perspectives are very different.

After differentiating the soluions to obtain the fields, they are for jerked partices. The LW equations are for accelerated particles. There does not appear to be anything wrong with them when the $\dot{\boldsymbol{a}}$ terms can be neglected, which is usually the case.

The location of the barycenter frequently needs to be known in order to formulate the retardaion problem, but the retarded potentials themselves should not depend on anything except the location of the partcle and its derivtives.

This calculation reduces to the LW equations if the avdot terms are assumed to be zero. The LW equations are the retardaion equatons for an acceleratd partile. There does not appear to be anythign wrong with them when the avdot terms are small enough that they can be negected.

Perhaps surprsingly, the first rotation is included in the LW equations, even though it is not visible in the first frame of referene. In the frame of reference of the particle, it acqires the velocity dv=av dt at time dt. There are no relativistic corrections that are first order in velocity. The equations are vector equations. There are relativstic corrections at time dt1+dt2 and the symmetric terms become important.

The symmetric terms are not included in the calculaions of the Thomas pression until they are differentiated. The avdot terms then become important. The avdot terms are small at first, but they grow faster than lower order terms when integrating in time. The avdot terms serve no purpose in the solutions for free particles when the speed of light does not matter.

While the Thomas equations themselves do not include the symmetric terms of the second infinitesimal step, they do appear in differential form after the equations are differentiated in time. Since the 4-potential transforms in the same way as the coordinates, the same equations apply to the retarded potentials.

That does not affect the location of an accelerted particle at time t+dt, but two points on the trajectory are requieed for computing both the locateion and velocity at time t+dt. The avdot terms have to be carried for computing the location of the third point, which has the effect of rotating the velocity vector at time t+dt. The rotation drops out of the solution if the obserever at the barycenter is not included in the problem.

The curvature is not visible for the distant observer until the solution is differntiated. However, the curvature is visible to the disamt observer if the loation of the partile at time deltfx2 is computed. Curvatrue is not representable with two points on the trajectory, even when the particle is accelerated.

The observer at the barycenter can therefore subdivide the dtf interval, making it possilbe to obtain a more accuracte estimate of where the particle was, although there is still some slipperyness in the equations.

With these selections, the radius vector rv1=rvf+rv0 extends directly from the distant observer to the particle, which is on the light cone. The particle appears to be approaching the distant observer at the point of closest approach, which is a contradiction. From the prespective of a second observer closer to the particle, that is as it should be, because the more distant observe perceives the particle earlier in its history, when ru1 dot vv was not zero and there was a radial velocity component. Due to the slipperyness of the light cone equation, the distant obeserver's calculations are for the wrong particle.

Two closely spaced events are required to represent the period of a doppler shifted monochromatic signal for one observer, which is the time aspect of the problem. The solution for two observers and one event is the space aspect of the same problem. The time and space aspects are connected by the light cone equation.

When the doppler shift is constant, the coordinates of one event, relative to the observer, are sufficient to define the doppler shift. Other points on the trajectory are linearly dependent. Two nearby events are required when the motion is not radial. When the transverse velocity is low, the solution for one event, applied independently at various points on the trajectory, is usually close enough, but the solutions are not all for the same particle.

There are additinal considerations for orbiting particles or orbiting observers.

An isolated and unaccelerated inertial particle would be half way to the destination in half of the travel time. Light cone solutions for one observer and one particle are for ghost particles unless other constraints are applied.

The distant observer should be able to obtain the solution without knowing the value of rv0.

The light cone soluton is suprficially valid forever. As a starting point, a solution that is valid just long enough to compute the first derivative would be useful.

dtf has dropped out of the solution, indicating that the solutin is valid just long enough to compute the first derivative. The solutions can be differentiated more than once, however the derivatives will be missing terms unless more events are considered in deriving the equations. r0 has also dropped out, showing that it does not matter where the other observer is if r0 is small.

Light cone solutions are for ghost particles until the transverse terms are better constrained. The acceleration terms in the equations are important, but the velocity terms do come first.

This degree of freedom is less visible in Cartesian coordiantes unless one of the coordintes is parallel to the retarded trajectory. If the solution is for the wrong particle, rotating the coordinate system only distributes the error amogntst the three coordintes.

It may seem that we have always known where the partile is at the simulteneous point. When the speed of light matters, we cannot know what is happenging there while it is happening. It is necessary to obsevere the trajectory for a while then extrapolate to the simultaneous point. The inverse calculaion assumes that the solution is known before working the problem.

The rotatin does not corrupt the distant obervers calculaions. The distant observer did not know where the particle was in the first place. Actually, the second observer does not know either, but the second obserevere is closer to the problem.

The distant observer can know the true transverse velocity of the partile, but not quickly with the light cone equation. The light cone equation is too slippery for calculatins that are both accurate and of short duration. For observtionso of short duration, the distant observer should rely on the more accurate calculations of an observer closer to the particle. The second observer does not know exactly where the particle was either, but the estimate is better.

While the radius vector should be rotated, and there would be some magnitude change in the rotation, it can be seen in Eq – below that the rotatin would have no first order effet on the scaler potential in the second frame of reference. The retarded potentials also depend on the velocity of the particle.

k has dropped out of the solution, demonatrating that the location of the particle is not adequtly constrained. A distant observer can know where the particle was, but not with light cone calcutions for short intervals. It takes a while to determine precisely where the particle was.

The location of the barycenter has dropped out of the solution, but its location does have to be known when applying the equations. The acceleration terms of the solution will need further development.

Ligth cone solutions are not necessarily for inertial particles.

The light cone solution of the distant observer is for a small family of particles. The distant observer needs to select a different particle from the family in such a way that the calculations of both observers are for the same particle.

The light cone solution of the distant observer is for a particle that defies the methods of reason.

There are additional considertions if the particle is in an orbit. It could be orbiting the observer. Probably not, but we would know if it is.

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r0 is zero when there is only one observer in the problem. The radial velocity of the particle is then zero at the point of closest approach to the observer, which is an agreeable conclusion. However, from the perspective of the second observer, the particle is approaching the distant observer at the point of closest approach. That is because the more distant observer perceives the particle earlier in its history when ru dot vu was not zero and there was a radial velocity component.

The distant observer could simply ignore the other observer, but there is another problem. The equations have to be for the same particle when applied at different time. The solution for two observers and one event or one observer and two events are not independent. They are different perpectives of the same problem.

It seems that equations that work for two obsevers and one event should also work for one observer and two events. More than two events would be needed if the equations are to be non-degenerately differentiated more than once.

From the perspective of the distant observer, the magnitude of ts1 is inconsequential. From the other perspective, it goes to infinity as r0 goes to zero. The solution for r0=0 is a removable singularity.

The distant observer has a second problem, one that is harder to see. The equations have to be for the same particle when applied at a different time.

When the speed of light matters, we cannot know what is happening at the simultaneous point while it is happening. One aspect of the problem is to determine precisely where the partile is when it is at a point on its trajectory where we cannot see it. Contradictions will arise if we assume that we know the answer before working the problem.

It is not yet possible to be sure of precisely which particle the solution is for. We would know its idendtity if it had idenfying marks, but small particles do not have them. One more step helps in minimizing the ambiguites of the problem, because it is necessary to observe the trajectory for a longer time to compute another derivative. We will eventually know which particle the equations are for, but not until a global solution is obtained.

The other terms can be neglected when the orbital radius is large, however the av and avdot terms are in the same order as the velocity terms, so they do need to be carried in the solutions for small orbits.

***

The advanced potentials in the frame of reference of the particle are the retarded potentials in the frame of refernce of the field point. In the second frame of refernce, the scalar potential at the field point does not depend on when the field point will be at that point. It only depends on where it will be. The light cone equation is only needed for computing the derivatives, but the retarded potentials exsit only for the purpose of being differentiated. They represent the integral of the derivative. the dtf interveal can be subdivied for the purpose of computing the derivatives, so it is appropriate to subdivde it for the purpose of computing the integral. The first derivative becomes free of error as the interval becomes small, but that does not imply that the equation has been reduced to first order.

Since our frame of refence is not special or different, the Thomas precession also exists in our frame of reference, although from a different perspective. The – term in the solution is closely related to the Thomas precession.

An equaton can be reduced to a system of several first order equations[2], but not in one step. For example two equatons are required to integrate around a circle. It cannot be done with one linear equation. The two linear equatons for integrating around a circular Newtonian orbit are .. .

It may seem possible to integrate around a circle with one linear equation if it contains π, but that is only because the 3-space integral was discovered long ago. Obtaining the integral in 4-space is more difficult.

Integration in the infinitesimal degenerates to multiplication, so there are no curvature terms in the infitesimal, but that does not imply that curvature can be negelected in the double integral of acceleartion. The spinning coordinates of the Thomas precession are for a single integral. That is not the right way to integrate around a circle unless the equation contains pi.

In 3+1 space, the two linear equations for integrating around a circular orbit are the Newton equatons, rv(n+1)=r(n)+vv(n) dt and vv(n+1)=vv(n) av dt. The $\dot{\boldsymbol{a}}$ terms are degenerate in 3+1 space.

****

dtf has dropped out of the solution. If the soluton is valid for the time t+dtf then is is also valid for the time t+0.

Even though dtf has dropped out, the solution is for one observer and two nearby events. The two events can be reconstructed by extrapolten the vector potential to a nearby point with the equation Av = A0+d A0/dt dt, which is sufficint for computing the first derivative. The second derivative will be missing terms unless the solution is for three events.

From a different perspective, the gradient of a vector is a tensor of the second rank, which represnts both symmetric and antisymmetric terms. A vector cannot be extrapolated without the chain rule unless the equations contain symmetric terms. The Lorentz condirion, ,,, is a symmetric vector equation but it is always zero the the LW solutions.

The gradient of the Lorentz conditon is another vector, howver the tensor of the third rank represents the second derivatives[6]. The third vector in the solution for two events is evidently incomplete.

drv and dtf have dropped out. The solution is valid just long enough to compute the first derivative in either space or time.

In the first frame of reference, the time and space coordinates are orthogonal, making the chain rule for differentianon unnecessary. The time and space derivatives can be computeed separately and independenlty. The retarded potentials are first-known in the second frame of refence.

Actually, our frame of refernce is neither special nor different, but it is mathematically convenient to work in a projection into 3+1 space.

The solution is for two events. The solution for one event can be obtained by setting dtf to 0 in Eq –. The solution for one event cannot be differentiated in the first frame of reference without applying the chain rule for differetntiaon. The LW solution is for one event.

XXXVIII. THE INCOMPLETENESS OF THE LW EQUATIONS

Being incomplete and being wrong do not have the same meaning. The equations for one observer and one event are missing terms, but they are not wrong.

The relativistic doppler equaion[20] seems to be for one event, but in being in differential form, it is actually for two events. It represents the transformation of the time interval dts at the source to the interval dtf at the field point. When the function is a straight line, the drivative is a constant, but the solution is still for two points, allowing the equations to seem simpler than they are. The actual reltainship is that integration in the infitesimal degenerates to multiplication. There are no curvature terms in the infinitesimal, but that does not imply that curvature can be neglected in the integral. The doppler frequency is approximately constant for a short time, allowing the doppler equation to be a good approximation for a short time. But for long times, the integral of the doppler frequency is for the wrong particle. The integral has to be of the total differential. The chain rule for differentiation is required for computing the total differential unless the variables are independent.

The solution reduces to the LW equations when the avdot terms are small enough that they can be neglected. They appear to be the correctly detrermined retardation equations for accelerated particles.

The avddot terms will be needed when integrating for a longer time. They are small at first, but they grow faster than lower order terms. The ax3 and avx2 terms are usually in the same orde as the avddot terms.

Since our frame of reference is not special or different, it should not matter which frame of reference is used for the calculations. The equations of the rotaions of the Lorentz group are more general than the Lorentz transform, so they may be relevant.

Due to the length of the calculations for jerked particles, they are only shown in the supplemental material. The following calculations are for accelerated particles. The retardaion equations of that order are the LW equations. The calculations for a jerked particle proceed in the same way, except that the light cone solution for the time t+dt1+dt2 is needed.

The zeroth order solution is not radiative, but there are eleborate near field velocity-dependent terms at the partice's closest approach to the field pont. The solution shown in the SOM. The same solution is obtainable as a special case with the LW equations.

The LW solution is only valid for an instant if the particle is accelerated. This soluion is only valid for a short time, but it is more than an instant.

dtf0 has dropped out of the solution, so it is not necessary to know when the time t=0 was. This behavior is the signature of a retardation equation. They can be applied without knowing what time it is. They are form invaraint for translations in time. However, the solution is only form invariant long enough to compute the first derivative. In the light time accross the system, avdot terms would inegrate to ave terms, which would have to be intetrated twice again to obtain the location of the particle. The partcile would be elsewhere if the avdot terms were significant. Extrapolating with the Taylor theorm is equivalent to integrating.

This solution could be obtained from a different persepcteve. If the retarded location of the particele is assumed to be already known at time t, then the total differential would be needed when integrating to a different time, even when the different time is t+dt. The chain rule for differentiaon is required for computing the total differential unless the variables are independent.

dtf is an infitsimal quantity in the frame of referene of the field point. dts/dtf can be arbitrarily large, so dts in the same solution is not an infitesimal quantity if the particle is accelerated. From the perspective of the other observer, it is necessary to integrate over an infinitesimal interval. Integration in the infitesimal degenerates to multiplication. There are no curvature terms in the infiteesimal, but that does not imply that curvature can be negleted in the double integral of accelertion.

In applying retardatin equations, the location of the particle is assumed to be already known, then the the derivatves are computed. In orbital calculations, the derivatives are integrated to obtain the location of the particle. The perspectives are very different.

dt has dropped out of the solution. That is the signature of a retardation equation. They are form invariant for translations in time. They work in the same way at time t+dt as they do at time t+0. The LW equations have the same characteristic, but they are for one obsever and one event. There are two events in the solutions for times t and t+dt. There are three events in the solution for the time t+dt1+dt2, but the solution for three events can be obtained by integrating the solution for two events unless it is missing terms. It is nearly impossilbe to show that an equation is not missing terms, especially in a cosmologial context, but the four points on the trajectory do constrain the slipperyness of the light cone equation better than three points can.

The location of the barycenterr has dropped out of the solution. It is nevertheless necessary to know where it is when applying the retardaion equations. As shown in the SOM, the solution reduces to the LW solutoin when the observer is at the barycenter. That location would be valid at the center of a current loop, but not for nearby points, making it impossible to compute the space derivatives.

The angular velocity of the particle depends on the retarded acceleration. Angular velocity and acceleration are uncoupled with the Newton equations.

Vector equations do not have symmetric terms. The second infinitesimal step does not work in the same way as the first step unless the equation is Newtonian.

From the perspective of the other observer, the particle is approaching the more distant observer. The two observers are not performing calculations for the same particle.

An isolated and unaccelerted inertial particle would be half way to the detination in half of the travel time. At least one of the three solutions is for the wrong particle.

Similarly, while the LW equations do not have to be for an accelerated particle, they can be.

The distant observer needs assistance in determining precisely which particle is the right one.

XXXIX. RIGHT EQUATION, WRONG PARTICLE

From the perspective of the distant observer, the radial velocity is zero at the point of closest approach, which is in accord with the methods of reason. However, the magnitude of R is at a quadradic mimimum at that point. The distant observer does not know precisely when the particle is at the point of closest approach. The solution is actually for a small family of particles.

Integration in the infitresimal degenerates to multiplication. When the equations do not behave that way, it means that the integral was not what it was thought to be.

If subdividing the infetessimal makes any diffrence at all, it does not mean that there is anything wrong with the theorms of the calculus. It means that the equations were not of first order in the first place.

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An equation can be reduced to a system of several first order equations[2], but not in one step. For example, The nonlinear Newton equation r = r0 + v0 t + 1/2 av tx2 can be reduced to the two linear equations ... . Numerically integrating with the two equations is equivalelnt to evluating the double integral int(int()).

Reducing 4-space acceleration to first order is more difficult. The second infitesimal step does not work in the same way as the first step.

An equation has to be free of first order error in each infinitsimal step before it can be integrated. It is not sufficient that it be free of error in the first step. The spinning coordinates of the Thomas precession [] assume that 4-space acceleration can be reduced to first order with two linear equaions. That is not true[8].

The pivot point for the Thomas rotation is the same as the origin of the coordinate system, which can be anywhere. The difference between a particle orbiting us and us orbiting a particle is important, but the Thomas calculations do not distinguish between the two cases.

... The coordinates in the last frame of refrence are the same as the were in the first. The 4-potential transforms in the same way as the coordinates. A potential is not a true potential unless the loop integral is zero.

In Eq –, there are no relativistic corrections when rv dot vv is zero. The scalar solution is the same as for a particle at reset at the same retarded disance. There are no terms that are identifialble with the transverse doppler effect[20] Both solutions are based on the Lorentz transform, but in representing dts/dtf, the doppler equaion is in differntial form, while the potential equaion represents an undifferentited function. It would be necessary to integrate the doppler equation and supply a constant of integration in order to directly compare the two equaions, but it does seem that there should be relativistic corrections.

For the observer at the field point, the same equation has to work for any particle. For an observer in the frame of refernce of the particle, the same equaion has to work for many observers at various locations, in which case the time at the field point is not an independent variable. From the perspective of the other observer, the chain rule for differentiaon is required when a varialble is not independent.

The particle can be thought of as moving along a marked course that is at rest in the frame of refernce of the field point. A synchronzed clock is at rest at each grid intersection. An second observer near the trajectory can report the time shown by the nearest clock when the particle passes by it. From this perspective, an unaccelerated particle should be half way to the destination in half of the travel time if it has inertia. The soluion is for a particle on the light cone, but it is not the soltion for an inertial particle.

A second observer near the trajectory could report the time that the particle passes by a synchronized clock that is at rest in our frame of reference, but that would not allow us to find out where the particle was any sooner.

The particle would also be half way to the half way to the destination in half of the travel time when it is on a collision course with the observer. The descrepance occurs when the trajectory is at a 45 degree angle to the line of sight.

From the perspective of an observer co-moving with the particle, the equations for the retarded potentials cannot depend on what time it is. That is because there can be many observers at many locations, and the same equation has to work for all of them.

Since the magnitude of R is the same from both perspectives, the particle is on the light cone from both perspectives, but it is not the same particle that the LW equations are for.

The LW solution appears to be the right equation for the wrong particle.

The solution for the time ts=-r1/c is the same as in Eq –. If the angular velocity of the particle did not matter, two points on the trajectory would be sufficent for defining the solution for an unaccelerated particle for all time. The angular velocity does matter.

Retardation equations have to work in the same way when applied in different places and at different times. They have to be form invariant for translations in space and time. That is because we do not know where we are, and we do not know what time it is. Form invariance has about the same meaning as the requirement that equations of physical significance should not depend on the choice of a coordinate system.

This is what the LW equations look like at time ts=-r1/(2 c).

We are always free to choose a coordinate system that is centered upon ourselves. But once the selection is made, the coordinates cannot be readjused as time progresses. It is not all right to choose a new coordinate system at time ts=-r1/(2 c).

Static and quasi-static terms are lost when integrating the time derivative, so the solutions obtainted there are necesarily missing terms, but the radiative terms are idential for either method of calculaion.

ts has to drop out of the solution if the solution is to be form invariant for translations in time.

The solution for toff is observer-dependent. It does not have a general meaning.

The particle is not half way to the destination in half of the travel time. The solution is for a particle on the light cone, but it is not the solution for an inertial particle.

We cannot know what is happening at the simultaineous point while it is happening. We can find out what happened there, but not until after waiting a while. It is difficlut to be sure of precisely when a particle is at its closest approach to the observer with delayed observational data. There would be no uncertainty if the observations were not delayed.

In this solution, the particle is visible forever. It is permanently on the light cone.

It is difficlut to be sure of precisely when a particle is at its closest approach to us when the obsrvational data are delayed. An observer co-moving with the particle could report the time that it is coincident with a clock that is at rest in our frame of refernce, but that would not allow us to find out where it was any sooner.

This is what the LW equation look like at time ts=-r1/(2 c). In Eq –, the particle is visible forever. It is permanently on the light cone. The time ts=-r1/(2 c) is not special. The solution is coordinae-dependent.

The solution is contradictory. drv/dts is vv, yet an unaccelerated particle does not travel at a uniform velocity. The solution is not for an inertial particle. An unacelered inertail particle will be half way to the destination in half of the travel time.

The loophole will be explained in a later version of this paper, but anyone adept with the metods of vector analysis can work through the details.

The vector from the observer to the particle is perpendicular to the velocity vector at the particle's closest approach to the observer. But it can be perpendicular at the time ts=0 and tf=+r/c, or at the time ts=-r/c and tf=0, or at any other time. It has to be possilbe to apply retardaion equations without knowing when the time tf=0 should be, because we do not know what time it is. It is not sufficient that equaions of physical significnce be invariant. They must also not depend on the choice of a coordinate system.

At the point of closest approach, the velocity vector is perpendicuar to the radius vector. A coordinte system can be selected such that one of the coordintes is perpendicular to the velocity. There are then no cross terms between the components of the velocity vector. The equations behave like vector equations, althoug they are not actually vector equations. A true vector equation is unaffected by a coordinate rotation.

They are for the right particle if ts is taken as the independ variable, but then the chain rule for differnetion is required for differentiating with respect to tf. It is only the derivattives of the retarded potentials that are of interest.

The particle is on a collision course with the other observer. The radial velocity of the particle is the only velocity that matters. By taking the time at the particle to be the independent variable in light cone solutions, an observer located elsewhere can compute the effects of a transverse velocity. The LW equations assume that the retarded radial velocity is the only velocity that matters.

This is the transverse doppler effect[20]. The chain rule for differention would be required if tf is taken as the independent varialbe in light cone soluions.

Solutions for two observers and one event or one observer and two consecutive events from the same particle are not independent. They are two perspectives of the same problem. Both cases can be accomodated by taking the time at the source as the independent variable, although there is always more than one way of working a problem.

We have no way of knowing when the time tf=0 should be. Retardation equaions have to work in the same way at any time.

There is another condition. The soluitons at times tf=0 and tf=+r0/c have to be for the same particle if it is on an inertial trajectory.

A space rotation does not affect the invariant quantity $\boldsymbol{r}dot\boldsymbol{r} - c^2t^2$. It is difficult to be sure the all of the calculaions of this section are in the same coordinate system from all perspectives.

The retarded potentials exist only for the purpose of being differntiaed. They do not necessarily have to be form inariant for a particle that is visible forever. It may be sufficinet that they are only form invariant for a short time.

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We cannot know what is happening at the simultaneous point while it is happening. We can find out what happened there, but not until after waiting for the light time accross the system. The LW equations assume that the retarded location of a particle is known before working the problem.

??In spherical solutions, inverting the sign of the radius vector is not the right way to rotate it by 180 degrees. The coordinate system in the soluiton is not specified, but it could be in a spherical system.

When $\hat{\boldsymbol{r}}vdot\hat{\boldsymbol{v}}$ is zero the equation simplifies to. This is not the right equation for the transverse doppler effect. The correct equation is[20]

When $\theta = 0$, the soluion to order v^3 is

ts and tf are not independent variables in light cone solutions. The solution depends on which is taken to be the independent variable.

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Right equation, wrong particle

From the perspective of the contravariant tensors, which have not been regarded very highly since the success of the Einstein tensor, the particle moves along a marked course that is at rest in our frame of reference. There is a syncronized clock at rest at each grid intersection. Other observers near the trajectory could report the time that the partcile passes by each clock, but that would not allow us to find out where the particle was any sooner. We cannot know what is happening at the simultaneous point while it is happening. We can find out what happened there, but not until after waiting for the light time accross the system.

This is not the right equation for the doppler shift either[20].

The is not the right equation for the doppler shift. The doppler shift depends on what time it is when there is a transverse velocity, but the doppler equation cannot, because we do not know what time it is.

These calculations are shown in detail in the SOM.

It is easy to see why the contravarinat tensors are not regarded very highly. With this method of calculation, they are coordinate-dependent. Coordinae dependencis can exist in either space or time.

The particle is at the simultaneous point when tf=ts. The LW equations assume that we already know where the particle was when the time at the field point is ts+r/c. There can be a particle at that place and time, but it is difficult to be sure if it is the right particle.

The particle is visible forever. If we could see to infinity, retardaion equations for the same particle would have to work in the same way forever.

In 3-space, the tensor of each rank is irreducible, with the highest order multipole increasing with the rank of the tensor[6]. There are other terms in 4-space, but a space rotation is still a space rotation. A space rotation does not affect the invariant quantity $\boldsymbol{rdotr} - c^2t^2$.

Tensors were originally develped for the study of the deformation of elastic media, and those calculation still provide an intuitive basis[2]. Those calculaions usually neglect terms beyond the quadrupole. The irreducibility theorm shows that there are octupole and hexadecapole terms when the deformations are large. The sum of three dipole solutions can resemble and octopole, but a true octopole cannot be synthesized by any linear combination of dipoles and quadupoles.

Since the transverse term was zero at the time +r0/c, it would be the correct value if the soluion were obtained for the time $t_f = r_0/(2c)$. However, we have no way of knowing when this time is unless we know when the particle was at its closest approach to us.

The LW equatons are invariant. However, they exist only for the purpose of being differentiated. It has to be possible to differentiate retardation equations without knowing what time it is.

When the particle is on a collision course with us,

This is not the right equation for the doppler shift[20]. The solution is missing a factor of $\gamma = (1 - v^2/c^2)^{-1/2}$. That would not matter if the potential solutions did not need to be differentiated in time. The LW equations are nevertheless known to be very adequate when the relativist corrections are small, which is always the case for stationary current carrying wires. The LW equations appear to be the correctly determined retardaion equations for the tensor of the first rank, a vector.

That does not mean that retardaion equaions should include γ. The LW equations are a transformed Coulomb solution, so it was already included in deriving the retardaion equations. The same correction should not be applied twice.

****

The particle is only on the light cone for an instant at the time ts=0. It emits a signal at the retarded location, then continues on to the simultaneous point. The particle arrives at the simultaneous point at the same time that the signal arrives at the field point.

The LW equations work the same way, but the LW solution is missing relativistic corrections, causing the trajectory to be for the wrong particle. Those calculaions assume that the simultaines point is already known, then the retarded point is computed. With these equatons, the retarded location is first-known then the simulteneous point is computed.

But when applying retardation equations, it is the simultaneous point that is first-known, so these equations have to be inverted. The particle is not visible at the simultaneous point, but it is an essential mathematical reference point for mapping the solution into 3+1 space.

The same correction should not be applied twice. When applying retardion equations, the location of the particle at the simultaneous point is assumed to be already known, even though the location is ficticous in a physical sense. A distant observer could know where the particle is then, but we cannot see where it is yet.

Potential equatins represent the perspective of a distant observer. They do not represent locally measurable relationships until they are suitbly differented. The speed of light is invariant, yet photons travel slower through the gravitaional field of a distant galaxy. Local and global pespectives are not interchangable. Neither is right and neither is wrong. They are just different.

γ is traditionally associated with the other frame of reference. It appears to have a role in our frame of refernce too. That is as it should be unless our frame of reference is special. However, that does not necessarily mean that reatardation equations should explicity contain γ.

The partilce is not visible at the simultaneous point, but for computaional purposes, its location can be assumed to be first-known at that point in space and time.

There are relativistic corrections when ru dot vu is zero. They are of a different symmetry than the LW terms, so they will result in multipole terms that are not present in LW solutions. That will make it easier to experimentally separate them from the much larger LW multipoles in the solutions for rotating electrical equipment. It is likely that there are quasi-static solutions to the Maxwell equations that are not obtainable with LW equations, even when the radiative terms are not solutions. Appratus design would be on firmer ground when familiar design equations can be used.

There is no requirement that potential equations be unique. If using the actual location of the particle rathere than its computed location provides the right derivatives then it shold be acceptable. The interpretaion would not be acceptable if the retarded potentials represented locally measurable relationships, but they do not. They exist only for the purpose of being differentited. The considerations would be different when integrating the derivatives to obtain an orbit.

In being the integral of the fields, the rearded potentials are arbitrary to within a constant of integration. The constant of integration depends on the history of the particle. There are many particles with diffent histories that have the same retarded velocity. It is not possible to determine which particle in a family of particles the retardaion equations are for until a global solution is obtained. The partcile could have been at the origin a differnt time but with a small acceleration parallel to the velocity vecotor and the retarded velocity would be the same.

The reasoning is inverted but the method of solution is computationally convenient. It makes it seem that our frame of reference is the only one that matters, which has a strong intuitive appeal.

The reasoning leading to Eq – is inverted. It is the retarded location that is known. The simultaneous point has to be computed. There are many accelerated partcies with different histories that have the same retarded velocity at a given place and time. From this perspective, the simultenous point is not uniquely defined until a global solution is obtained.

But when applying retardtion equations, the simultaineouls point has to be assumd to be already known in order to compute the retarded location. The retarded potentials exist only for the purpose of being differentiated. If applying a relativistcie correction to Eq – provides the correct derivatives then that should be acceptable.

This is the series exanson for gamma, and it remains that if more poweres of velcoity are carried. That does not necessarily mean that retardaion equatons should explicitly contain gamma. If it is independly comptable then it was there all along.

When appying retardaion equatios, the location of the particle is assumed to be already known, then the derivatives are computed. On the other hand, the derivatves are integrated to obtain an orbit. The persecteives are very different.

tf is the appropriae indepent variable in our frame of refernce. ts is the approraite time from the perspective of the other observer.

Thus, even in the first frame of reference, there are two perspectives on where the particle was.

The radius vector from the field point to the particle rotates as the particle moves. The angular velocity in Eq – does not appear to be the correct value for light cone events.

These relationships can be inverted. If the particle location is first-known at the retarded intersection then the simultaneous point can be computed.

The particle is not visible at the simultaneous point, but the light cone equation can be solved for a particle at that place and time. If the -r0/c time drops out of the solution, then the equation does not depend on what time it is.

This proceedure may seem unnecessry for unaccelerated partiels, but there can be no harm in insuring that the calculations can be performed without knowing the identity of the particle. The considerations would be differnt if the objective were to compute an orbit by integrating the derivatives. In that case we would have to know the identity of the particle.

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This is not the right equation for the doppler shift. There is no tansverse doppler term. The actual problem is worse than it seems to be in this solution. The equation for the doppler shift is different at time tf=0. The doppler shift changes with time when there is a trnsverse velocity, but the doppler equation has to work without knowing when the time tf=r/c or tf=0 should be, because we do not know what time it is. The relativstic doppler equation does work that way[20].

The doppler equaion and the LW solution are both based on the Lorentz transform, but the doppler equaion is in differential form while the LW solution represents an undifferentiated function. The two equations cannot be directly compared.

The missing terms would not matter if the retarded potentials did not need to be differentated in time, but they exist only for the purpose of being differentiated. We do not even care what the retarded potentials are, so long as their derivatives are right.

In being the integral of the fields, potential equaions are arbitraty to within a constant of integration. It is not necessarily true that the constant of integration needs to be known in order to compute the derivatives. From a different but related perpective, potental equations are subject to gauge transformations. For these reasons, the retarded potentials do not represent physical quantites. Physical arguments should not be applied to them until they are suitably differentated.

In deriving the LW equations, the simuleneous point is assumed to be known, then the retarded locaton is computed. The reasoning can be inverted. The retarded location can be assumed to be known, then the simultaineous point computed. The particle is not visible at the simultaneous point, so this persepective is more commensurate with measurable relationships.

There are many accelerated paraticle with different histories that have the same retarded velocty at a given point in space and time. Unless the equations contain accelertion terms, the retarded potentials are for a family of accelerated particles. The identity of the particle is not known until a global solution is obtained.

The procedure may seem unnecessary for unaccelerated particles, but there can be no harm in insuring that the retarded potentials are computable without knowing the identity of the particle. That is important, because we do not know the identity of the particle unless we know where it is at the simultaneous point, but we cannot see where it is then. A distant observer could know where it is then, but we do not.

The consideratons would be different if the objective were to integrate the derivatvives to compute an orbit. Even in that case, it is not always true that we need to know which particle the soluton is for. There could be more that one particle in the same orbit. It is usually sufficient to know the shape of the orbit rathere than which particle the solution is for. The identity of the partice would be imporatant when integrating from standstill, but that would be a hard way of obtaining the solution.

There is no possibility of computing an orbit if the equations are for the wrong particle. It is better to be unsure of the particles identity than to assume that more is known than is knowable.

Even though we do not know where the simultaneous point is yet, a connection to the equaions of 3+1 space has to be established, because that is the space where the theorms of Eucludian calculus exist. Eq – provides this connection. There are additional consideratons for accelerated particles, but the velocity terms do come first.

It is therefor necessary to derive the equaions the retarded potentials without knowing the identity of the particle. Its identity will become known after a global solution is obtained, but not before then.

That has the effect of applying relativistic corrections to the 3+1 space solution. The calculatin is capable of being logically circular, so further evaluation will be needed.

The reasoning leading to Eq – is inverted. We can see the partcile when it as at the retarded intersectin. We cannot see where it is when it is at the simulteneous point. A distant observer near the trajectory could know where it is then, but we cannot.

Since we cannot see when the particle is at the simultaneous point, it is difficult to be sure sure of the identity of the particle that the equations are for.

If the equations are to represent observable relationships, the coordinates should be taken as first-known at the retarded point, even though we are not yet sure which partile is at that place and time.

The LW equations do not depend on the acceleration of the particle. If this relationship holds true, then there is a family of acelerted particles with different histories that all have the same retarded velocity at a given place and time. In that case it has to be possible to derive the retardaion equations without knowing which particle the equations are for.

The simultaineous point is know known, but the partile is

The coordintes should be taken as first-known at the retarded point. That poses a problem, because the retarded point is not computable without assuming that the simultenous point is already known.

But since the calculaions leading to the LW equations are known to work well at low velocites, it should be all right to rely on them for obtaining a premininary estimate of the retarded location, although the algorithm is capable of being logially circular. There are probably other terms at cosmological distances.

The radius vector from the field point to the particle rotates as the partile moves if there is a transverse velocity.

This rotation appears to be closely related to the Thomas precession[7].

The simultaneous point is not the same as in Eq –. We did not know where is was in the first place, so that should not be a problem.

We tend to favor coordinate systems that are centered upon ouselves. It is only coordinte differences that matter, so that is all right, provided that the coordinte system does not follow us when we are orbiting a particle. These calculations are for a distant and detached observer.

The solution can then be iteratively refined, which amounts to obtaning the solutoin by the method of succesive approximation. There are probably several other ways of obtaining the solution.

If this relationshp is true, then it has to be possilbe to derive the equations for the retarded potentials without knowing the identity of the particle. Its identity will evntually be known, but not until a global solution is obtained. Similarly we will not know where the simulteneous point is until a global solution is obtained. This is a difficult concept to accept, but attemting to know the unknowable leads to endless frustrations.

If this relationship is true, then it has to be possilbe to derive the retardaion equations without knowing the identity of the particle. The identiy of the particle will eventually be known, but not until a specific global solution is obtained.

By knowing when the particle is at the simulteneos point, its identiy at other times is established. However, the location of the simultaineous point is not observable by us. A distant observer near the trajectory could know where the particle is then, but we cannot.

The radius vector from the field point to the particle rotates as the particle moves. The tajectory cannot be that of an inertial particle if the pivot point is an unobservable mathematical abstraction.

The calculations ssem to be firmly anchored in our frame of reference. Thet is not the referene point used to establish the identiy of the particle at two different times in the trajcctoty. There is no measurable way of insuring that the solutions for two times are for the same particle.

The coordinates should be taken as first-known at the retarded point. There is a peanilty for doing so. At first, we have know way of knowing which particle the solution is for, although a small family of particles can be selected by the methods used in computing the retarded location for the LW equations. The advantage is that the location of the partcile at time ts+dts becomes relative to its location at time ts. The initial conditions are uncertain, because we do not know which particle in a small family of particles the solution is for, but the integral thereafrer is accurate. It is not necessary to re-introduce the uncertainty at each infitesimal step, as is done when the location of the simultaneouls point is assumed to be already known at each step in the integraion.

Even at a velocity of 0.1 c, relativistic corrections are only about 0.5 percent. Equations for the wrong particle can work well even at moderately high velocites.

The following calcualions are the same as the LW solution if only one infitesimal step is taken. They apper to be the properly determined retardaion equaions for the tensor of the first rank, a vector. They are usually the only retardaion equations needed.

When the time that a particle is at the simultaneousl point becomes known, the identity of the particle becomes known. It can be determined if a light cone event for a different time is for the same particle if it is on an inertial trajectory. However, the simulteneous point is not visible for us. Since we cannot know when a parcile is at the simulteneous point, we cannot know precisely which particle a light cone event is for.

The reasoning leading to the derivation of the LW equations is inverted. The particle is visible at the retarded intersection, so the coordinates shold be taken as first-known at the retarded intersection. If the location of the simultaneousl point is needed, it would ideally be recomputed as needed, although that can be difficlut to do.

There is a penality for taking the retarded locaton as first-known. We have no way of knowing precisely which particle the solution is for until a global solution is obtained, becauses we do not yet know precisely when the particle will be a the simulteneous point. The problem is logically circular. The light cone equaton provides a good estimate of the relationships for inertial particles, especially at low velocites, but the soluton is not precise.

Intuive understanding is useful but more precise equations are needed.

The problem can be work backwards by discovring a constant of integration that provides the correct derivatives. It is not necessarily true that the soluiton obtained this way is unique.

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If it is to be possible to apply the retardion equaions without knowing what time it is, they have to look the same at time t+dt as the do at time t.

The second infitesimal step should be taken if the second derivatives are needed.

The gradient of a vector is a tensor of the second rank. The relatiistic doppler equaion would be no simpler than the retardaion equations if it included both time and space derivatves.

The relativistic doppler equation appears to be correctly determined, in which case we can rely on it to select the right particle, even though we cannot see which particle is the right one.

The doppler equation is in differtntial form. The time at the particle is the integral of the doppler equation, plus a constant of integration. Selecting the ritht particle is therefore the same as detrmining the constant of integration. It is the nature of retardation equations that the constant of integration depends the history of the particle, as perceived by the oberver. The constant of integration is observer-dependent. The retarded potentials would exist it we were not even here. Selecting the right constant of integration cusomizes the retarded potenials for our use.

Kinematic solutions do not have to be for light cone events, but they can be, in which case the same considerations apply to them. There is no possibily of computing the an orbit if the equaions are for the wrong particle.

The retarded potentials would exist if we were not even here, but retardation equantions are observer dependent. The constant of integration customizes the retarded potentials for a specific observer.

If the particle is known to be on an inertial trajectory, the light cone equation provides a good estimate of the relatinships between the retarded and simultneousl points, but the solutions are not exact, because both observers have to make unobservale assumptions in order to obtain a solution.

When the time that a partile is at the simultenous becomes known, the identity of the particle becomes known. If it is on an inertial trajectory, it can be determined if a light cone even at a different time is for the same particle. But since we canot see the partile at the simulatinus poit, there is some uncertainty in the identity of a particle at a light cone event

Intuitive perspectives are useful, but more precise equations are needed.

The constant of integration depends on the history of the particle. The perceived history of the particle depends on where the observer is, relative to the partilae. Retardaion equations are intrinsicaly observer dependent.

The simulteneous point is not a satisfacory reference point for euations of physical significnce. A distant observer near the trajectory can know where the particle is then, but we cannot. The other obsever could report the coordinates to us, but that would now allow us to find out where the particle is any sooner.

The space and time derivaves of the retarded potentaals gnerally cannot be considered indpendently. According to the Maxwell equaions, $\boldsymbol{\nabla}\times\boldsymbol{A}$ has an independent physical meaning, but $\boldsymbol{\nabla}\times\times Av$ does not. Only certain ways of differentiating the retard potentials are meaningful, but the light cone equation is not a potential equation.

From the perspective of the other observer, the particle is traveling backwards in time, which is not the solution for an inertail particle.

Since the simultaneous point is a mathematical abstraction, the retarded location be taken as first-known.

If we did not already know about the retarded poitentials, the solution would have to be obtained by integration. It is easier to discover a light cone equaion that provides the right doppler shift, but conceptually is amounts to evaluating a constant of itntegation.

The transformed coordinates are normally relative to the simultaneousl point. In this solution they are relative to the retarded point. The conclusions drawn do not necessarily apply to other problems.

All times in the equations are in the same frame of reference. The retarded velocity dr/dts is vv0 at all points on the trajectory, yet the particle is not half way to the destination in half of the travel time when there is a transverse velocity. The solution does not consistently represent the trajecory of an inertial particle.

The particle is on the light cone at this time, but it is not the same partile that the LW equations are for.

When there is only one observer, the same equaions have to work for a different particle. it is difficult for one observer to distinguish between the solutions for one observer and two particles or two observers and one particle.

Solutins for two consectutive events from the one particle and one observer are not independ of the solution for two observes and one event. They are two perspectives of the same problem. It is difficult for one obsever to be sure that all of the events are for the same partile. If they are not, then the solution is not for an inertial particle.

The same equatons have to work for a second particle at a different location. The second partcile could be the first partile at a diffeent time. This relationship makes it difficult for one observer to distinguish between the solutions for two observes and one event or one observer and two consecutive events from the same particle.

A transverse velocity for one observer can be a radial velocity for a second observer at a different location. The equations for one observer have to work for a second particle at a different location. The second particle could be the first particle at a different time. This relationshp makes it difficult for one observerr to decide which equation to use. Some assistance from a second observer would make the problem easier.

There is a false curvature term in the solution that needs to be taken out.

This relationship is difficult to interpret, because taking the curvature out of one equation can have the effect of putting into another equation.

Thus, the LW equaions actually do predict the existence of a transverse doppler term. The problem with the equatons is that they do not specify when the particle is at its closest approach to the observer. What looks like one solution is actually a family of solutions. It is difficult to show that there is wrong with the LW equations, because one member of the family is usually the right member. In some sense, there is not anythign wrong with them, but more specific solutions are needed when there are relativistic corrections. The LW equatons are nevertheless very adequate when relativstic corrections can be neglected.

The tensor irreducibity theorm[6] implies that retardation equations are always for a family of solutions, but the practial consequences of the theorm will need further study.

An unaccelerated inertial particle should be half way to the destination in half of the travel time. The solution is for a ghost particle unless the kx2 terms are taken out.

The equation is singular at r0=0. That is because the angular velocity of the particle at the point of closest approach goes to infinity as r0 goes to zero. Exact solutions are not likely to exist.

The observer does not actually care where the simultaneous point is. It is the retarded point that matters.

For vector equations, ts0 can be taken as zero. To the extent possible with true vector equations, they are free of error. However, the gradient of a vector is a tensor of the second rank, so they are missing terms that are not representable with true vector equations. The first derivative of the LW equations is technically not computable. What looks like one soluton is actually a family of solutons. The correct member of the family is usually contained somewhere in the solution, making it difficult to show that there is anything wrong with it. The LW equations are not wrong. They are too slippery. That is not important when relativstic corrections can be neglected which is often the case.

By comparing the space derivatives of the doppler equation to the light cone equation, it should be possible to insure that the solutions for times t and $t + dt$ are for the same particle. When integrating an equation, it is important that the equation be for the right particle. When integrating numerically in n steps, terms must either vanish as $1/n^2$ or be carried. After integration, terms that vanish as $1/n$, no matter how small they are, are in the same ordere as the solution. It is not sufficnt that terms vanish in the limit. They must vanish quadradically.

There are two solutions. One is ts=0, which leads to the LW equaions. They appear to be the correctly detrmined retardaion equations for the tensor of the first rank, a vector. However, the gradient of a vector is a tensor of the second rank, so they cannot represent the symmetric terms.

There is a particle at that place and time, but it is not the right particle, which would cause the retardaion equations to be for the wrong particle.

gamma is traditionally associated with the second frame of reference. Unless our frame of refernce is special, it should exist in our frame of reference too.

The soluintion indicates that if retardation equations in 3+1 space are to satisfy the equaion f=1/lambda then the equaions sometimes need to include gamma. It is possible that there are other interpretaions, but one of the considerations is that 3+1 space does not intrinsiclly restrict velocities to less than c. The limiation has to be projected into 3+1 space. 3+1 space and our frame of reference do not have quite the same meaning.

In an earlier time, 3+1 space was thought to be a real space, but we now know that it is a mathematicl abstraction. The space does not intrinsically limit partcile velocites to less than c. The limitaion has to be projcted into it. As Eq – illustrates, neglectitng a facor of gamma can cause trajectores that look straight in 3+1 space to look curved in 4-space, meaning that the solutions are not for inertial particles. Despite its limitations, 3+1 space is a convenient space to work in.

According to the Newton equatins, the particle arrives at the simultaneous point at the same time the signal arrives at the field point.

Light cone solutions can be obtained by finding the roots of a polynomal or by the method of succesive approximation. The perferred method depends on the problem. There are two roots in this solution. Selecting the root for which the time at the partilce is less than the time at the field point and expanding in a series in v

The other root is for the advanced potentials, which are required when a charged particle is at rest in our frame of reference and the other observer is moving. There are more than two roots if acceleration and other terms are carried.

According to the Newton equations, the trajectory of the particle is vv ts = k tf r0/c,

When there is a transverse velocity, the k^2 terms show that Newton euaions contain false acceleration terms that have to be taken out if the solution is to be for an inertial particle. The particle would be on the light cone if they were not taken out, but it would be the solution for a ghost particle.

The retarded location should be assumed to be first-known, then let the simultaneous point fall as it may. We already know from the relativistic doppler equation where the simulteneors point should be.

The LW equations asume that we already know were the simultaneous point is in our frame of refercne. we cannot see where the particle is then, so it is difficult to be sure. However, for the purpose of establishing a connection to 3+1 space, it is necessary to assume that the location of the simultaneous point is already known, because the location of the retard location would not otherwise be computable in 3+1 space. It might be computable in some other space, but not in 3+1 space.

If we assume that the simulteneous point is known then the computed retarded point is wrong, and conversely. One of the points needs a relativstic correction. It could be that the observer in the other frame of refernce knows where the simultaneous point is. We can find out if the other observer does, because we already know where it is from the relativistic doppler equation.

The speed of light is the same in all frames of reference, however the two frames of refernce are disconnected. The two observers do not necessarily agree on where the simultaneouls point is.

The point of no doppler shift occurs slightly before the pont of closest approach, then becomes a redshift at the point of closest apporach. The equations do not work that way in 3+1 space, but it may be possible to project the solution into 3+1 space.

Since the two frames of reference are disconnected, the simulteneous point is not a satisfactory refernce ponint for the retrarded potentials.

One of the considerations in interpreting the solutions is that the light cone euaion is not a true vector equaion. The light cone soluton for the sum of two velocity vectors is gnerally not in the same directiomn as the vecotor sum of the velocites. The behavior of the solutions is therfore not necessarily explainable with the methods of vector analysis.

The coordintes seem to be spinning. The spin is due to the angular velocty of the field point. The calculaions of the Thomas prcession[?] represent closely related relationsips, but they do implicitly assume that absolute simultataity exists.

It should be possible to extend the calculatons to obtain the solution for time dt1+dt2.

There would be a second rotatation if the dt1, then tree rotaons in the third infitesimal step. There are only three Euler anges, suggesting that a cosmologial term will become appropriae at some point in the progression. The expansion factor of the general theory of relativity is a likely candidate.

The solutions shown in this section could be incomplete in more ways than neglecting the vx4 terms. Some of the relationships are still being investigated.

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From the perspective of an observer in 3+1 space, the coordintes apper to be spinning about the second observer. The spin is related to the rotaion of the Thomas precession[7]. One of the differences is that the Thomas calculations implicitly assume the existence of absolute simultaneity.

Many of the equations in this version were typeset by unsophisticated software and need cosmetic improvements.

The ts0x2 terms represent the spin of the system. They vanish for radial motion, where ru dot vu is zero. The spin terms are singular at r=0 because the angular velocity at the particle at the point of closest approach goes to infnity as r0 goes to zero.

When ru dot vu is zero, the angular velocity of the field point is v/r. The v/r (v/c)2 terms represent a relativist correction to the angular velocity. These terms vanish for radial motion, where ru dot vu is pm 1.

Dipoles are the only multipole that vector equaions are capable of representing. The light cone equaion can represent other multipoles, which is why this soluton is not obtainable with true vector equations.

The eqation is singular at infinity, but we cannot see to infinity, so that should not be a problem. Mathematical singularites at infinity are not uncommon. Intuitivele, a refrecne point at infinity may seem like a good choice, but it beyond the range of validity of many equatons.

The more general soluton is more important, but the equations are also harder to solve from that perspective.

However, if the equaion is for the wrong partle in our frame of refernce it will still be for the wrong partile in the othr frame of refereence. It is not always the other observer who does not know where the particle is.

That is usully interpreted to mean that it is the other observer who does not know where the particl is. The other observer could be us next time.

This method of construction reduces to the basis of vector equations when the particle is on a collision course with the obsever, which is not the preferred location for the observer.

One of the observers does not know where the particle was. That is because the solutons of the Lorentnz transform do not exhibit absolute simultaneaty.

It is indicated that it is not the other observer who does not know where the partile is (or was).

By rotating the coordinate system so that the equaion for a family of particles becomes the equaion for one partile, it shoud be easier to integtrate the relativeis doppler equaion. It will not be easy, but it should be easier.

From the perspective of an obeserver in the frame of refence of the particle, the doppler euation is for one particle. From the perspective of an observer at the field point, it is for a family of particles. One member of the family can be selected by rotating the coordiantes to the point where the velocity vector has only one component. It may then become possilbe to integrate the equation.

The Lorentz transform does not exhibit absolute simultaineoty cite x,x. However,the particle is at rest in the other frame of reference. The other observer is not the one who does not know where the partile is.

The particle is at rest in the other frame of refernce. The obserer who does not know where the particle is is not necessarily in the other frame of reference.

It is not necessarily true that this solution provides the actual location of the particle. The retarded potentials exist only for the purpose of being differentiated. It is suficitnt that the derivavies be consistent. The derivatvies would have to be integrated to obtain the actual location of the particle.

The location in this soluton is not necessarily the actual location of the particle, but since the retarded potentials exist only for the purpose of being differentiated, it does not have to be.

Both observers are now in agreement on where the particle was. However this is not the true location of the particle.

The solutions of the Lorentz tansform do not exhibit absolute simultainey cite ,. The soluton will not be for the rigth particle unless the simultaneous point is where is thought to be. It canmnot be adsumed that the solution is automaticllyfor rigth particle. The location of the particle in our frame of reference is not an independent consideration.

This location is not the true location of the particle. When applying retardaion equaions, the location is assumed to be already known, based on a light cone calculation, then the derivaatves are computed. It is only the derivatvies of the retarded potentials that are of interest. On the other hand, the derifataices would have to be integrated to obtain the locaion for a trajectory calculaion. The intrgral would require a constant of integration, making it dificult to be sure of where the particle actually was in a general way.

It is not true that we always need to know the idntity of a particle. [shape]

Due to the absence of absolute simultainty, it cannot be assumed that the solution in the other freame of refernce is automatially for the right particle. The location of the particle in our frame of referene is not an indpendent consideration.

Because it is not possilbe to predict the future perfectly, if the coordinates are first-known at the retarded point, there is some uncertainy in prediting where the partile will be when the signal arrives at the field point, especially if the calculations are performed with a truncated Taylor series.

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For the purpose of projecting the 4-space first derivative into 3+1 space, it appears justifiable to interpret the solution as being for the wrong particle when tf0=0. The discrepancy is due to a coupling of the angualr and translational velocites. There is no coupling in 3+1 space. The only options are to abandon 3+1 space or assume the solution is for the wrong partile.

The assumption that the soltuion is for the wrong particle is not completely arbitrary. The integral of time dialation is a time offset. Consequently, whem comparing events at the simultamneous and retardee points, one will be offset in time. A clock will not pass a reference point at the expected time when th clock runs slow between the events, making difficult to be sure which particle is the right partiel. The time dialtion terms are recovered after the potential solutions are differentiated.

For reasons that are not independent, the Lorentz transform is also capable of producting solutions for the wrong particle.

rx0 has the meaning of a constant of integration. The calculations constrain rx0 by the method of undetermined coeffients.

Calculation based on the Lorentz transform are rampant that implicitly assume the existence of absolute simultaney, while at the same time denying that it exists.

The time shown by a clock depends on the history of the clock, making it unlikely that absolute simultany does exist. The loop integral of a true potential should nevertheless be zero, even when part of the path is in the other frame of refercne. A zero loop integral does not require absolute simultenety if two frames of refernce can be consistently connected at two different times.

It is nearly impossible to use vector equations to show that there is anything wrong with vector equations. Vector calculaons will usually show that the two frames of refernce are already consisently connected.

These shortcommings preclude their use in practial problems. To order vx2, the soluons are nevertheless idential to those of the LW equations. They appear to be the corect departure point for the next term of the series. The LW equations are obteined by settein gamma to 1 in this solution.

With true vector equaions, the Lorentz transform will always halt a particle in a single step. These are not true vector equations. The partile is still moving.

With this intepretaion, the LW equaions achive a utilitarian goal by dropping terms that should not be dropped. Furthrmore, the gradient of a vector is a tensor of the second rank, which includes symmetric terms. The LW equations do not have any symmtric terms, indicatiin that they are for the tensor of the first rank, a vecor. The vx2 trms of the solutons do not appear to be meaningful, which is why the predited doppler shift of eq – is the wrong value.

True vector equaions are technically not dfferenteable. It is the contravariant tensor of the second rank that represents the first derivaaties.

In some cases, the equations acieve good accuray in a pecular way by dropping therms that should not be dropped.

The true order that the equations belonng in is visible in the doppler equaion, which is missing the vx2 terms.

The coordiantes in the light cone soluton appear to be spinnigng when there is a transverse velocity. The spin represnts a relativisci correction for the angular velocty of th particle.

The coordnates seem to be spinning about the observer, but not really. It is just a relativesstic correction for the angular velcocity of the particle. There are no corrections for true vector equaions, but that is becuse true vetor equations are not diferentialbe. The gradient of vecor is a tensor of the second rank, which includes both symmetric and antisymmetirc terms. There are no symmetric terms in true vector equations, meaning that they are not differentiable.

The second infitesimal step would be required if the vx4 terms are to be carried. The soluton for no steps at all is a subset of the LW solutoins. The LW equaions appear to drop terms that should not be dropped, which is a pecular way of improving the accuracy of the equations. The pecular way only works in special circumstance. The LW equatons are of great practical importantce, but they are not a satisfactory basis for further theroretial developments.

Unlike the soltions for true vector equations, the particle is still moving in the second frame of reference.

It seems that the obserer in the first frame of refeence did not know where the partile was in the first place. That is not necesarily a problem. When applying retardaion equaions, the location of the particle is assumed to be already avalable, based on a light cone solution. The solution is then diffrentited. The retarded potentials exist only for the purpose of being differentited, so the actual location of the particle is a secondary consideration. On the other hand, th derivatves would have to be integrated to obtain the actual location of teh partile. The integral would require a constant of integration, which makes is difficlult to be sure of which partile the soluton is for in a general way.

Absolute simultaniey is not thought to exist cite ,. This soltion does not imply that it does. It is rather that thre coordinte differnces beween the retarded and simultaneous poins are consistent between the two frames of reference.

The integral of time dialation is a time offset. Potental equaions therfore contain coordinte offsets rathr than rate terms. The rate terms are recovered after the soluitions are differentiated. Potential equations do not require abslolute simultanety. It is suffient that the coordinate offsets beteen the simulteneos and retarded locaions be self consistent. It is only coordinate diffrences that represent measurable relationships.

The infitesimla transform can halt the particle in a single step, no matter how high its veloctу is. The full transform has the same capability, but something is amiss. It is much too easy to halt the particle.

As the flying clock experimennts show cite, the time shown by a clock depends on it history. There is little possiblity of kowin ghte absolute time in the other frame of reference. When working the retardaion problem, we do not need to know the absolute time. The difference in the displaytd clock numbers between the retarded and simultaneous points is all that matters.

When using the Lorentz transform to compute the diffrence between the two times, the absolute time does affect each of the two transforms. If we were lucky, the absolute time would drop out of the differnce. The prowess of the infitesml trnsform imply that we ran out of luck long ago.

The full Lorentz trasform will also halt the particle in a single step, but the ease of halttng the particle causes its prowess to be suspect.

Equally important, if the ... from the rear Pseudovecors and vectors do not work in the same way. The tensor irreduciblyt theorm reprensts the differntial angular relationshps of pseudo-vectors.

The transform veloicy and the partile velocity do not have to be the same. The events are simuteneous when the partcle is at rest in the first fraeme of refernce.

If we are lucky, the absolute time in the other frame of refernce will drop out of the sllution for the differnce between the soluton for the same partice at two different times. The prowess of the ininitesiml transform implies that we ran out of luck long ago.

The partile has now been halted in the last frame of reference, even though it was not moving in the first place.

The flying clock experiments cite show that the time shown by a clock depends on the history of the clock, so there is little possiblity of knowin the absolute time in the other frame of reference. Yet it is not possible to apply the transorm without knowing the absolute time. If we are lucky, the absolute time will drop out of the soltion for the diffeerences between two solutins for the same particle at two differnt times. The prowess of the infitesiml transform imply that we ran out of luck long ago.

It is therefore not posslbe to compute the retarded time at the particle with the equation ts=tf-r/c unless the radial and transverse velcityis are separaelty known.

The problem with the Lorentz transform

This soluton neglects the expandion factor of the cosmos. The exansion factor stretches the coordinaes within a mass shell isotropically cite. We know that we cannot see to infinity, so the equaions should not assume that we can. It is not necessarily true that the expanson factor has no effect on laboratory measurements. It is difficlult to be sure, because we cannot perform experiments without the cosmologial term. We are not detached observers.

The retardaion equaions will not be developeld at this time, but other investigatos can avoid some frustration by being aware that the solutions are not solutions to the Maxwell equaions. Refer to Ref. - for some useful background material.

It is only coordinate differnces that represent measurable relationships. The absolute value of a coordinte is not of physical significance. The coordinates in the second frame of refernce are relative to other points in that frame of refence. All things measurable are relative, but it is not the clock in our frame of refernce that they are relative to.

The absolute value of a coordinte is not of physical significance. It is only coordinte differences that represent measurable relatioonshpp. The time shown by the clock in the other frame of referece has to be in relation to something. Due to the ambuities in the doppler equaion, the clock in our frame of refeence is not a satisfactory time standard, because the equaions do not specify precisely which partile the equation is for.

The moving clock can be synchronized to a clock that is at rest in our frame of refernce at the moment it passes by the clock. Thereafter, due to the effects of time dialation, the moving clock will run slower than the at-rest clock.

These relationships are not independent of the concept of absolute simultanety. Many calculaions, including the derivation of the LW equaions, assume the existence of absolute simultaney without stating that they do.

The ambiguity of the doppler equation and the absence of absolute simulatenaty are not independent concepts. They are two ways of being unsure of where the particle was.

The full transform also halts the partile in a single step, but since the infitesmal transform has the same capability, there is not much reassuance that the solution is for the rigth particle.

The transform velocity and the partilcle velocity do not have to be the same.

The absence of absolute simultianiey and th ambiguity of the doppler equation are not unrelated phenomeana. They are two parts of the same system.

It is not possible to apply the Lorentz transform without knowing the history of the clock. If the readial velocity were the only velocty that mattered then the time in the other frame of refernce could be compted by the equaion ts=tf-r0/c, but the doppler equaion shows that the redial velocity is not the only one that matters.

There are special cases where the history of the clock is known, so it may be possible to approach the problem from other perspectives.

It is plausible that the locaion of the simultines point is more important in the other frame of refrence than it is in ours, which could provide yet another approach. There is always more than one way of working a problem. In any case, the absolute value of a coordinate is not a measurable quantity.

The quadrupole terms are essentially invisible to Maxwellian methods of detection. However, the enegy density of a wave is quadradic in field strength. Two nearby non-Maxwellean transmitters radiate more when they are in phase than when they are out of phase, so there will be a beat frequency within each transmitter when the two frequencies are slightly different. The terms will be too small to detect in statioary currrent carrying wires.

This problem does not appear to be with the transform itself. It is rather that if the location of the partile is not known in the first frame of refernce then it will not be known in the second system either.

On the other hand, it might be that the constant will emerge naturally as an uncomputable constant of integration.

After rotating once about each of the three coordinates, it would be possible to start over and rotate three more times. A solution for three rotations is not necessarily complete when the rotations are parameterized by other quantities.

XL. THE METHOD OF RETARDATION

The absolute value of a coordiate is not a measurable quantity. It is only coordinte diffeences that are of physical significance. This limitation is not just for us. It also applies to the observer in the other frame of reference. The methods are the same either way. All things measurable are relative, but it is not the clock in our frame of reference that they are relative to.

As the flying clock experiments show[7], the time shown be a clock depends on its history. In special cases, it would be possible to syncronize a moving clock and then read the number that is shows at a later time. In other cases, in not knowing the history of the clock, we do not know what time it is in the other frame of reference.

The light cone equaion does provide a specific time, relative to the time on our frame of reference, but the equation does not include the transverse doppler effect, so it is not known which particle the solution is for. In being relative to the time in our frame of refernce, the computed time is disconnected from the actual time shown by the other clock.

It arrives at the simiultenous point at the same time the signal arrives at the field point – maybe. We cannot see the particle when it is at the simultanoous point, so it is difficlut to be sure of where it is then, especially in the presence of the cosmologial expansion factor. The expansion factor streteches the coordinaes isotropically within a stationary mass shell[8]. It is possible that the exansion factor affects laboraty electrial measurements that include mass terms, in which case we can find out if it does, but it will be necesary to defer the question for a while yet.

The absolute time in the second frame of reference is not yet known. It is only delta ts that matters. The equatons are simpler if the coordinaes are translated in time so that events that were simulteneous in the first frame of referce are also simultaneous in the second system.

The moving clock can conceptually be synctronized with the other clock at the instant the signal arrives at the field point. The moving clock runs slower, so it will not remain synchronized. The integral of time dialation between the simultencous and retarded points is a a time offset.

Consequently, pontential equaions do not contain rate terms. They contain offsets.

The acceleration term is not real. It is an observer-dependent consequene of the angular velocity of the particle. Retardaion equatins themselves are intriscally observer dependent, but their basis should be equations that are the same for all obervers. The false acceleration term has to be taken out.

Dividing by velocity in this step makes it necessary to carry twice as many powers of velocity in the intermediate calculaions as will be needed in the final soluion. The behavior is very odd for a series calculaion. When in doubt, carry another power of velocity.

Since the x component of the vector potential does not contian any coordinate-dependent terms, it is possble that the equation for the x component works in the same way for other orientaions of the coordinae system.

The soltions of the Proca equaions include exponential terms that represent the range of the fields[11]. The range of the fields is assumed to be infinite in the following calculations. We know that we cannot see to infinity, so the additional terms will need further consideration. The expansion factor within a stationary mass shell streches the coordinaes isotropically[8]. The expansion factor of the cosmos is a likely candidate for assimilating the range of the fields into retardaion equations.

If is possible that the expandion factor has an effect on laboratory electical measurements if the euations include mass terms. We cannot perform experients without it, so it is difficult to be sure, but we will eventually know.

It is conceivable that the expansion factor affects the location of the simultaneous point, in relation to the retarded point. The particle is not visible when it at the simulenous point, so only indirect methods of evaluation are feasible. These calculations tentatively assume that the expansion factor is zero.

The doppler equaion appears to be missing terms when the point of closest approach is nearby. The difficulty is that the angular velocity goes to infinity as the miss distance goes to zero.

As shown in the SOM, the Ax component behaves like a compoent in a true vector equaion, except that it does not contain any coordinate-depeendent terms. The Ax soluton is the whole vector solution.

k is not an independent variable. Enough powers of k have to be carried that more powers have no effect at all on the final solution. k allows the solution to be developed as a pertubtion of the LW solution. The LW solution would be obtained by setting k to 1, except that the equaions are singular for k=1. The singularity is removable in low order solutions.

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The solution depends on both where the particle was and when it was there. The solution is not usable as a retardaion equation. It has too many degeres of freedom. No way has thus far been found to expess the solution in 3+1 space.

However, the retrded potentials are not real in a physcical sense. They exist only for the purpose of being differentited. It does appear possilble to project the potentials into 3+1 space.

It is shown in the SOM that the soltion is rotationally invariant. The soluton for Ax can be used for any orientaion of the coordinate system.

That poses a problem for the Lorentz transform. It is not possible to apply the transform unless the time in the other frame of refernce is asumed to be known before working the problem.

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Time tf1 has dropped out of the solution. It is important that it drop out, because we do not know what time it is.

There is, of course, no restriction to motion along the y axis. The solution for Ay becomes the whole solutoin if the subscript is dropped. vector equations just work that way.

The tensor of the second rank represents the first dervatives. The highest order multipole that it can represent is a quadupole. One of its decomposion productes is a vector, so assistnce from the tensor of the first rank is not needed. The tensor of each rank stands alone. The vecor represents the E field. According to the LW equaions, the magnteic field is a transfrmed E field. In being a derivved auantity, the magnetic field is not one of the decompositon productes.

In applying to just one instant in time, the LW equaions cannot be consstenly differentiated. Actually, the can be differentiated, but the derivavies are all degenerate. Being degenerte does not have the same meaning as being wrong. Degenerate derivatives can be useful.

If the light cone equaion were a true vector equation then it could be reliably concluded that two events in the history of the particle are connected by the Newton equaions. The light cone equaion is not a true vector equaion and the Newton equations do not work well when the speed of light matters. A stronger demonstration is needed that two consecutives events are for the same particle.

There is a large family of accelerated particles that have the same coordinates at two points on the trajectory. This soluton does not specify which member of the family the solution is for. Three points on the trajectory are needed to better constrain the identiy of the particle. The acceleration terms are important, but those terms cannot be developed until acceleration and velocity become distinguishable.

The same ambiguity exists in the first frame of reference. The following calculations are in the first frame of refernce, but it is important that the equaions not depend on which frame of reference is used for the calculaions, so it should also be possible to perform them in the second system.

The equations themselves do not provide any asurance that both solutions are for the same particle. If they are not, then neither solution is for an inertial particle.

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Dividing by the velocity in the intermediate steps of the derivation has an odd effect on the behavior of the equations. It is necessary to carry twice as many powers of velociy throughout the dervivaion as will be needed in the final solution.

The time required for a particle to reach a specified marker goes to infinity as the velocity goes to zero. A singulary at zero velocity is fundamental when the equatons are in terms of the integral of the derivative. The singularity is capable of upsetting computational methods appropate for the derivatives. No matter how small the veocity is, the equations can contain an error of the first order. This behavior is not at all alien to the therems of Eulcludian calcululs. It is rather that those theorms are its basis. An equaion can be reduced to a system of several first order equaions[2], but not in one step. The equaion has to be of first order if it is to be integrated. The behavior looks difernt when the equations are in terms of the intgral in the first place, but its basis is the same in both cases. Even though the first derivative is free of error, there are not many equations than can be reduced to first order in one step.

More powers of velocity are shown in the SOM. With guidance from the relativistic doppler equaion, the solution was obtained by the method of undetrmined coefficients

Another point on the trajectory is needed to better constrain the identity of the particle. Accelertion terms are important, but there is no hope for them until accelertion and velocity terms become distinguishable.

tf0 has dropped out of the solution, as it must, for we do not know what time it is.

The folowing calculaions are in the first frame of refernce, but there are related consideratons for the second system.

The angular relationships in the top panel are distorted by the projection into 3+1 space. The angular relationships are correct in the bottom panel.

The following calculaions are for the first frame of refernce, but there are related considerationos for the second system that need further attention.

Both events would be for the same partcile if the light cone equaion were Newtonian, but it is not.

The solution is for a particle that was on the light cone, but we have no way of knowing which particle it was. Determining the identity of the partcle would usually be a difficult orbit determination problem. The retardation solution for two points in the orbit could be connected with the Newton euaions if the orbit were Newtonian, but it is not. The identity of the particle (or device) will be somewhat better constrained if it emits two consecutive signals.

The acceleration terms are important, but it is necessary to distinguish between angular velocity and accelertion before they can be developed.

The solution does not provide any clues as to which partilce is the right particle. In 3+1 space, the solution for a different particle in the stream of particles could be obtained by simply translatng the coordintes in time. The problem can be projected into 3+1 space, but the reverse projection does not work.

The identity of the particle becomes even better constrained if it emits three consecutive signals. Accelertion terms are important, but they cannot be devloped until it becoes possible to write the equation for a straight line.

Time is the independent variable in the top panel, which causes the angular relatonships in the projecton into two space dimensions to be distorted. The angular relationshps are correct in the bottom panel, but they are not usable without referring to the top panel.

It is not necessarily true that it should be possilble to transform to the Hubble distance in a single step. Many small consecutive transforms are probably a more appropriate approach for connecting the two regions.

The solution can now be for one partcile at two times or two particles at the same time. It does not matter which way it is. However, there could be a third particle midway between the other two.

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The velociy vector is not restricted to being parallel to the x axis. The solution for A_x becomes the whole solution by substitutint $\boldsymbol{A}$ for A_x.

The result is the LW equations. There are several ways of deriving the equations. This method is just one of the ways.

The altrnative perspective is important, because the gradient of a vector is a tensor of the second rank, which includes both symmetric and antisymmetric terms. The solutions of the LW equations do not have any symmetric terms. The Lorentz condition, ... is a symmetric vector (or pseudo-vector) equation, but it is always zero in the LW solutions.

There can be a stream of particles that passes by the point of closest approach, with all of the particles being on the same trajectory. The solution for a different particle in the stream of particles can be selected by translating the coordinates in time. In a fully general solution, the solution for the same particle at a different time could also be obtained by translating the coordntes in time. This soluton is not fully general. It is only valid at the point of closest approach. The solution for the same particle at a different time is not obtainable with it.

Actually, the solutions can be differentiated, and the derivatives are approximately correct at low velocites, but they nevertheless belong in a space with four mutually orthogonal coordinaes.

Discussion

Neither field equations nor retardadation equations form a complete representation. Field equations constrain the fields without specifying the physical proceesses responsible for them. The physical processes are not necesarily what they are thought to be. Due to the non-orthogoanlity of the coordinates, it is generally not possible to attribute a field as being purely associated with either the space derivatives or the time derivatives. The multipoles of the solutions of field equations are inherently incapable of distinguishng bewtween the solutions for charge and mass, except that the terms are orthogonal in vector equations. Retadation equations do specify the physical processes, but they are inept in other ways.

Neither panel is right and neither is wrong. It is the connection between the two slices of 4-space that matters.

However, there could be a third partcle midway between the other two. The job is not done yet.

After factoring a polynomial in the denomenaor, it becomes the square root of a a quantity squared.

The angular location of a transmitter should be obtained with a directional antenna. The direction is perpendicuar to the wavefront. The space derivatives of the doppler equaion would be required for determining the angle of the wavefront. The space and time derivatives are usually both required. Inferring the location of the souce from the time derivatives alone is inaproprate, and unnecessarily challenges the intellect.

Any one observer perceives a virtual acceleration term in the trajectory. A field of observers would nevetheless conclude that an unaccelerted particle moves at a uniform rate along a straight line. Retardation equations are intrisially observer dependent, but their basis should be equations that are the same for all observers.

An accelerometr in free fall would not register an acceleration, but a field of distant observers would still conclude that it is accelerated.

The angles in top panel are distorted in projecting one space coordinte and one time coordinate into two space coordinates. The angles in the bottom panel are correct.

Time is the independent coordinate in the light cone solution in the top panel. The space coordinate is the independent variable for the same solution in the bottom panel. The angular relationships in the top panel are distorted in the projection of one time coordinate and one space coordinte into two space coordinates. This distortion does not occur with the Newton equaions, but it is important when the speed of light matters.

when the range of the fields is infinite, the particle is visible forever. It is permanently on the light cone. Fig – shows the case where the partilce is receding and tf1 is positive. The disconnect beteen the time and space angular relationships is less intuitve in that case.

Is the coordinte difference contaminated by coordinate absoluteness? It is difficult to be sure. The location r=0, t=0 in the first frame of referene is the same in the second frame of refernce. Translating the coodinates to this location does not necessarily insure that the coordntes are not contaminated by absolute cross terms, but it is a more conservative choice. Restricting the coordintate translation to an infinitsimal amount, while possibly unnecessary, is still more conservetive. With further development, it should be possible to perform two consectutive infitessmal translations to obtain a more general solution.

Retardation equations do not contain time as a parameter. They look the same at a second time as do at the first time. They are form invariant for translations in time. Translating the coordinates will not have any effect on them if they are correctly determined.

Within limits, that makes it possibe to apply the equations without knowing what time it is.

This solution is only valid for one instant in time. If it is valid for one instant then it is also valid for a second instant, but it does not provide a way of determing if both solutions are for the same particle. If they are not for the same particle, then neither soluton is for an inertial particle.

The family member that the retardaion equations are for is not known until a global solution is obtained, but the identity of the particle does become better constrained than it is for one event in the history of the particle.

The solutons of the Proca equaions contain exponential terms that are interpretable in a cosmological context[11]. Those terms are neglected in the followng calculations, but they will need further attention.

It is conceivable that the expansion factor affects the location of the simulteneous point, in relation to the retarded point. If so, it may be a satisfactory basis for assimilating the exponential terms into the retardaion equations. It is not necessarily true that it should be possilbe to transform to the Hubble distance in a single step. A large number of small consecutive transforms would be a more reasonable approach.

No arguments are presented that the simulaneous point is where it is assumed to be in the following calculaions, so the assumption will need furthr evaluation, especially when the expansion factor is not zero.

This solution provides no means of determining if all of the events are for the same particle. The solution could be for a ghost particle and we would not know. At the point of closest approach, the light cone equaion doe not impose a first order constrint on which particle in a stream of particles the solution is for. If the solution is for the wrong partiel at the point of closst approch then it is always for the wrong particle.

That does not matter in a universe consisting of one observer and one partile. It does matter if there are two observers or two particles.

This solution is only for the first derivtive. Three consectutive events would be required for computing the second derivative.

The derivtives in this soluton contradict the function they are obtained from. When that happens, it means that the function should be discarded and the solution obtained by integration. Reducing a problem to first order, then obtaining the solution buy integration, is a traditional approach. In neglecting the second derivatifes, this solution is not acutually of first order, but the same consdiertions apply.

The soluton appears to be the retardion euaions for the tensor of the first rank, a vector. History skipped this solution. History also skipped a step the gravitional solutions, probably because of the limited obserational capability of an earlier era. It is sometimes best to begin at the begining, which is easier to do when the solutons are already known.

This solution appears to be the retardion equaion for the tensor of the second rank. The tensor of the second rank includes symmetric terms, but, in the order of the quarupole, they are not associated with the motion of charged particles.

The soluton is only valid for one instant in time at the point of closest approach. It cannot be differentiated.

Each of the slanted lines terminates at an observer. We do not know which of the observers is us yet, and we do not need to know yet, because the retardation equations are the same for all of them. We would know which of the observers is us after a global solution is obtained and the identity of the particle becomes established by its history. The equations for one observer and one particle at one instant in time are too slippery to be useful without further processng.

the chase

The prowess of the infitesimal transform is impressive but not credible. The trasform velocity does not have to be the same as the particle velocity. It is possilbe to transform with a small fraction of the particle velcoity, and in that way chase the particle until it is eventually halted. The calculaion is analogous to obtaining a solution by integration.

The assumption will need further evalution, but it is probably all right to neglect the expansion factor in low order solutions.

Neithrer soluton is in agreement with the relativistic doppler equaion[20]. The solutions are missing a factor of γ.

Furthermore, if the location of the simultaneous point in the other frame of refercne depends on what time is is in the first frame of refernce, then the particle must be moving in the other frame of reference, even though the Lorentz transform shows no indicatation that the partice is in motion. We know that the particle is moving in our frame of reference. It is supposed to remain halted in the other system. There can be many observers of the same particle. They should all be able to agree on where it is, even though none of them can see it when it is at the simultaneous point.

The location of the particle in the secnd frame of referene should not depend on what time it is in the first system. Furthrmore, if it did, the particle would have to be moving in the second system, even though the Lorentz trnsforms shows no indication that it is in motion. The perceived motion is an oberver-dependent effect. It is not real in a kinematic sense.

There can be many observers of the same particle. The location of the partcile in the second frame of reference should not depend on which observer is watching it.

There can be several observers at various locations for the same particle. From the perspective of an observer in the frame of refernce of the partcile, the solution would be unphysical if the location of the particle depended on which of the distant observers is watching. The equations for the other observers are observer-dependent an require a correction. Being observer-dependent has about the same meaning as being coordinate-dependent. Retardaion equations are intrinsically observer-dependent, but their basis should be equations that are the same for all observers.

The solutons of the Lorentz transform usually do not exhibit absolute simultanety cite. This solution is a special case of more general solutions.

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In an orthogoal coordinte system, the two ways of computing dtf would be the same. The four coordinaes are not mutually orthogoanl in 4-space. 4-space can be projected into 3+1 space, but the gradient of a vector is not obtainable in a flat 3+1 space.

Light cone equaions are correct, but in being unaffected by a space rotation, they are slippery. The light cone solution in Eq – is not for one particle. It is for a family of particles parameterized by the angle ru dot vu. One member of the family has to be selected, but there is no possible way of making the selection that is not observer-dependent. Observer-dependent solutions depend on the coordintes of the observer, so they are also coordinate-dependent.

Eq – is also for a family of particles, but it is a smaller family. There is a family of accelerated particles that all pass though two points on the trajectory. This solution does not specify which family member is the right one. One member of the family can be selected by obtainng a specific global solution, but the history of the particle is not well constrained.

dt has dropped out, showing that the equation works in the same way at time t+dt as it does at time t. That is as it should be, because we do not know what time it is.

This is not the right equatio for the transvers doppler effect.

When rx is ¿¿ ry the solution approaches Eq – is for the reciprocl of this term. It shows that this solution is approximately twice the correct value. This soluton is missing a factor of γ.

This would be the right answer in a flat 3+1 space. Howver, the gradient of a vector is not obtainable in 3+1 space.

dt has dropped out, showing that the equaion works in the same way at time t+dt as it does at time t. The equaion is form invariant for translations in time, although only for a short time.

The events are equally spaced along the verticle dotted line, but they are not eually spaced in time. The rate that the clock runs at depends on where the observer is. Watching a clock should not affect the rate that it runs at. The basis of this inconsistency is that the construction assumes that the four coordinates are mutually orthogonal in 4-space. That is not true.

However, the distance rx0 in Fig – is not v ts. It is gamma v ts. It is as though the moving clock were running slow by a factor of gamma.

The time that a particle is at the simultaneous point depends on how far away it is. That is usually not a problem for one observer, but it would be a serious complication if two observes at different locations needed to reconcile their observtions. This relationship can be misleading, because it does not matter how far way y0 is. Regardless of what y0 is, the inconsistency occurs at points between here and there. There is a false curvature term in the equaions. The false term is invisible unless there are two observers or two consecutive events from the same particle.

In this figure the retarded locaton of the particle is relative to the simultaneous point rather than relative to the observer.

As the flying clock experiments show, the time shown by a clock depends on its history[7]. The history of the clock is usually unknown. The clock is on the light cone at time ts0 in Fig –. The clock is visble to an observer at the fiedl point then, and the number that it displays can be used to establish a connection to the time shown by an at-rest clock in the observer's frame of reference.

The calculatons are not in a real space. They represent a projection of 4-space into 3+1 space, which is an abstract mathematical space. Unlike real clocks, moving clocks run at the same rate as stationary clocks in 3+1 space.

The sloutions of the Lorentz trnsform generally do not exhibit absolute simultaniaty cite. This relationship is not independent of when and how the clocks were synchronized.

This result is unreal.

It is only coordinate difference that matter. ts can be taken as zero and tf as +ry/c at this instant.

These would be the coordinates of the moving clock at the time tf=0 if it were synchronized at the time tf=+ry/c. The moving clock can be reset when it is at this location. Resetting the clock will not affect where it is.

In eq –, both clocks show zero when the other clock is at the point of closest approach. Both clocks now show zero when the moving clock is on the retarded light cone at the time tf=0. In a simultaneous system, the time shown by the field clock at this instant is ... = -ry/c. In a conventional sense, the clocks are not synchronized. While is is possible to synchronize the clock, it will not stay synrornzed. It is as though the moving clock were runnng slow by a factor of γ. That is not possible in 3+1 space, but it does seem that way.

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There is no transverse doppler term when ru dot vu is zero. There is a doppler shift at other times, but it is not in agreement with Eq –.

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As the flying clock experiments show, the time shown by a clock depends on its history[7] A moving clock runs slow by a factor of gamma.

Unless the history of a clock is known, it has to be synchronized before the number that it displays is usable. This limitaion applies even when all the clocks are in the same frame of refernce. More generally, the absolute value of a coordinate is not a measurable quantity. It is only coordinate differences that are of physical significance.

The Lorentz transform imlicitly contains the assumption that the clocks have been satisfactorily synchronized. The transform will hold the qunatity $\boldsymbol{r} dotrv - c^2t^2$ invariant regardless of the accuracy of the synchroniztion.

In practice, the number shown be the other clock would be recorded and used for computing subsequent time differences. It is computationally simpler to reset the other clock to zero at that instant. We did not know what time it was in the other frame of reference in the first place, so resetting the clock at the beginnng of an experiment should have no effect on measurable relationships. The other clock is not visible when it is at the simulteneouls point. It is not physically possible to synchronize the two clocks at that time.

The following calculations represent a projection of 4-space into 3+1 space. 3+1 space is not a real space. It is a mathematical abstraction. Straight lines in 3+1 space do not have uniformily spaced tick marks along their lenght indicatating the ticks of a morving clock. Consequently, it is not possible to tell if a straight line has been nonuniformly stretched. Unlike real clocks, moving clocks seem to run at the same rate as stationary clocks, because we cannot tell the difference in 3+1 space.

This computational model for a moving clock would not be acceptable if the retarded potentials were real in a physical sense, but they are not. It is only their derivatives that are of interest in the laboratory. The integral of the derivatives is also a measurable quantity, however the space and time terms cannot be integrated independently. Except possibly in static solutions, neither the scalar nor the vector potential has a separately identifiable physical meaning. Static solutions cease to be static if the observer is moving, in which case the scalar and vector terms again become inseparable.

dt has dropped out of the solution, showing that the equation works in the same way at time t+dt as it does at time t. That is important, becuase it should not matter when we reset the elapsed time clock at the beginning of an experiment.

These are the LW equations when dtf is zero. They appear to be the correctly determined retardaion equations for each particle in a stream of particles. However, the retarded potentials exist only for the purpose of being differentiated twice. Provided that the vx4 terms can be neglected, the missing terms for the first derivatvie can be set aside.

These are the LW equations. They appear to be the retardaion equaions for any particle in a swarm of particles. However, they do not explictly specify the connection between two consecutive events from the same particle.

An inconsistency

Since the acceleration term is not real, it cannot yet be concluded that the particle actually does arrive at the simultaenous point at the same time the signal arrives at the field point. The solutions of the Lorentz transform do not exhibit absolute simultaneity cite, making it difficutl to be sure of exactly where the simultenouls point is. Retardaion equations will be for the wrong particle if the simultendouls point is not where it is thought to be.

A particle at that location is visible at the field point at the time +ry/c, but since a moving clock runs slow by a factor of gamma, a corretion for the clock rate would be required for computig the retarded location of the clock at the time tf=0.

The connection between the retarde and simultaneous points would be more elaborate when the particle is accelerted. If the location of the partcle is first-known at the retarded point then the simultencous point is in the particl's future, even though it is not in our future. It is not possible to predict the future perfectly. In general, a Taylor expansion with an infinity of terms would be required to connect the retarded and simultenous points.

It is possible to synchronize the clocks with sufficient accuracy without assuming that events far into the future can be predicted accurately.

This solution asssumes that the solutins at times t and t+dt are connected by the Newton equation. The Newton equations work well at low velocites, but corrections are needed at higher velocites.

There could be several streams of particles at variouls miss distances, in which case the soluton is for a swarm of particles.

Fig – shows the light cone solutions for a stream of equally spaced particles that are all on the same trajectory. Each partcle transmits a signl at a different time, but all of the signals arrive at the field point at the time tf=0. Each particle is moving left to right, but each particle transmits only one pulse during its traverse.

it is also possible for one particle to transmit a continuous monochromatic signal. The particle is then permanently visible and this discontinuity becomes undetectable. It is only in the space derivatives that the gap is present.

The gradient of a vector is a tensor of the second rank, which includes both symmetric and antisymetr terms. The symmetric terms are not obtainable by differentiating in time in 3+1 space. Misinterpreting the true nature of 3+1 space can result in it being unfairly peanalzed. The decompositon products of the second rank contravariant tensor are a scalar, a vector, and a quadupole[6].

There is no vector in the tensor equaions for defromed elastice media[2]. Depending on the perspective, that is either because the solutons are static or because they do not include charged particles.

The loctions at the simultaineous point so denely saced that the locaton of th partile at that time is nearly useless for predicting where it was at other times. The problem is worse than that. ... The equations contradict themselves. There are ambiguites in projecting 4-space into 3+1 space.

There is no first order round trip constraint on the first derivative. The anchor point for the derivtion of the LW equations is much too slippery. The fault does not necessarlly lie in 3-space itself. It is rather that singular points are best avoided. Solutons near either side of a singular point are not trustworty either.

dt has dropped out of the solution. Since the value of dt does not matter, if this is the right solution for the time t+dt, then it must also be the solution for the time t+0. Equations that behave in this way are rare. They are not retardaion equaions unless dt does drop out, because we have no way of knowing when the time t=0 should be. The LW equations have the same characteric, except that the solutons for times t and t+dt are the same.

The basis of this relatinship is that the first derivarive at a displaced point is one of the contributors to the second derivative. When the coordinares are first-known at the retarded point, we are always one step behind.

Obtaining retardation equations that are valid forever for each particle in a swarm of particles may be too ambitious a goal.

A clock at that place and time is visible at the field point at a later time, but by then the clock has moved on to anothre location. It is not physically possile to synchronize a clock when it is at the simultaneors point.

The solutions of the Lorentz transform do not exhibit absolute simultenety cite. There are no indications that the clocks are inherently synchronized.

Retardaion equations will be for the wrong particle if the unobservable simulteouls point is used as an anchor point and it is not were it is thought to be.

For a short time, but not necessarily forever, the retarded location of one particle in a swarm of particles is

Because the time and space coordintes cannot be integrated independently, the assumption that moving clocks run at the same rate as stationary clocks is only valid for a short time.

section – A swarm of particles

The soluton for the first derivativve is for a tiny swarm of particles, but it is a better departure point than the solution for a string of partices.

Due to a coupling of the radial and transvese compoinents of a velocity vecotr, gammma tends to be invisble in the first frame of reference, but it is no less important than in any other frame of refernce.

this term looks like a second dervative, however the secnd derivtive is required for computing the first derivative at a displaced point. The retarded point is displaced relatie to the simulteneous point. When working at the retarded point, we are always one step behind – sometimes two steps.

An observer at the field point can read the number shown by a moving clock, then use the number for computing coordinate differences in subsequent calculations.

Clocks do not show absolute time. They show elapsed time. The displayed time is not usable until the clock is synchronized. This limitation applies even when all the clocks are in the same frame of reference.

The significance of this relationship is that it is not possible to integrate around a circle with one linear equaion unless it contains π. The only reason it is possilbe at all is that the 3-space integral was discovered long ago. Nonlinear equations for integrating around a circle in 3-space are easily obtained. It is nevertheless possible to integrate numerically around a Newtonian orbit with the two linear equations .. . More genereally, an equation can be reduced to a system of several first order equations cite, although it is doubtful that 4-space can be fully reduced to first order.

Singular equations are sometimes capable of yielding specific soutions, but the singularity does need to be either removed or avoided. Applying l'Hosptial rule to this solutin to remove the sinngularity would be approximately equivalent to formulating the problem in terms of the second derivatives.

The LW equations do not look the same at time t+dt as they do at time t+0. That probably means that the chain rule for differention is needed for obtaining consistent derivatives, with that being because the time and space coordintes are not independent varaialbes in light cone solutons.

It is computationally more conentian to obtain an artifical potential soluton that does not require the chain rule. It is only the derivatives of the retarded poentials that are of interest in the labortory. There is no requirement that they be real in physical sense.

There is no magnetic field in this frame of reference, but there would be in the frame of reference of a moving test partcicle. The partcile must hover at a fixed point if the magnetic field is to be neglected.

The Coulomb soluton is not accurate for moving charged particles. The particle must hover at a fixed point in this frame of refence if the Coulomb solution is to be applicable. The basis of this relationship is tha the time and space coordinates are not independent in light cone solutins. The chain rule for differentiation would be required to halt the partile in the second frame of reference.

The retarded potentials are not real in a physical sense. They are an artifical mathematical construct. They will be more convenient to apply if the first derivatives behave as though the four coordinates were mutually orthogonal. The approach represents a projection of 4-space into 3+1 space. The reverse projection does not work.

Eqs – and – would be the same in an orthogonal coordinate system. The two paths are analogous to exchanging the order of differentiation.

The solution for a point that is displaced in both space and time can be reached by another path.

The particle seems to be halted in Eq –, but it is actually still moving. That is because the clocks have not been corretly synchronized.

In this soluton, the time shown by the clock when it is at the simulteous point depends on when the oserver at the field point synchronized it when it was at the retarded point. When there is more than one observer, the time depends on which observer is watching.

The particle is not visible when it is at the simulteneos point, but it is nevertheless an essential mathematical anchor point when applying the Lorentz transform to the problem.

The particle is permanently on the light cone in this solution. k is a free parameter. The particle is at the point of closest approach when k is 1. The magnitude of radius vector is at a quadradic miminum at the point of closest approach. Consequently, there is no transverse doppler term.

The rate that a clock runs at in 3+1 space is a free parameter. That is not very important until straight lines are non-uniformly stretched. The non-uniformity is invisible with unadorned straight lines.

This limitaion applies even when all the clocks are in the same frame of reference. Clocks do not display absolute time. They display elapsed time.

The derivation of the LW equations assumes that the time at the particle when the signal arrives at the field point is inherently known. The time is not inherenetly known. Indeed, it is not even knowable until the clock is synchronized.

The figure represents a projection into 1+1 space, where the space and time coordinates are mutually orthogonal. An observer in the second frame of reference would perceive the relationships differently. The derivatives cannot necessarily be reliabley inferreed from the figure, with the considertion being that the chain rule for differentiation is generally required when the varibles are not independent.

Since clock is visible at the time +R/c, the clocks could be synchronied at that time, except that an observer in the second frame of reference might not agree on when that time is unless the clocks have been synchronized.

The magnitude of R is at a quadradic minimum a the point of closest approach. Consequently, the light cone equation does not impose a first order constrant on when the particle passes that point. The second derivative is required to impose a first order constraint. The LW solution is for a particle on the light cone, but it is the right equation for the wrong particle.

There are no indications that this solution is the last term of the retardation series.

The simultaneous point is not in our future, but it is in the particle's future. The future location of the particle would not be expected to affect the retarded potentials, suggesting that the solution is applicable to accelerated particles.

This soution represents an infitessimal integral of the first derivative. The solution for the double integral of the second derivative will not necessarily be the same.

It is not difficult to carry more poweres of velocity in this solution, but there is no point in doing so because the vx4 terms would be different at time dts1+dts2. The compactness of equations that look like they are exact is nevertheless useful. The 'exact' derivatioon is shown in the SOM. It is

The angle tau does not show where the partice was when it emitted the signal. It shows where it will be when the signal arrives at the field point. The particle will be at the simultneous point when the signal arrieves at the field point. The location of the simultaneous point is not important for light cone solutions, but it is an essential reference point for the Lorentz transform.

This solution can be differentiated more than once, but derivatives beyond the first will be incomplete if the differentiations are performed in 3+1 space.

This is the LW soluton when dtf1 is 0. However, rr0 is not where the particle was when it emitted the signal.

gamma is usually associated with the second frame of reference, but it also has a role in our frame of reference.

The figure represents a projection into 1+1 space. The time and space coordinates are independent variables in 1+1 space, but not in light cone soluitins. For this reason, the derivatives cannot be reliabley inferred from the figure.

In fig – the time at the field point is the independent variable in the light cone solution. It shows that the time of closest approach is poorly constrained. Indeed, as illustrated by Eq – there is no first order constraint.

Consequently, it is possible for light cone solutions to be for the wrong particle if the time that it reaches the simultaneous point is assumed to be known before working the problem.

Despite these contradictions, the LW equations appear to be the correctly detrmined retardaion equations for the tensor of the first rank, a vector. The Coulomb solution is the retardation tensof for the tenssor of the zeroth rank, a scalar. The tensor of rank n+1 is the gradient of the tensor of rank n.

The tensor irrecucibility theorm cite represents the differential angulr relationships between the tips two or more vector in 3-space. There are other terms in 3+1 space, but a space rotation is still a space rotation. The theorm implies that exact retardation equations cannot exist in 3+1 space.

The LW equations are nevertheless the only retardation equations that are needed when relativistic corrections can be neglected, which is always the case for stationary current carrying wires, even when the current is high enough to melt the wires.

The primary interest in the relativistic corrections is not that of a small correction to the LW solutions. It is rather the souitions contain multipole terms that are not present in the LW equaons. Multipole terms that do not exist in the LW solutons are easier to experimaentally separate from the much larger LW terms. The additional multiples will be larger for rotating electrical equipment or in electron beam experiments.

The particle velocity cannot exceed c, but angular velocity is unbounded. Angular vecocity is more important in tiny systems, so they will also be of experimetnal interest.

A tiny system in gravitational solutions is vastly larger than in electrical solutions.

The equation is not of first order unless the residual vanishes as 1/nx2.

The second partcile could be the first particle at a later time, in which case there should be a connection between the solution for two nearby particles and the doppler equation.

If the simultaneous point is not where it is thought to be then retardaion equations can be for the wrong partile in another way, which would not necessarily be an unrelated way.

The Coulomb solution is usable in quasi-static solutions, but it is not accurate unless the particle is at rest. The nearby particle has to be halted when good accuracy is needed for the first derivatives.

The behavior is more eleborate when the signal is monochromatic and the receiving antenna is many wavelengths in diameter. The wavefront is parallel to the trajectory of the transmitter. A directional antenna will produce a maxmum response when it is aimed at the simulteneous point, but that was not the actual retarded location of the transmiter. This effect is insignificant for natural sources unless their angular velocity is high, which is only possible if they are nearby. It is a near field effect that is not relevant for distant sources. The behavior of the near terms of antenna solutionos are always very different from those of far field inverse square law radiation.

Fig –. When the velocity is hight, a directional antenna will produce a stronger response when it is aimed at the simultaneousl point than when it is aimed at the actual retarded location of the transmitter. This effet is insignificant unless the transmitter is monochromatic and nearby. The wavefront angle is perpendicular to the propograion direction in the far field.

Unintuitively, the perceived anglular location of the source is where it will be when the signal is received rather than where it was when the signal was emitted.

All antenna solutions contain near field terms that beheve differently than inverse square law radiation, although the near field E and B terms normally decay as

1/rx2 or faster. This effeect is unconventional in that it decays as 1/r. 1/r terms are radiavive in some sense, yet the wavefront angle becomes increasing perpendenticalr to the propogation direction at great distances.

The anglar velocity of astrophysical sources is always neglignvle, even when the transvere velocity is near c, but VLBI angular measurements are very precise, so the term may be detectable that way.

However, because the transverse particle velocity canont exceed c, the wavefront angle becomes progressively more perpendicular to the direction of propogation at great distances.

The basis of this inconsistency is that the LW equations confuse the angular velcoity of the particle with the retarded Newtonian accelertion. That does not matter when the angular velociy is low, which is always the case for stationary current carrying wires. The LW equations serve well when the velocity is low.

The simplificaln should be usable when the orbital radius is large, but the accelertion terms do need to be carried when the angular veocity is high.

There is only one vector in 3-space, but are two in 3+1 space, with those representing the E and B fields. grad L defines a third vector, but in representng the second derivatives, it is probably incomplete in the solution for the first derivtive.

It should be possible to remove the singularity with l'hospital's rule. Obtaining the soliton for the second derivative would have a similar effect. It would then be necessary to integrate the second derivative to obtain a global solution for the first derivative. The singularity is unique to our frame of refernce. It is easer to obtain the soluton for the first derivatvie from the perspective of the other observer than it would be to integrate the second derivative.

Consequently, no matter how small dt is, and no matter how low the velocity is, the first space derivatives cannot be reduced to first order in the first frame of reference.

Applying the chain rule for differentiaon to the problem would help, but then the equations would contain space-time cross terms representing the second derivatives.

A vector extrapolates as three scalars. The extrapolated vector is still a vector, but the two closely spaced vectors are connected by a tensor of the second rank[2,6]. In general, Eq – is not the right way to extrapolate a vector, but it is the only way that is availalbe in the projection into 3+1 space. There are no tensors beyond the first rank in a flat 3+1 space. 3+1 space is not a real space. It is a mathematical abstraction.

The solution cantans a transveres rvddot term that does not decay with distance. The LW equations contane a similar term for radial motion. If a particle is approaching the observer and decelerateing, then a second observer furthre from the prticle perceives it earlier in its history when its velociy was higher. The result is a potential that does not decay with distance. However, a prticle cannot decelerte forever, so the extent of the radiated pulse is bounded in global solutions.

The transvees term has the opposite sign on the other side of a radiatvie current loop. Due to the propogation delay accross the loop, the signalls from the nearside and farside of the loop are not exactly out of phase, so there is a net contribution, but the global solution is well behaved.

The space-time cross terms do not exist for vectors in 3+1 space. The equations will be easier to apply if they are projected into an orthogoanl coordinate system.

In 3+1 space, a scalar extrapolates to a nearby point with the equation psi=psi0+grad(psi0) dot dr + d psi/dt dt. An extrapolation to a nearby point in both space and time should include space-time cross terms. It would represent the first derivative at a displaced point, which is one of the contributors to the second derivative. Those solutiions are more general, but they are not the equations for the first derivave in 3+1 space, where the space and time coordnates are orthogonal.

From Eq –, d psi/dt is

grad(psi) dot [dx,dy,dz] is

It is usually more convenient to retard the potentials then differentiate the solution to obtain the fields, but it is possible to directly retard the E and B fields.

For light cone solutions, what looks like one step in space is actually two steps, one in space and one in time. Two consectutive first order extrapolatios represnt the second derivtve in an equation that an observer in the first frame of reference would view as a first derivative.

The particle does not move when obtaining light cone solution for the dislaced point. It is frozen in time for the first step in space.

In the first frame of reference, a vector extrapolates in space and time with the equaions Av = Av0 + (dr dot vnapla) A0 vdot drv + d A/dt dt.

Since there are no space-time cross terms in the first derivative, the time at the displaced point has to be the same as the time at the central point.

When the displaced pont is closer to the particle than the central point, the signal arrives at the displaed point sooner than it does at the central point. It is now necessary to wait a while until the displaced point comes onto the light cone at the time dt.

There are other terms in the equations when the tips of the vectors are moving, but a space rotation is still a space rotation.

The second time derivative would also impolse a first order constraint. However, it the light cone solution is for the wrong partcile in the first place then the result is logially circular.

Consequantly, light cone solutions can be for a small family of particles rather than for a single particle. One member of the family is automatcally selected when a global solution is obtained, but it is not necessarily the right member.

More points on the trajectory are better for constraining the transveres location of the particle, which requires that the equations be differentiated again.

The velocity of a partcile cannot exceed c, but angular velociy is unbounded, so nanomaeter scale circutey may also be of iterest, such as in the solution for a nanometer scale phased array antenna.

It is not necessary to know how to build a receiving anteena for non-Maxwellian radiation. When two transmitters are operated at slightly different frequencies, the mutual interaction energy will result in a beat frequency in the current flow within each antenna. The mutual inteaction energy obeys the inverse square law when the antennas are widely separated. All antenna solutions also contain near field terms that behave differenely.

There are other terms in 3+1 space, but a space rotation is still a space rotation.

Since an extrapolated vector is still a vector, it is possible for vector solutions to exist that are not obtaiable by the methods of vector analysis.

But since the Lorentz condition plays no role in the Maxwell equations proper, which are in terms of the fields rather than the potentials, it may be justifiable to set this inconsistncy aside in some cases, since the inconsistency is invisible to the Maxwell equations themselves.

This inconsistency implies that the retarded space and time coordinaes are not really as independent as they seem to be. It is rather that the depency is not visible in the first derivatives of the coodinates. It is visible in the near field region of the retarded potentials, because angular velocity is more important in the near field region.

While the interation has not converged perfectly, the first derivatve is nevertheless exact for an arbitrarilly small region of space and time. That is because it is not possible to choose a small value of dr1 without also choosing a small value for dt1, and conversely. Since the coordinates depend on each other, the error in either coordinate seems to vanishes quadradically. In some sense, the exactitude is illusionary, but it is the best than can be done with the first derivative in 3+1 space.

Since this inconsistency is perfectly invisible to the Maxwell equations themselves, it is all right to simply ignore it when the objective is to obtain a solution to those equations.

The velocity of conduction electrons in stationary wires is so low that the inconsistency is undetectable in those solutions. Indeed, relativistic corrections can frequently be neglected even at moderately high velocites. In being the gradient of a scalar, Newtonian gravitaional acceleration is in the same order as the coulomb solution, yet those equations continue to serve well. Choose the right equation for the job is important.

Performing more iterations has no effect at all on the solution. The method of successive approximation refuses to provide an exact solution for the simulteneous time.

This behavior is not unique to light cone solutions. When integrating around a unit circle in 3-space ...

Even though the interations failed to converge perfectly, the first derivatve is exact for an arbitrarilly small region of space and time. That is because it is not possible to choose a small value of dr1 without also choosing a small value for dt1, and conversely. Since the coordinates depend on each other, the error in either coordinate vanishes quadradically.

There are no space-time cross terms in the first derivative. There are cross terms in the equations, but the first derivative at a point displaced in either space or time is one of the contributors to the second derivatvie. For the first derivative, the displacements are just in space, or just in time.

?? This term is included in the calculaions of the next section, and it should be included. However, that is not the way vector equations work in an orthogonal coordinate system. For true vector equaions, the first derivvie is computed by extrapolations just in space or just in time. For those equations, the first time derivave at a point displaced in space is one of the contributors to the second derivtive.

??? When there is only one step in space and one in time, it is all right to combine the two steps into a single composit step in space-time. But when there is more that one step in either coordinate, it is necessary to proceed just one step at a time, as the method of successive will otherwise converge sooner than it should.

While the first derivative is for a displacement that is just in space or just in time, the first dervative at a point displaced in both space and time has to be obtained before the fully general second derivatives can be computed.

The more general second derivatives can then be integrated to determine what the first derivatives should have been in the first place. The total differential is harder to obtain when the chain rule for differentiation is required. These calculations do not explicitly utilize the chain rule, but it lurks within the equations.

Fortuously, dr and dt have both dropped out of the soluton. They have to drop out if the solution is to be valid, becuse retardaion equations have to work in the same way withoug knowin what time it is, or where we are. Most equations do not work this way, because most equations are not retardation equations.

For the first derivative, the translations are just in space, or just in time. There are no space-time cross terms in the equaions. The first derivative at a point displaced in either space or time is one of the contributors to the seccond derivative. Cross terms do occur in the equations, but not untill they are differentaited again.

However, unless the first derivative is the total differentail, it could be that it is missing terms.

These calculation assume that the retarded locatin of the particle does not depend on its retarded accelertion. It is possible that the assumption is an oversimplification. The details are not yet clear, but it is clear that additional considerations in some form will be required to obtain convergent soltuions for the d5/dt5 terms. In an intuitve sense, this limitation might be related to the fact that there no equatin for the fifth root of a polynomial. Another possibity for obtaining convergence might be to include the gravitational expansion factor. For now, the d5 terms will have to be set aside for future investigations of the solutions for hammered particles. The r3 terms are for jerked partciels, the r4 terms are for yanked particles. The LW equaions are for accelerated particles.

This would normally be the solution for the first derivavive in either space or time. But since the LW equations are in terms of the first time derivatve of the coordinates, one more step in time is required for computing the potenials at a point that is displaced in time.

The solutions for jerked particles can be obteined by dropping th d4 terms. The soluioins for accelerated partciles can be obtained by dropping he d3 terms. The solutons cannot be reduced further unless the angular velocty of the particle is zero.

These would be our equatons if we were at a point displaced in space and time. But since we did not know where we were in the first place, it is difficutlt to be sure they are not ours.

The particle velocity cannot exceed c, but angular velocity is unbounded. There are no indications exact retardation equations are obtainable.

If these are then right retaraion equations when we are at the location drv rather than at the origin of the coordinate system, then we would also have to use then when we are at the origin, because we have no way of knowing when we are at the origin.

The first derivative is a good approximation at low velocites. However, solutons for an arbitrarily low velocity can contain a small error of the first order. For example, the Coulomb solution is usable in quasi-static solutons, but since the magnetic field is of order v^1, the soutions contain an error of the first order, no matter how low the velocity is.

Thus, the LW equaions are not form invariant for translations in space. It is doubtful that this soluton is either, but it is once removed from an absolute coordinate system.

Consequently, it is possible for light cone solutions to be the right equation for the wrong particle.

In these calculaions, te is the time shown by an elapsed time clock that is reset just before the experiment begins. The outcome of the experiment must not depend on when the timer is reset, so te has to drop out of the solution for the retarded potentials. The retardaion equations are valid for a longer time if more powers of te are carried. The particle is visible forever. Retardaion equaions would ideally be valid forever also, but that may be too ambitious a goal.

In the soluton of this section, te is set to zero. The retardaion equations are valid just long enough to compute the first deriative. They are not valid long enough to compute the second derivaive.

Carrying more terms in th Taylor expansion helps, but it also exhaborates the rate at which the cross terms grow, to the extent that it is not feasible to compute the first derivative at a point that is displaced very far in either space or time.

While the solutons of this order technicall cannot be differentiated at all, the errro is small enougyh at low velocities that the equations can be differentaed anyway in practical problems.

Once the vx3 term drops out, it is gone forever. The surving terms are for accelerated particles.

This characterisc of the Taylor theorm tends to be invisible if we do not know how to look for it.

The method of successive approximation has again converged in just one step. If the calculation seems too easy that is because it is.

The $\ddot{\boldsymbol{R}}$ term reperesnts the trajector of a jerked particle. Discarding the term is bothersome, but there is no choice. It is not mathematically possible to retain the term in the first derivaties of 3+1 space. The cross terms always vanish quadradically. The surviving terms are for accelerted particles. After discarding all of the cross terms, the soluton simplifies to

The LW equaions are in terms of the fist time derivative of the coordinaes, so one more step in time is requird for computing the retarded potentials at the time dt.

Again, the method of succesive approximation has converted in just one iteration. More iterations would be required if the solution was for a nearby observer.

No matter how many terms are carried in the series expansion, the cross terms are of higher order and grow faster. Carrying more terms in the Taylor series helps some, but equaions are so ill behaved that it is not feasilbe to compute the first derivative at a point that is dislaced very far in either space or time.

Carrying more terms in the series expansion helps some, but it also exhabearate the problem by creating new cross terms that grow even faster. The equations are so ill behaved that it is not feasible to compute the first derivave at a point that is displaced very for in either space or time.

It may seem that there is no ambiguity in determngi which particle is the right particle when the motion is purely radial, but that is not true. The correctons of the relativise doppler equation[20] are missing in Eq –. The ligh cone equaion does predict a doppler shift, but it is the wrong equaton.

There seems to be no ambiguity in the identity of the partilc when the motion is radial, but that is not necessarily true. The doppler shift predicted by Eq – is not in agreement with the relatistic doppler equation cite. The light cone equation is missing a factor of γ.

Clocks do not show absolute time. They show elapsed time. If a timer is rest to zero a the start of an experiment, the outcome of the experiment must not depend on when the timer was reset, even though the retarded angular relationships do depend on when the timer of the reset.

Some soluions is shown in the SOM, but the more elaborate solutions should be recomputed on an as-needed basis to insure that that there no software compability issues.

The light cone equaton is for a partile on the light cone, but it is difficutl to be sure it is the right particle. A second opinion from another observer helps in identifying the particle.

In the calculations of this section, another obsrerver is at the origin of the coordiate system. We have to develop the retardaion equations for our use without caring where the other observer is. While the equations are for our use, we still need the second opinion provided by the other observer for resolving the ambiguites of the problem.

dr1 has dropped out of the solution. While we require a second opinion in establishing the identity of the particle, we do not care where other observer is, provided the other observer is nearby.

It would be possible to direcly retard the second derivatives, but it is easier to retard the potentials and then differentiate the solution.

We are using the wrong equation if the measurements seem to depend on when the timer was reset. Uing the right equation for the wrong particle is no better.

The tensor of rank n+1 is the gradiant of the tensor of rank n. The tensor of the zeroth rank is a scalar. The retardain equatin in that order is the Coulomb soluton.

The first derivative in 3+1 space is misleading in this respect. Again, the method of sucessive approximatin has converged in with only one iteration. The first derivative in 3+1 space is misleading in this respect. The equations seem to be more accurate than they are.

The solutions for the third and fourth steps are shown in the SOM, but the more elaborate solutons should be recomputed on an as-needed basis to insure that there are no sofrware compability issues.

The tensor of the zeroth rank is a scalar. The Coulomb solution is the retardion equation in that ordre. The zeroth order retardaion equation is usable in quasi-static solutons when the magnetic field can be neglected.

The gradient of a scalar is a tensor of the first rank, a vector. The LW equations are the retardion equations of that order.

The basis of this inconsistency is that the light cone solution is not for one particle. It is for a small family of particles.

For the first derivatvie, any member of this family is a solution at the time of closest approach. If the solution is for the wrong partaile at the time of closest approach then it is for the wrong partile at any time. (Eq – is probably not the right equaion. The equation was selected for illustrative purposes.)

The magnitude of R is at a quadradic minimum at the time of closest approach, so a smaller family can be selected with the second derivative. However, becaues the tensor of each rank is irreducble cite, it is doubful that the solution for a single particle is obtainable.

The tensor of the zeroth rank is a scalar. The gradient of a scalar is a vector (actually a 4-vector in this case).

???In 3-space, the vector is normally taken as the starting point, then its gradient is the first derivative. But in 3+1 space, the 4-vecotor is the first derivativve, then its gradient is the second derivative. The 4-gradient of a vectoe is more closely related to the 3-space tensor of the third rank.

??? The basis of this relationship is that computing the first derivative at a point displaced in space requires two steps, one in space and one in time, while there is only one step in 3-spae. There are mathematicl similarites between two consecutive steps in space and one step in space followed by one step in time.

However, the retardion equaions of this order are incomplete. There are three vectors in the decompositon products for the third rank tensor cite. There are only two vectors in this solution.

The equations for the retarded potentials must not depend on when the timer is reset, because we do not know what time it is.

Clocks do not show absolute time. They show the elapsed time. In the followin calculations, tf is the time shown by a timer that is reset just before an experiment begins. The outcome of the experiment must not depend on the elapsed time since the timer was reset. The timer is used for computing time differences. The absolute value of the number shown by the timer is of no interest. For physical reasons, it has to drop out of the solution.

It does not matter when the experient begins, so long as it is not too long after the timer is reset. The retardaion equations of this order are not valid long enough for the experiment to begin at time dt1+dt2.

There are other terms in 3+1 space, but a space rotation is still a space rotation, so the decomposition products of the 3-space tensors are a useful reference.

Since a vecctor is the gradient of a scalar, from the perspective of the tensor series, the gradient of a vector can be viewed as being a second derivative.

There is still some uncertainty in the actual location of the partile in this solution, but, in being of higher order, the location should be better constrained.

Now matter how small dr and dt are, the soluton contains an error of the first order. Retardaion equations are intrincially observer dependent, but the trajectory of an inertail particle does not depend on which observer is watching. The solution is for the wrong particle.

No matter how small dr and dt are, the soluitn is not of first order. It is first oder in dr and first order in dt, but the total differential is not just in space, or just in time.

Retaration equations are intrinsically observer dependent. No one obsereve can be sure of which partile is the right partcle. A field of observrs could be sure. With prolonged observtions, one observer can also be sure.

The problem would be harder if more powers of tf were carried.

When integrating numerically in n steps, an error that vanishes as 1/n does not vanish at all.

The observer at the field point is not necessrily at the origin of the coodinate system. A more general solution can be obtained by reprameterizeing the soluton for coordinates that are first-known by an observer at a different location.

The Newton series is not an orthogonal series when the speed of light matters. The trms ... are all in the same order for light cone solutions. For close encounters, it is posible for the velocity to be arbitrarly high when the accelertion is low, but in orbital calcations it is not meaningful to carry excessively high powes of velocty while neglettein the powers of the other Newtonian terms.

This relationship is of no consequence for an observer in the first frame of reference. It does not matter whethere the light cone solution is obtaine in one or two steps. But due to the non-orthogonalty of the coordiantes in the second frame of referned, the other oberver cannot take the computatonal shortcut.

In Eqs –, dr is of first order and dt is of first order, but it does not necessarly follow the equaions represnt the total differential. The derivatives are always exact – until they are differentiated again.

There is no known series where the terms are orthonaoal at both ends of $\boldsymbol{R}$.

To some extent, the problem can be orthogonalzed by obtainin the retardartion soluton one term at a time. With this approach, the soluton for the avdot terms is developed as a perbubation of the solution for the vv terms.

By proceeding this way, it may be possilbe to obtain euaions that behave as though the four coordntes were mutually orthogoanl, even though they are not. The retarded potentials are an observer-dependent mathematical abstraction. There is no requrement that they be real in a physical sense.

A computer program was used to re-arrange the vector components to coordinate-free vector equations. The algorightm used in the calculaions was emprical, but it is closely related to the the methods used for obtaining solutions by the method of undetermined coeffients. The calculaions for more elaborate solutions are much more efficient if the solutions are obtained several passes, with each pass being for only one power of velocity.

The two contributors to the E field are not experimentally separable in local obervtions of short duration. Each contributor behaves like a quasi-potential in this respect. The full solution is

The Lorentz condition behaves like a quasi-potential. It is not locally measurble until it is differentiated.

The first derivative of this souton seems to represnt the first derivtvie at a point displaced in both space and time. But since we did not know where we were in the first place, it is dificult to be sure.

When the partile is acceerted, its location should evaluated relative to the solution for an unaccelerated particle, rather than relative to us. The retrded potentials would exist if we were not even here.

From the perspective of an observer in the frame of reference of the particle, the location of an accelerted particle should be evaluated in relation to the locaton of an unacclerated partciele rather than in relation to a distant observer, because the solution depends on where the distant observer is. The soluton would be unreal if the location of the particle depended on who is watching.

These three steps in space-time can be combined into a single composite step and the solution is the same.

Some solutions for the retaded E and B filds are shown in the SOM. The solutions are exactly the same as the solutons of the LW equations.

The observer is not necessarily at the origin of the coordinate system. It is now necessary to reparametrze th solution by the coordinates first-known by an obsrver at a different place and time.

However, the location of the simultneouls point in the first frame of reference has no special significance for the other observer. The solutions of the Lorentz transforem generally do not exhibit absolute simultaneity cite.

If the problem is not worked this way, there are angular velocity terms that masquerade as acceleraion terms, causing the soluton to be for the wrong particle.

The av terms grow as tx2; the avdot terms grow as tx3. The av avdot cross term gows as tx5, placing it in the same ordere as the av ddd terms. Attempting to improve the accuracy of the series expansion by indluding the avdddot terms would not help. The .. terms are nearly meaningless. The first derivative at a displaced point is one of the terms that is required for computing the higher order derivatviies at a point that is not displaced. Attempting to improve the accuracy of the derivatvies at a point that is not displaced by carrying more terms in a Taylor series representaion of the trajectory is futile.

The consequences of the cross terms can be minimized by developing the trajector of an accelerated particle as a pertubation of the trajectorey of a constant velocity particle.

The terms in this series to not have the same physical meaning as the terms of the Newton equations, but the Taylor theorm has a meaning in its own right.

If dt1 actually is an infitesimal quantity in light cone solutions, then integratating over the dt1 interval will have no effect, because integration in the infitesimal degnerates to multiplication.

The accelertion terms should be carried in the following calculations. Velocity terms are of lower order than accelertion terms, so the accelertion of the partcile can be neglected when the conditions are suffiently mild. The accelertation terms are important, so not much should be expected of the solutions of this order, but the veclocity terms do come first.

If it actually is an infitesimal quantity for light cone solutions, then integrating over the dt interval is harmless, because integration in the infitesimal degenerates to multiplication.

The accelertion terms are important. The solution obtaied by dropping them is of too low an order to be of much interset, but the velocity terms do come first.

This behavior would be contrary to the therorms of Eucludian calculus if the retarded time and space coordinates were independent variables, but they are not. They depend on each other. The chain rule for differentaion is noramlly required when the variables are not independent, but it is not clear how it should be applied when they depend on each other.

The proceedure can be viewed as chasing the partile in many small steps, and adjusting the direction of the steps as necessary to eventually bring the particle onto the light cone.

The integration would hav no net effect if Eq – reprewsented the total differntial.

In obtaining a solution by the method of succesive approximation, a more cautious approach is to chase the particle with many small steps rather than with a single large leap through space and time. The equations are ill behaved at high velocites. For a radially approaching partile, dts/dtf goes to infinty as v goes to c. There are special cases where infinity can be reduced to first order in one step, but those cases are very restrictive.

is not real. However, the soluton illustrates that it is possible of angular velocity to masquerade as an acceleration term, and conversely.

This algorithm would exponentially approach the solution, but it would never converge. A convergenc factor can be used to obtain larger steps near the end of the chase, where the residual is already small. With i running from 1 to n-1, a satisfactoey convertgence factor is (i/(n-1))x2. The convergence factor is an emprical equation, and there are many choices for it. It affects the the speed of the convergence, but it has no effect at all on the assymptotic soluton, provided that all of the steps are small.

From the perspective of the obervr at the field point, ininity can be reduced to first order in one step. An observer co-moving with the particle would disagree.

The first derivative at a point displaced in both space and time is required for computing the second derivatvies, so there are cross terms in the second derivavives but not in the first. For exmple, the cross term ... is one of the contributors to the Maxwell equations.

Fortutously, the method of successive approximation has converged in a single step.

There are now rvddot terms in the solution, shoing that there are some similaritiss between two steps in time and one step in space followed by one step in time. But there are also differences. There are three directions in space, but only one in time.

Retardaion equations are ill behaved when the velocity is high. From the perspective of an observer at the field point, dts is neverthess of first order in the sense that subdiving dtf has no effect at all on the form of the equations. But from the perspective of an observer co-moving with the partille, since dts can be arbitrarily large, there is ample time for accelertion terms to affect the location of the particle when the dts interval is mapped into the dtf interval. From the perspectivew of the other observer, the solution at time dtf should be obtained by integration, even though dtf is arbitrarily small.

When integrating over the dtf interval in n steps, Eq – becomes

There is ample time for quadradic terms to affect the trajectory. No matter how small the dtf intrval is, it is necessary to integrate over the intrval to retain the curvature terms. However, integration in the infitesimal degenerates to multiplicaion, so the integration would not accomplish anything. It would be necessary to differentaed the equation, integrate it, then supply a consatnt of integration. There may be other ways of obtaining the solution, but it is a hard problem in the first frame of reference.

It is possible for there to be a second observer who is not at the origin of the coordinate system. The solution can be translated in space to obtain the solution for the other observer. Since we have no way of knowing where the origin of the coordinate system should be, the other observer could be us.

An infitesial translation in space in the frame of reference of the field point is an infitenaal roation in the frame of reference of the particle. There is no known way of integrating around a circule with one linear equation unless it contains π, implying that the solution for the second infitesimlal rotation will be different than this one.

It is likely that there is a connection to the rotations of the Lorentz group cite.

XLI. THE FIRST ROTATION

The light cone constraint becomes stronger if more derivatives are carried. However, invariant equations are not for intertial particles unless they are for the right particle. When they are for the right partile, more that one observer can agree on its identity.

The space derivatives are effective in localizing the retarded location of the particle.

Retardaion equations are intrinsically oberver dependent, yet they have to work in the same way for all observers. That is because the other observer could be us next time.

These appear to be the retardaion equations for a nearby observer. But since we did not know where we were in the first place, they could be our equations too. There could be a third observer in the problem. It is difficult to be sure that the third observer is not us.

There are no space-time cross terms in the first derivaives of 3+1 space. As exemplified by the Maxwell equations in potential form, there are cross terms in the second derivatives, but not in the first. The cross terms always vanish quadradically in the first derivatives.

There could be a third observer in the problem. Since we do not know where we are in the first place, it is difficult to be sure that the third observer is not us.

In the calculations of S –, the coordinate system was centered on the observer. In this section, the coordinae system is centered on a second and nearby observer. Ideally, the retarded potentials should not depend on where the origin of the coordinate system is. That goal may be difficult to achieve, but we can at least acknowledge the existence of other observers.

Fortuosly, the method of succssive approximation has converged in a single iteration. (However, the calculaion assumes that the location of the particle at the time t=0 is already known.)

The space derivatives are helpful in localizing the retarded location of the particle.

If an observer at the origin of the coordinate system does not know when the time t=0 should be then an nearby observer probably does not either, but the other observer does provide a second opinion.

The acceleration terms will need further evaluation. However, there is no hope for the acceleration terms until the angular velocity and acceleration terms become distinguishable.

Since we have no way of knowing where the origin of the coordinate system should be, the nearby observer could be us.

The acceleration terms will need further evaluation. However, there is no hope for the aceleration terms until the velocity terms are right.

The calculations of this section take some shortcuts that should not be taken, but the solution nevertheless appers to be a valid term of the retardation series. The equations of this order are vector equations. There are no symmetric terms in the solutions.

The accelertion terms should be carried in the following calculations. But since the velocity terms are of lower order than the acceleration terms, the acceleration of the particle can be neglected when the conditions are sufficiently mild.

The Lorentz condition is a symmetric vector equatin, but it is always zero in the solutions of the LW equations. (It is actually a pseudo-vector equation. True vector equations do not have symmetric terms.)

The equaions of this section are for an observer who is at the origin of the coordinate system. The solution for a nearby obsrever is shown in S –.

However, in observations of short duration, one observer cannot see that the equaions are for the wrong partile.

True vector equaions cannot represent symmetric terms. Vectors and pseudo-vectors behave differently for sign inversions. The Lorentz condition is a symmetric pseudo-vector equation.

For an observer at the orgin of the coordinate system, the cross terms vanish quadriaclly. They have no effect on the first derivatives. There are cross terms in the second derivatives, but not in the first.

We are always free to choose a coordinate system that is centered upon ourselves, in which case the cross terms vanish. We are not free to choose a coordinate system that follows us as we orbit the sun, but, for a short time, it is nevertheless all right to choose a coordinate system that is centered upon ourselves.

The cross terms vanish quadradically as dr and dt both become arbitrarily small, so the Rvdot terms can be negelected in a small region of space and time at the field point.

In an orthogonal coordinate system, the cross terms vanaish quadrdically as dr and dt both become arbitrarly small, so they can be negelected in a small region of space and time at the field point. (On the other hand, there would not be any cross terms in an orthogonal system.)

The gradient of a scalar is a vector. The gradient of a vector is a tensor of the second rank cite. The scalar is R. From the perspective of the tensor series, the gradient of a vector is the second derivative, not the first.

On the other hand, concluding that the cross terms vanish quadradically in an orthogonal system is contradictory, because there are no cross terms in an orthogonal system.

However, the equaions are linearly dependent if the dt interval is subdivided. Integrating the first derivative over the dt intrval would not accomplish anything. It would be necessary to integrate the second derivative, then supply a constant of integration. The constant of integration would represent a corrction to where we thought the particle was.

The soluion at locaton dr and time dt would be for the same particle as the solution at dr=0 if the equains were of first order, but they are not, because rotations cannot be reduced to first order in one step.

If the equaions were of first order then dt and dR would be linearly dependent and the magnitude of dt would not matter. However, rotations cannot be reduced to first order in one step.

dt has dropped out of the solution, showing that, no matter how high the velocity of the particle is, the infitesimal transform can halt it in a single step. It could not possibly be the right particle.

Our frame of reference is the only one we can ever measure anything it, which makes it important. However, the other observer should be able to work the retardation problem without knowing where we are.

It is possible to either obtain a global solution for the retarded potentials and then differentate it to obtain the fields, or to directly retard the E and B fields. As developed in §xx, the equations of this section are for the wrong partile, but they are nevertheless known to work well at low velocites, and they are usually the only retardation equations needed. The velocity of condution electrons in statinary copper wire is so low that the inconsistencies are not detectable in those configurations.

We need to know how to compute the retarded potentials for a second nearby observer. Because we have no way of knowing where we are in space and time, the nearby observer could be us. Ideally, we would be able to obtain the solution of another observer anywhere in the universe. A less ambitous goal is to proceed one small step at a time.

As shown in the SOM, the derivation of the LW equations does not depend on whether the time at the source or the time at the field point is the independent variavle. The solution is the same either way. However, due to the constantly changing doppler frequency when there is a transverse veclocity, the derivatives do depend on which time is the independent variable. The correct equatioins are derived in §xx. The derivatives of this section appear to be for the wrong particle in the sense that it is not a particle on an inertial trajectory, but they are an excellent approximation when the velocity is low.

However, as developd in §xx, the rapid converegence is a consequece of obtaining a light cone solution for the wrong particle.

This inconsistency indicates that the calculations are not working in the expected way. The basis of the inconsistency is developed in §xx.

The midpoint shift vanishes quadradically as dt becomes small. Consequently, the midpoint is at the middle in that case so the shift has no effect on the first derivative. It does affect the retarded second derivative.

This relationship is not contrary to the theorms of Eucludian calculus. It is rather that the chain rule for differentiaon is required when the variable are not independent.

The tensor of the zeroth rank is a scalar []. The retardation equaton of that order is the Coulomb solution. According to the LW equations, the magnetic field is a transformed E field. There are no velocity terms in the Coulomb solution, so there is no magnetic field, but the equations are nevertheless usable in quasi-static solutions.

The LW equations appear to be the second term of the retardation series.

Fortuously, the method of succesive approximation has converged in a single iteration. However, as shown in §–, the solution is for the wrong particle. The solution is nevertheless a good approxmation when the particle velocity is low. Indeed, the velocity of conduction electrons in stationary copper wire is so low that the descrepancy is undetectable in those configurations.

The tensor irreduciblity theorm [] represents the diffeential angular relationships in 3-space. There are other terms in 3+1 space, but a space rotation is still a space rotation. The irreducibilty of the 3-space tensors implies that exact retardation equations do not exist. The basis of the irreducibilyt theorm is that the highest order multipole in the solutins increases with the rank of the tensor.

At low velocites, the R2dot terms reduce to Newtonian acceleration terms and the R3dot terms reduce to avdot terms. The solutoins are for jerked particles. It may be possible to obtain solution for yanked particles by carrying the R4dot terms in Eq –.

The calculatin seems to have converged in a single interatation, but not really, because the solution is for the wrong particle. It is not sufficient that the partcile be on the light cone. It must also be on an inertial trajectory if it has mass. The inconsistency is illustraed in Fig – for a radially apporaching particle.

Even though the solution is for the wrong particle, the error is small when the velocity is low. Indeed, the velocity of conduction electrons in stationary copper wire is so low that the error is undetectable even at currents high enough to melt the wire.

In the top panel, cos tau is the same as rhat dot vv. The relativistic doppler equaion [] is parametereized by cos tau rather than rhat. There are no terms in the LW equations that are identifiable with the transverse doppler effect – in Eq –, there are no relativstic corrections when R dot v is zero.

All things measurable are relative, but it is not us that they are relative to. In the calculations of this section, the time at the particle is the independent variable. The time at the field point is the dependent variable.

It may be possible to integrate the solution for the retarded fields in a general way to obtain the equaion for the retarded potentials. Not all equaions are integrable, so the integral might or might not exist.

Since dr has dropped out of the solution, it does not matter where the other nearby observer is. Consequently, the other nearby observer could be where we are.

If a particle moves along a marked course with a synchronized clock at rest at each grid intersection, then an assistant near the trajectory could report the time that the particle passes by each clock. The time that the assistant would report is not the same as the time computed with the light cone equation if the time at the field point is the independent variable.

The times that the assistant would report represent the trajectory of an inertial particle. The computed source times do not, because an unaccelerated particle is not half way to the destination in half of the computed travel time.

gamma is usually associated with the second frame of reference. However, in so far as is known, the only thing special about our frame of reference is that it is ours. It should not matter which frame of reference is used for calculations.

Unless our frame of reference is special, it should be possible to obtain the missing terms in the other frame of reference.

Since the tensor of each rank is irreducible [], it might be that the soltion obtained by directly retarding the derivabves will be differnt than the solutio obtained by retarding the potentials and then differentiating the solution. In any case, the velocity terms come first.

Since the 4-potential transforms in the same way as the coodinates, the calculions of the Thomas precession are applicable to the retarded potentials. Those equations are used in Ref – to compute the first time derivatives of the retarded potentials, then the solutions were integrated in time for the curent loop and dipole solutions. To order vx3 only, the radiative terms of those solutions are identical to these solutions. The static and quasi-static terms are lost when integrating the time derivatives, so the near field solutions are necessarly different. The calculation implies that it is possible to obtain the solution for missing terms in the other frame of refernce. That is as it should be unless our frame of reference is special.

When a short dipole antenna is at the location y=0 and the field point is also at y=, dtf=dts and there is no doppler shift. In the figure, the time at the field point is then +ry/c, but the solution is unaffected by translating the coordinates so that the time at the dipole is -ry/c and the time at the field point is zero. When retardaing several partices, the time at the fieeld point is takaen to be zero for each partilce, so the coordinate translation is different for each partcle. In practice, the coordinats are not translated – the time at the field point is taken to be zero in the first place, so that the signals from all of the particles arrive at the field point at the same time.

The time interval at the field point is longer that at the partcile, so a periodic signal is redshifted. This is the transvers doppler effect in Eq –.

The angle beta in the top panel is the same as the beta in Eq –. (+++drawn on wrong side).

The calculations in the reference imply that the missing terms are computable in the frame of reference of the particle. That is as it should be unless our frame of reference is special.

If there is a short dipole antenna parallel to the x axis at the location x=0, then a charged particle in the antenna is always near x=0, regardless of its instantaneous velocity.

The space vectors are based on the angular relationshiops in the middle panel. The slanted lines in the bottom panel do not represent 3-space angular relationships.

The doppler shift in the top panel is ff/fs = dts/dtf. For an unaccelerated transmiter, the maximum rate of change of the doppler shift occurs at the point of closest approach. In Fig –, the maximum rate of change occrs before the point of closest apporach.

The maximum rate of change could of course occur at any time if the transmitter is accelerated. When the transmitter is in a distant circular orbit, it is accelerated, so the point of zero doppler shift does not occur at the point of closest approach. Similarly, the point of zero doppler shift does not occur when the trasmitter is at the maximum distance from the receiver on the farside of the orbit. The acceleration contrbution to the perceived orbital phase is negligible at low velocites.

It is always possible to rotate the coordinates so that the trajectoey is parallel to the x axis at the point of closest approach, and it is always posible to translate the coordintes in time so that the time is zero at the field point at the point of closst approach, so the figure is more general than it seems to be. The figure can nevertheless be misleading at high velocites if the trajectory angle is first-known at a point far away and the point of closest approach is nearby – that causes the observer to be near the target.

In applying the LW equations to apparatus design, the particle moves along a marked course. Since all of the clocks are in the same frame of reference and the magnitude of R is at a quadradic minimum at the point of closest approach, there is no transvese doppler term.

Clocks do not show absolute time. They show elapsed time. It may be justifibale to assume that a clock moving along the marked course needs synchronization in the first frame of reference.

In 3+1 space, the equatios would work in the same way at time tf=+ry/c in Fig –. There should be a transvese doppler term at that point , but there is none. However, the four coordinates are mutually orthogalal in 3+1 space. The solution is not in 3+1 space, and the vector from the field point to the partcle is not a true vector. It is a pseudo-vector. Vectors and pseudo-vectors behave differently for sign inversions. There are no multipoles beyond the dipole with true vector equations.

Being unable to obtain the right doppler shift at the time tf=+ry/c is not necessarily a restriction. The velocity can be of either sign, and the particle is always on the retarded light cone at some point in its history. Fig – is just one slice of the projection into two space dimensions.

The solution at the time tf=0 could be extrapolated to time tf=+ry/c, with the Taylor theorm, but the extrapolation cannot be performed by simply translating the coordinates without annoying the observer in the frame of reference of the particle.

The vector from the field point to the particle is not a true vector. It is a pseudo-vector. The solution for the observer's backside cannot be obtained by inverting the vector and translating it.

For any given choice for the coordinate system in which the time at the field point is zero, it should be possible to develop the problem as a series expansion parametrerized by the retarded Newton series. The $\boldsymbol{a}$ and $\dot{\boldsymbol{a}}$ terms of the series are important, but there is no possibity of representing them until the velocity terms are right.

The time at the particle, as evaluated in the first frame of reference, is now compatible with the Newton equations, and the midpoint of a line is at the middle. It is possible to nonuniformly stretch a wire so that the midpont is no longer at the center, but the derivatives would be more eleborate.

The same equation should apply to the acoustic wave, but only if there is no wind.

Light cone solutons normally include both radial and transvese velocies. There is no radal velocity at the point of closest approach, allowin gamma to become visible in the first frame of reference. gamma is usually associated with the second frame of reference. Unless our frame of reference is special or different, it should also be visible to us.

It may be possible to work this problem with the time at the field point as the independent variable. If the problem is worked that way, it should be kept in mind that $\dot{\boldsymbol{a}}$ terms integrate to acceleration terms.

The retarded velocity usually has both radial and transverse compoinents in light cone soltuions, tending to obscure the role that gamma plays in the first frame of reference. The radial velocity is zero at the point of closest approach, allowing gamma to become more visible.

In the figure, vector equations would not be affected if the coordinates are rotated about the z axis, or indeed about any axis.

There appear to be differences of opinion that are not reconcilable with vector equations, but it is the transform that matters.

The solution at the time ry/c in Fig – is computable, but it cannot be obtained by translating the coordinate sysem in the first frame of refernce without annoying the observer in the frame of reference of the particle.

It does not matter when the time tf=0 is chosen to be. If the solution for the time tf=+ry/c in fig – is needed, then then tf=0 should be chosedn to be at that point, and the solution would look the same.

The Maxwell and LW equations are nevertheless of a good form when the particle velocity is low, and they provide an essentil refernce point. The are usually the only electrical equations that are needed.

Newton gravity is the gradient of a static scalar, as is the E field of the static Coulomb solution. The Maxwell equations are of higher order than the Newton gravitational equations.

(The chain rule for differentiation would be required of perfoming these calculations in differential form.)

The magnitude of R' is at a quadadic minimum at the point of closest apporach, so dR'/dt is zero at that point. (In 3+1 space, the point of closest apporach would be the same in both frames of reference.)

The potentials are not traditionally viewed as representing physical quantites. They are subject to gauge transformations, so there is not necessarily a requirement that this computational model correspond directly to physical processes, provided that a gauge transform can be discovered that provides the same solutions for the E and B fields, and that is also interpretable in a physical context.

Those calculatins suggest that the soltons are obtainabgle in the other frame of reference, where there are transveres doppler terms.

3-space vectors are based on the angular relationships in the top panel. The slanted lines in the bottom panel do not represent 3-space vectors. In the top panel, $\cos\beta$ is the same as the $\hat{\boldsymbol{R}}vdot\boldsymbol{v}/c$ of Eqs. –. Vector equations throughout are unaffected by rotating the coordinates by the three Euler angles.

Since the computational models are different, it is conceivable that they are connected by a gauge transform. In any case, the transform is the solution. Each perspective is one aspect of the problem.

In Fig –, the time at the particle is taken as the independent variable. An unaccelerated particle is now half way to the destination in half of the computed travel time, which makes the Newton equations easier to interpret. The accelertion terms are important, but the velocity terms do come first.

While the particlre can be at the point of closest aproach at the time ts=0, in the following calculations it can be anywhere.

These relationhips will be developed further in later versions of this paper. Refer to Ref. – for some relevent background material for the Proca equations.

The solutions of this section appear to be missing relativistic corrections. The corrections are neglible at low velocites. The LW equations are of a good form at low velcites, and they are usually the only retardation equatons that are needed. The velocity of conduction electrons in stationary copper wire is so low that the corrections are undetectable in those configurations.

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The expansion factor inside a stationary mass shell streteches the coordinates isotrotropiclly. The time and space coordinates are affected equally, so the locally measured speed of light is not affected[8].

It is conceivble that the expansion factor affects the location of the simultenoeuls point, in relation to the retarded point. The particle is not visible when it is at the simultenos point, so there would be no direct way of observerving the relationshp.

It is not yet known if the expansion factor affects laboraty electrical measurements that contain mass terms. There could be no effect inside a stationary mass shell. However, we cannot know whether it is the particle or the shell that is moving. It is mathematically possilbe for there to be a vector in the interior region of a moving mass shell that is parallel to the shell's velocity. An effect on massless charged particles would not be expected, but there are no known particles of that form.

A solution that is spherically symmetric in the interior region of a mass shell is not spherically symmetric in the frame of reference of a moving particle. Measurable relationships should not depend on which frame of reference is used for calculations. Our frame of refernce would be special if we knew which one it is, but we do not, and cannot. An observer at rest in the interior region of a mass shell is not in a preferred frame of reference. It is probably all right to neglect the expansion factor in low order solutions, but the assumption will need further evaluation, especially in a cosmological context.

The calcultons in this second are for a second nearby observer. Since we cannot know where we are in space and time, we cannot tell if the other solution is for us, but it does not matter, because the equations are the same for both observers.

dr has dropped out of the soltion, so we cannot tell which of the two observers is us. That is important, because we have no way of knowing where the origin of the cordinate system should be.

The light cone equation can be solved with either ts or tf as the independent variable. In Fig –, the time at field point is the independent variable. This figure corresponds to the application of the LW equations to practial problems.

The LW equations can be derived with the Lorentz transform[9]. As shown in the figure, the time at the particle in the first frame of reference is -gamma ry/c when deriving the retardation equations if the particle is unaccelerated. If the particle is accelerated, then the solution is for a different particle moving tangentially to the trajectory at the retarded intersection, in which case the particle will not be at the location x=0 when ts is 0.

Even though the trajectory is not that of an inertial particle, the particle is at rest in the second frame of refernce, so it is not possible to tell that the solution is unphysical until it is differentiated. That is because, as shown in Fig –, the locaion of the particl in the first frame of reference is the same when ts is the independent variable. The retarded potentials do not depend on whether ts or tf is the independent variable until the solutio is diferentiated.

Acceleration terms are important, but there no possiblity of representing them correctly until the velocity terms are for an inertial particle.

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In being the terms of an orthogonal series, each of the vectors of the Newton series is in an independent direction and the multipoles of each order can be obtained separately.

There are necessarily other ways of working this problem. The calclations were performed with vector equiations, but the underlyng relationships are assuridly not vector equatins.

No straigforward way of performing this calcuation with the Lorentz transform has thus for been found. More general invariant coordinate transforms are known, and they may be relevant to the retardaion problem.

The dipole solution is not the same as the solution in Ref.[11]. The discrepancy needs to be resolved, but I have not had time to do it yet.

Attempts at obtaining this solution with the undifferentiated Lorentz transform have not thus far been successful. A traditional approach is to reduce the problem to first order, then obtain the solution by integration. It is doubtful that the four dimensional space can be fully reduced to first order, but it should be possilbe to develop the problem as a series expansion. Other invariant coordeinates tranforms are known, and they may be relevant to the retardaion problem.

When computing an orbit, the derivatives are integrated to obtain the location of the particle. When applying retardation equations, the location of the particle is assumed to be already known, then the derivatives are computed. The location of the particle has to be known by independent means before retardation equations can be applied.

The LW equatons are known to work well at low velocites, so that method of calcualtion can be used to esitmate an approximae initial location of the particle. Thereafter, the location should be obtained by integrataion.

It is not yet known if the cosmological expansion factor affects laboratory electrical measurements. It may be possible to find out if it does by including it in the retardation equations. There is no possible way of recognizing cosmological terms without the appropriate equations, as they have been there from the beginning. The expansion factor is already known, which would faciliate evaluation of the equations.

There is a possiblity that the cosmological constant will emerge naturally as a free parameter in the form of a constant of integration. An uncontrived solution would be the best of all ways.

Thus, while the scalar solution is different than the solution in the reference, it appears to be because this perspective applies to all phases. This additional degeree of freedom appears to be inaccessible in the textbook calculaions of the Thomas precession. However, all of these calculations are preliminary.

It is indicated that the textbook calculatins of the Thomas precession are missing the terms that would represent an integration to a different place, but for the same time.

The meaning of this degree of freedom is not yet clear. It is entirely possible tht it is due to inadequate calculatons, but it is conceivable that the solutions contain degrees of freedom that can only be constrained by laboratory measurements. On the other hand, there is no known role for emperical pure number quantities in geometrical equations. The ratio of the electrostatic to gravitaional force between two electrons is a pure number quantity. Is is a computable or an emperical quantity? We do not know yet. For now, T will be carried as a free parameter, but that cannot be.

There will be other multipoles for the other Newtonian vectors. In being a monopole, it should be possilbe to separate the cosmological expansion factor from the other terms.

The acceleraion of the particle is frequently not weak enough that it can be neglected, so those solutions will be of limited practial interest. However, the acceleration term is not the first term of the series.

If both observers are included in the problem then three points on the trajectory are required to compute the retarded velocity. We are one step behind.

The acceleration of the particle during the second infinitesimal step can be neglected if it is weak enough, although it does make a first order contribution to the location of the particle at the end of the second step.

From the perspective of the other observer, the field point is accelerated. The field point can in be accelerated, but not when the solution is for a distant and detached observer.

The other observer may be right. The field point can be accelerated, but retardation equations represent the perspective of a distant and detached observer. They are not usable in an accelerated frame of reference.

From the perspective of the other observr, the partcle seems to be accelerated. When the particle is known to far from all fields, both observers should be abe to agree that it is not accelerated. When there is a weak acceleration, neither can compute the orbit of the particle if there is a velocity term that looks like an acceleration term.

If two observers are in radial free fall, one behind the other, then each will perceive the other as being accelerated, so it is not true that the acceleration terms in such solutions should always be zero. They should be zero when both observers are far from all fields. They should be small when the fields are weak.

When tf1=-tf0 the particle must be where it was in the first place.

The soluton is the same as the LW solutions if the avdot terms are not carried. The LW equaions appear to be the correctly determined equaions for accelerateed particles. This is the equation of the retarded jerk.

The particle is initially at rest in the second frame of reference. Its veloicty at time dt is dv = av dt. There are no reatvistic corrdections that are first order in velocity. The equains for acceleratd particles are vector equaions. There are relativisc correctons in the frame of refence of the particle at time dt1+dt2. The eqations for the second derivatve are not true vector equations, and the second derivative is required if the equaions are to be integrated – unless trajectory is Newtonian.

The solution for a yanked partilce is obtained by including th avddot terms.

In these solutions, the acceleration is the gradient of the expansion factor. According to the Einstein tensor, the exansion factor of the gravitaional wave is zero at infinity. The radius of curvature is infinite at infinity, so it is difficult to be sure what exists there.

From a global perspective, the soluiton exhibits the symmetriy of a quadrupole, while the LW equaions exhibit the symmetry of a dipole. The local and global perspecives are not at all independent. Multipoles of all orders exist, so this solution is in the lowest order of interest. The tensor irreduciblity theorm[6] specifies the relationships between the local and global perspecties of 3-space rotations. Those relatonships are intuitively more clear in the equations for the deformations of elastic media[2]. There are other terms in 4-space, but a space rotaion is still a space rotation.

Neither retardaion equations nor field equations form a complete representaion. Field equations constrain the fields without specifying what causes them. It is sometimes posssilbe to guess what causes them, but it is difficult to be sure. Retardaion do specify the physical basis of the solutin, but they grave limiations in other ways.

The radial and transverse velocites have now been separated at the point of closest approach. Unlike the doppler equation, there is no longer an ambigity in identifying the partile, so the equation can be integrated. (The doppler equaion would also be less ambiguous if the space derivatiees were included.)

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It is possible that the quantum theories have developed a peculiar way of viewing the symmetric terms. If our frame of reference is the only one that matters then it makes no differnce at all whether the sun is orbiting us or we are orbiting the sun.

The gravitational wave does not contain acceleration terms at infinity[8]. However, we cannot see to infinity.

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XLII. FUTURE WORK

The $\boldsymbol{a}$. $\dot{\boldsymbol{a}}$, and $\ddot{\boldsymbol{a}}$ terms terms would drop out of the solution if the particle was at the origin at the time tf-r0/c, but that is not where it was.

The potential equations in Ref. ... have two static solutions, which are shown there. Both have terms that appear to be cosmologial terms. A vector transforms as three scalars, so it should be possible to transform the static vector potential solution.

It is conceivable that the retardaion series could become degnerate or divergent at some point in the progression.

The expansion factor within a stationary mass shell stretches the time and space coordinates by like amounts, so the locally measured speed of light is not affected[8]. The cosmological expansion factor may need consideration when the the particle was not where it was thought to be.

The particle could have an accelerometr attached to it, and the accelerometer dial could be read from any frame of reference. Both observers should be able to agree on what the dial reading is.

The LW equations are nevertheless known to serve well at low velocities. The tools used should be appropriate for the job.

XLIII. THE LIGHT CONE EQUATION

In fig –, the particle emits two photons at the same instant. The two photons are presunabley engtangled. One of the photons propagets to ... The angular relationships of this solution are those of true vector equations.

The assistant is usually not available, making it more difficult to determine both where a particle was and when it was there.

The perceived angular relationships in an expanding universe need more attention than they have received. Intuitive extrapoltaions from low velocity vector equations are not reliable.

There can be a stream of particles, all on the same trajectory, that pass by the simultaneous point at different times. If the location of the simulteneous point had an absolute significance then we would know which particle in the stream is the right particle. But because the angular relationsips are observer-dependent, one observer cannot quickly determine which particle in the stream is the right one.

A field of observers would know which particle is the right one. If one observer keeps prolonged records of angular relationships, then one observer counts as several observers.

If the simulteneous point is taken as a reference point, then the equations need a relativistice correction.

It is the transform that matters, not which perspective is the right one.

There can be a stream of particles, all on the same trajectory, that pass by the retared point at different times. In 3+1 space, we would know which particle is the right particle without performing any calculations. The problem is not in 3+1 space. Eq – is for the wrong particle.

tx0 has to drop out of the solution, because it has to be possible to apply the equations without knowing what time it is. With true vector equatins, tx0 will always drop out. The light cone equation is not a true vector equation.

If the location of the simulaneous point is known for one observer, then its location for any other observer in the same frame of reference is easily computable with simple vector equations.

Consequently, the location of the simultaneous point becomes relative to the observer in a manner that is not simply related to the simulteneousl point for other observers. One manifestation of this relationship is that the Lorentz transform does not exhibit absolute simultanety []

The basis of this inconsistency is that the angular relationships for a distant oberver have no special significnce for an observer co-moving with the particle – there could be several distant observers at various angles.

The procedure converges in one step, no matter how many powers of velcocity are carried, implying that the solution could be obtaine in simpler ways.

In 3+1 space, the light cone equation for unaccelerated particles can be written as

Translating the coordintes in either space or time does not affect the solutions.

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When the angle between the simultaneous and retarded points is large, the location of the simultaneous point depnds on both how far away the particle was and the angle between the line of sight and the velocity vector. The angular relationships for one observer have no special significance for any other observer, including the one co-moving with the particle.

A vector cannet be rotated about both ends at the same time. The first rotaion is about the head of the vector. The particle is frozen in time for this rotation.

In 3+1 space, the events at both ends of drv1 have to be simulteneous when computing the first space derivaties, so dts1 is not a free parameter.

The first space derivtives are specified by the equation dAv = (drv dot del) A. del A is a tensor of the second rank. Its decomposition products are a scalar, a pseudo vector, and a quadrupole []. The scalar is del dot a. The pseudo vector is del cross a. The quadrupole has no role in the solutions of the Maxwell equaions.

Pseudo vectors and true vectors behave differently for sign inversions and rotations [].

In 4-space, the scalar becomes the Lorentz condition, ..., and there is the anothr vector, -dA/dt ..

Eq – contained Rv0 and vv0 terms, so they were reintroduced in these calculations. The solution has to be obtained by the method of successive approximation. All of the calculations are repeated until all of the origninal variables drop out. The number of iterations required depends on the maximum power of vecocity carried, but the convergence is rapid.

The terms of this series are mutually orthogonal in Newtonian solutions. There are cross terms in light cone solutions. The ax2 and avdot terms are usually in the same order.

The radius vector from the field point to the particle cannot be rotated about both ends at the same time, so the rotations have to be performed in two steps. The first rotation is about the tail of the vector.

It is possible that this inconistency is due to the neglect of the avdot terms in Eq –. The avdot terms are more important in tiny orbits.

The avdot terms drop out when there are only two nearby observrs, which has the effect of restricting the equations to experiments of short duration.

It would be possible to obtain accuarate solutions when the simulteneous point is take as a reference point, but a relativitic correction would be required.

This solutoin is not necessarily in conflict with Eq –, since we have no way of knowing then the time tf=0 in the equation should be.

While one observer could resolve the ambiguites in the doppler equation with prolonged observatins of the angular relationships, the relationships of space and time are not independent, so several coordinated observers at differnt locations have the same capability. The observations do have to be coordinated, since several disconnected observations are no better than one observation.

Clocks do not show absolute time. They show elapsed time. It does not matter when a timer is reset to zero. However, once it has been reset, it cannot be reset again for the duration of an experiment. The calculaions of this section correspond to an elapsed time of toff in Eq –. toff is an infinitesimal time.

In 3+1 space, the events at both ends of drv have to be simultaneous when computing the space derivatves, so dts1 is not a free parameter.

It is possible to perform an accurate calculation, but beause the simultenous point is relative to the observer, the solution would need a relatistic correction if there is another observr in the problem.

In Eq –, we have no way of knowing when the time tf=0 should be, because we do not know what time it is. The calculaions of this section are for the time toff, where toff is an infitisimal quantity.

In 3+1 space, the solutions at both ends of drv have to be simulteneous when computing the space derivatives, so dts1 is not a free parameter.

In 3+1 space, the differentions for the first derivatives are just in space, or just in time. There are no space-time cross terms in the equains. The dr dts2 cross term is important, but it is in the same order as the second space derivatevs and second time derivatve, so it has to be dropped in the solution for the first derivatives. Consequently, the solution for the second derivatives will not be obtainable by differentint the first derivatives in 3+1 space.

Even when an observer alredy knows where the simulteneous point is, they still cannot corretly compute the location of the retarded point, at least not with simle vector equations.

When del R is small, the solution for both observrs is required for computieng the first derivatives of the retarded potentials. Since the retarded potentials exist only for the purpose of being differentated, it is not necessrily true that eithr observer needs to know the actual retarded locaation, provided thate the derivatives are correct. Even for kinematic solutions, if the integral of the derivatives, plus a constant of integrations, works out correctly, then it is still not necessariily that the actual retarded location needs to be known at first.

These complications seem to vanish if the coordiates are first-known at the retarded point, but the simplification is illusionry, because we have no way of knowing when the time tf+0 in Eq - should be.

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Eqs – an – are not for the same particle. The difference is closely related to the fact that the Lorentz transform does not exhibit abslute simultaneity []. Since the retarded potentials are first-known in the frame of reference of the particle, it is more important that the equations be for the same particle from the perspectives of an obserer co-moving with the particle.

In 3+1 space, the differentions for the first derivatives are just in space, or just in time. There are no space-time cross terms in the equaions. The cross terms nevertheless need to be carried if the first derivatives are to be differentiated.

In Eq –, if we knew when the time t+0 should be then we would also know which particle in the stream is the right partcile. The right particle cannot be selected that way, because we do not know what time it is.

The LW equaions can be applied to two closely spaced points on the trajectory for the purpose of computing the first time derivative. Since we are not sure of which particle in the stream is the right particle, there are no assurances that the two solutions are for the same particle. There is a loophole in the derivatives of the LW equations.

As shown below, the LW equaions can be derivd by writng ts in this solutin as ts=dts1+dts2. Most of the terms in the equation drop out in this case. To first order, the locatin of the particle at time dts1 does not depend on its acceleration, but the locatin at time dts1+dts2 is required for computint the velociy of the pareticle at time dts1 for transformint the the feame of refernce of rhe particle. The LW equations are for accelerated particles. There are no acceleration terms in the potential equaions, but that is because the potentials are the integral of the fields. The acceleraton terms do not emerge until the potential soltions are differntiated.

In Eq –, the retarded coordinates of the particle can be first-known at any point on the trajectory. If we knew when the time tf=0 should be then we would also know where the particel is then, but we do not know what time it is. Equivalently, in a global solutions we would know which particle is the right one, but we do not have a global solution yet.

There can be a stream of particles, all on the same trajectory, that pass by the retarded point at different times. Because the angular velocity and acceleration terms of the solutin cannot be quickly separated, it is not possible to quickly determine which particle in the stream is the right particle. In a global solution, we would know which particle is the right particle, but we do not have a global solution yet.

By writing ts=, then the two points on the trajectory used for computeint the first derivatives of the retarded potentials are explicityl for the same particle. In principle, this correction could be applied in 3+1 space by using the chain rule for differention. Applying the chain rule to the retardaion problem is tricky, becuse the space and time coordintates depend on each other. It is simpler to obtain the right soluton in the first place.

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The doppler effect

The acceleration is not real. It is an observer-dependent aberation, but it nevertheless makes it impossible to integrate the doppler frequency until the two contributors to its rate of change are separated.

A steadily decreasing doppler frequncy can mean that a particle is approaching radially and decelerating, or it can be due to the transverse velcoty of an unaccelerated particle when the angle between the velocity vector and the line of sight is greater than 90 degrees.

Similar equations apply to the whine of a low flying jetliner, except that the acoustic wave behaves differently if the wind is blowing. The aether does not have a wind.

... Interplanetary ... ... The precision of the navigations has established that the equations correctly accomodate the abberation.

If a particle is accelerated, or could be, angular relationships cannot be neglected when integrating the doppler frequency.

This is the relativistic dopper equatin []. The interpretaion of the transvers doppler effect in the refernce is from the perspective of the other observer. Due to the absence of absolute simulttnety [], the interpretaion does not apply in our frame of reference.

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The solutions for the first derivatives therefore contain a loophole. The loophole is not very important when the velocity is low. Indeed, the velocity of conduction electrons in stationary copper wire is so low that the loophole is not even detectabvle in those solutions.

If the the ts in the equain – is written as tx0+dts1+dts2 then the solution for the first derviaves is explicitly for the same particle, closing the loophole. The solutinos of this order are for a jerked particle. The loophole can probably be closed in other ways, but not in 3+1 space, at least not without the chain rule for differnrion.

However, there is now a new loophole for the first derivatives of the first derivatives.

This loophole can be closed by writing the equation as ts=tx0+tx1+dts1+dts2. The equaions of this order would be for a yanked particle.

In 3+1 space, we are always one step behind – sometimes two steps.

There can be a stream of particles, all on the same trajectoy, that pass by the retarded point at different times. In eq –, the retarded coordinates can become first-known at any point on the trajectoy. Due to the ambiguites in separating angulr velocty and acceleration, we are no longer sure of which particle in the stream is the right particle. In a global solution, we would know which particle is the right particle, but we do not have a global solution yet.

By writing the ts in eq as tx0 + dts, it becomes possilbe to determne that the soluions at time tx0=0 and tx0 are for the same particle, making it possilbe to unambiguousl compute the first derivatives of the first derivatives.

Proceeding further, by carrying both the tx0 and tx1 terms, it would become possible to compute the second derivaties of the first derivatives, while remaining confident that all of the points on the trajectoey really are for the same particle. The solutions would be for yanked particles.

When tx0 is included in the equaions, then we do know that the solutions for two nearby points on the trajectory really are for the same partcile, but we are only sure of its identificain for a time just long enough to compute the first dereivatives of rhe first derivatives.

There are no acceleration terms in the LW equaions, but that is because the potentials are the integral of the fields. The acceleration terms do not appear in the solutions until they are diffrentriaed.

There can be a stream of particles, all on the same trajectory, that pass by the retarded point at different times. In Eq –, the retarded coordinates can be first-known at any point on the trajectory. Since angular velocty and acceleration of the particle are not instantly separable, we cannot be sure at first which particle in the stream is the right particle. After a global solution is obtaine, we would know when particle is the right one, but not at first.

Consequently if tx0 and tx1 are both zero in eq –, then the equaions for the first deriatvies can be differentiated in time, but since we do not yet know which particle is the rigth particle, there are no asurances that both points on the trajectory are for the same particle.

The following calculaons are for a jerked partcle. The third power of velocity is the minimum power in the solutions of this order. More powers of velcoity are sometimes meaningful. Due to the length of the expressions, the vx3 terms are not shown here in the intermediate steps. They are shown in detail in the SOM, and in the soluton.

When tx0 and tx1 are both zero, the first derivaties cannot be differentiated in 3+1 space to obtain the second derivates. Actually, they can be differentiated, but since, at first, we are not sure of which partcile in the stream is the right particle, there are no assurances that the two points on the trajectory are for the same particle. When tx0 is explictly in the equations, then we know that they really are for the same particle.

Similarly, when tx0 and tx1 are both carried, it would become possible to differentiate the first derivatives twice to obtain the third derivatives. The additinal degree of freedom in the solutions of this order would allow each partcile to have a richer history, while at the same time remaining uniquely identifiable.

The terms in this series are mutually orthonal with the Newton eqhuaions, but there are cross terms in periodic light cone solutions. The a3 adot2 and dddot terms are all in the same order.

Similarly, the retardain equains for jerked particles do not have any avdot terms unitl specific global solutons are differentiated to obtain the field.

There can be a stream of particles, all on the same trajectory, that pass by the retarded point a different times. When the data points are delayed by the light time accross the system, it is not possible to instantly determine which particle in the stream is the right particle. The right particle can be identified after a specific global soluion is obtained. The retardation equatonns themeslves are not capable of instantly specifying which particle in the stream is the right particle.

Carrying more of the t_0 terms imposes fewer constraints on the history of the particle, allowin it to have a richer history in specific global solutions.

Proceedng furhter, if this term is written as to1+to2, then it would become necessary to carry the avddot terms in orer to consistently compute the first derivtives at a point twice displaced in time, which are contributors to the third derivatives. The velocity at a point displaced three times in time would still be required for the Lorentz transform, so the solutons would be for yanked particles.

Because global solutions are obtaiable by integrating from the infitesimal, there are limits to how far this series can be extended in a simplistic manner without developing the exponential terms that represent the range of the fields []. The expontental terms could significantly influence high order local solutions. It should be all right to neglect them in low order local soluions, where simple inverse square law terms are a good approximation. However, for exponential terms, the distance to which an approximate inverse sqare law equaion remains accurate can depend on how many times it is differentiated. It is conceivavle that, in some oder, there is no distance small enough that the exponential terms can be neglecd. The exponential terms are important, but they can be set aside for a while.

The exponential terms can be expanded as a series in the radial distance, so it is not necessary to obtain complete solutonos. The series expansions represent the soluton in the interior region of a sphere. Their behavior is very different than the solutons for the exterior region.

The $\ddot{\boldsymbol{a}}$ terms represent the trajectory of a yanked particle. They drop out of the solution for the first derivative in 3+1 space, so they are not carried in the following calculations, but they will be needed for the second derivatives. For periodic light cone solutions, the $adot^2$ terms are in the same order as the $\ddot{\boldsymbol{a}}$ terms.

The $\dot{\boldsymbol{a}}$ terms reprsent the trajectory of a jerked particle. The a^2 terms are in the same order as the $\dot{\boldsymbol{a}}$ terms.

The $\boldsymbol{a}$ terms are for an accelerated partile. The v^2 terms are in the same order as the $\boldsymbol{a}$ terms. Carrying more powers of velocity in peridic solutons is harmless, but they do not actually mean anything. They can be meaningful in non-periodic solutions. The soluion in this order is the LW equations. In practical problems, they are frequently the only retardaion equains needed. The solutons of this order are always solutions to the Maxwell equations.

Powers of t_o beyond the first could be carried in the calculations, but doing so while neglecting the $\ddot{\boldsymbol{a}}$ terms would cause terms in the same order as the t_o^2 to be dropped.

Retardion equatons have to work in the same way without knowning what time it is, hence the need for t_o. It is possible to derive invariant equaions that are only valid at the time $t_o = 0$. However, we have no way of knowing when the time $t_o = 0$ should be, so the solutions are coordinate-dependent.

In 3+1 space, it does not matter when the time $t_o = 0$ is chosen to be, so the coordinate dependency is not visible in 3+1 space if there is only one observer, or if two observers independelty use the same equation. If two observers do independently use the same equation, then they are disconnected from each other.

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dts1 has not dropped out of the solution. In 3+1 space, the equaions cannot be applied without knowing what time it is. Coordinate dependencies can exist in either space or time.

The computed location for the retarded point is wrong.

The basis of the difficulty is that the location of the simulteneous point is realtive to the observer, introducing an element of logical circularity in the solutions. Newtonian calculations would not be logically circular.

The derivation of the LW equations [] assumes that the retarded location of the particle is computable with Eq –. The equation would be the right equation in 3+1 space. The acceleration terms are important, but there is no hope for them until the velocity terms are right.

This solution is for the first derivaties. The second derivatives obtined by differentianting the first derivatives in 3+1 space should be a useful approximation when the conditions are mild, but the solutins will be missing terms unless the differentianons are performed in 4-space in the first place.

? As shown in the SOM, carrying more powers of velocity would have no effect at all on the final solution of this section.

This completes the solution for the first derivative. The avdot terms would be required for the second derivatvie. In 3+1 space, we are always one step behind – sometimes two steps.

Thus, the two observrs obtain different results for the locatin of the retared point, and they are both wrong, yet neither observer can see that anything is wrong until they compare results. The basis of this inconsistency is that the simulteneous point is relative to the observer.

Neither observer can correctly compute the location of the retarded point, yet neither observer can see that anything is wrong until they compare results.

In 3+1 space, two at-rest observers can be simulteneous with each other while at the same time being simulteneouls with a moving particle. In 4-space, the two observers can still be simulatenous with each other, but they cannot also both be simultneous with a moving particle at the same point on its trajectory. The basis of this limiation is that the location of the simulteneous point is relative to the observer.

Now that the soluton is in 3+1 space, the sign of ru can be inverted.

In this solution, rv0 is where an obsrver in 3+1 space at the location drv=0 thinks the particle should be.

In 3+1 space, two observers can be simulteneous with each other and also be simultaneous with moving particle. The problem is not in 3+1 space.

Even when the observers know where the simultenous point is, they cannot corrctly compute the retarded locatin, and each observer obtains a different solution.

The two observers are now on the light cone at different times, but the solutions have to be for the same particle.

The two observers are now simulteneous with each other, and with the same particle at different points on its trajectory.

Both observeres can now consistently compute the retarded locatin of the particle, but since neither observer could obtin the right soluion in the first place, the solutions would be consistently wrong.

The retarded potentials exist only for the purpose of being differentiated, so we do not necessarily care where the partcile actually was, provided that the derivatives are correct. If the derivatives are correct, then they can be integrted and supplemented by a constant of interation to obtain the actual location. In being the integral of the fields, the potentials are always arbitrary to within a constant of integration anyway. When only the derivatives are needed, the constant of integration is not of interest, although it would be required in kinematic solutions.

While the retarded location could be computed, the acceleration of a particle would integrate to a velociy term at the retarded point, which would complicate the calculaions for accelerated particles, although it could be done.

The calculations are easier if the particle is left in place, but viewed from the field point at a later time when it is on the light cone.

The equation could now be used to compute the first derivatives of the retarded potentials. It will be more convenient to set the soluton for the location dr=0 aside and defer the differentaions until after a specific solution is obtained.

When computing the first derivatives in 3+1 space, the differentions are just in space or just in time. There are no space-time cross terms in the equtions. The cross terms are important, but they are contributors to the second derivaves.

The soluton for the location drv and time dtf therefore has to be set aside until the solution of all of the second derivaves is obtained. The second space dervative and the second time derivatvie are in the same order as the cross term. It would not be consistent to carry the cross term without them, so the dr dts2 terms have to be dropped in the solution for the first derivative.

Because the cross term is actually two steps in time and one step in space, we will still be one step behind, but it will be a smaller step. The terms of this series have the form of a multivariate Taylor series in space and time [].

The first derivarivs of the retarded potentials could be computed from these equations. It is computainally more convientent to defer the differentitions until a specific solution is obtained.

All three points could be simulteneous in 3+1 space, but the problem is not in 3+1 space.

Since neither observer can correctly compute the retarded location of the partiel, the location obtained this way is not for the right particle. But since the retarded potentials exist only for the purpose of being differentiated, that does not matter if the derivatives are correct.

The basis of the connection to the Thomas precession is evidently the coupling of the angular velocity and accelertion terms in Eq –.

Even for kinematic solutions, it is not necessarily true that we care where the prticle actually was, provided that the integral of the derivatives, plus a constant of integration, works out correctly.

The retarded location computed this way is not the actual location. But since the retarded potentials exist only for the purpose of being differentiated, that does not matter if the derivatvies are correct.

When computing the first derivatives in 3+1 space, the differentions are just in space, or just in time. There are no space-time cross terms in the equations. The cross terms are important, but they are terms of the second derivatives. Consequently, for the first derivatives, there is no requirement that the solution at location drv and time dtf be for the same particle as the soluion for the location drv=0.

The retarded locatin computed this way is not the actual location, but it is a convenient way of obtaining potential solutions. It is only their derivatives that are of interest. The correction has effect of projecting the first derivatives into 3+1 space.

The Proca equatios are in terms of the second derivatives, so there will be some inconsistencies for the second derivatves, since the second derivatives obtained by differentiating the first derivatvies in 3+1 space are for the wrong particle.

It is possible to differentiate three times and then integrate once, so there are no assurances that the second derivatives will be complete either.

Since the location of the simulteneous point is relative to the observer, one observer's calculaions for the location of the retarded point do not mean anything for other observers.

Two observers can be simuleneous with each other. One observer can be simulteneous with one particle. Two observers cannot be simultaneous with each other and a moving partcile at the same time. They could in 3+1 space, but the problem is not in 3+1 space. These relationships are shown in Fig –.

The acceleration terms need to be carried in the following calculatins. They will be developed in a later version of this paper. In being of lower order, the velocity terms do come first.

Since the location of the simulteneous point is relative to the observer, each observer's calculation for the location of the retarded point is logically ciucular.

Thus, if two nearby observers compute the retarded location of a prtcle, then the two soltuons are not for the same particle, and neither solution is for the right partcile. The obsrvrs cannot see that there is anything wrong with their calculations unitl they compare results.

This solution does not represent a contradiction. The first observer did not know where the particle was in the first place. The observer needed a second opinion.

However, because angular velocity and acceleration are not quickly separable, observtions of short duration are ambiguous.

The solution does not represent a contradiction, since observtins of short durtion are ambiguous. Two observers can resolve the ambiguities more quickly than one observer can.

An equivalent procedure is to obtain a potential equation that differentiates correctly in 3+1 space.

This solution is only for the first derivarives, but the solutions obtained by differentiaring it in 3+1 space should be a useful approximation if the conditions are mild.

When computing the first derivatives in 3+1 space, the differentions are just in space, or just in time. There are no space-timee cross terms in the equaions. The cross terms are important, but they are terms of the second derivativees. Consequnetly, for the first derivaves, there is no requirment that the solution for the locfation drv and time dtf be for the same particle as the solution at location drv=0. The Maxwell equaions are in terms of the second derivatives, so these relationships will need further development.

But since the solutoin for a nearby observer is for the wrong partilce, it becomes necessary to apply a relativisc corretion for the nearby observer, which has the effect of projecting the first derivatives into 3+1 space. The Maxwell equations are in terms of the second derivatives, but it is best to begin at the beginning.

If the two observers do independeltly assume that they are simulteneous with each other and with the particle, then the two computed retarded loctions are not for the same particle. Neither observer can see the discreancy until they compare observarions.

When computing the space derivatvies in 3+1 space, two nearby observers have to be simulteneous with each other. They also have to be for the same particle, but the two solutions do not map into the same point on the trajectory.

A related limiation is that angular velocity cannot be localized. Since the angle between the simulttaneous and retarded points does not depend on how far away the particle is, the correctins are of first order, no matter how small the distance is. The corrections are not visible in a simultaneous system.

Since the location of the simulteneous point is relative to the observer, each of these calculations is logially circular, causeing each soltuion to be for the wrong particle, although in a way that neither observer can see. The ideosyntractic nature of the solutions does become visble if the two obsrvars compare observations.

There can be a stream of particles, all on the same trajectory, that pass by the simulteneous point at different times. In 3+1 space, an observer would know which particle in the stream is the right partcile without performing any calculations. The problem is not in 3+1 space. In 4-space, one observer cannt quickly determine which partile in the stream is the right particle. Two coordinated observers can select the right partice in less time than one oberver can.

Consequently, for the first derivatives, there is no requiremt that the solutions be simulteneous at both ends of drv at time dts1+dts2.

In this solution, each observer exists in their own special world. Neither can see that their soluton for the location of the retarded point is wrong until they compare observations. Since the location of the simulteneuus point is relatie to the observer, each calculaion is logically circular and self fullfilling.

The relationsips of space and time are not independent, so if one observer keeps prolonged records of angular relationships, then one observer counts as several observers.

A related limitation is that, since the angle between the simulteneous and retarded points does not depend on how far away a prticle is, there is no distance that is small enough to localize angular velocity when the particle is nearby. No matter how small they are, the relativistic corrections for angular velocity are first order in distance. Cosequently, they are also of first order in distance if the equaion is integrated, so choosing a smaller distance does not accomplish anything. The integral still contains a small error of the first order.

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In the top panel, coordinates are first-known at the retarded point then the simulteneous point is computed. In the bottom panel the coordinates are first-known at the simulteneous point. When the velocity is low, the two perspectives are approximately equivalaent. When the velocity is high, each of the obsrvers in the bottom panel exists in their own special world, a world that revolves about them.

This velocity is not real. It is an observer-dependent abberation, with its basis being that the loction of the simulteneouls point does not have an absolute meaning.

When applying retardaion equaions to engineerng problems, it is convenient to assume that the location of the particle at the simulteneous point is already known, then use it to compute the location of the retarded point. But since the location of the simultenes point is relative to the observer, solutions obtained this way represent the perspectve of an obserever that exits in their own special world, a world that revolves about them.

While Eq – is not real in a physical sense, it does correctly represent the perceived angular relationships as a function of time, provided that we know which particle in a stream of particles the solution is for.

An assistant near the trajetory could report the time that the partile passes by an at-rest synchroinized clock. We would then know which particle in the stream is the right particle, but not until after waiting for the light time accross the system.

In being intrincisally observer dependent, the retarded potentials can evidently be viewed as being an observer dependent abberation. In any case, local physical arguments should not be applied to them until it is shown than they cannot be gauge transformed.

But since the location of the simulteneous point is relative to the observer, the soltuins obtained this way are for an observer that exists in their own special world, a world that revolves about them. This problem can be circumventee by applying a reativisic correction to the solutins of engineering problems, which has the effect of projecting the solutins into a 3+1 space. 3+1 space is not a real space, but it is a convenient space to work in, provided the reverse projection is not attempted.

If the observers assume that the simultenous point has an absolute signficance, then the two calculatins for the location of the retarded point are not for the same particle.

However, the two observers are not simultaneous with each other. They need to find a way to coordinate their observations if they are not shown where the particle is.

This problem can be circumvented by applying the equations to practical problems as though the simultenous point has an absolute signifcance, then applying a relativave correction to account for the fact that it does not.

A solution for two observers and one particle is shown in Fig. –. The particle is on the retarde light cone for both observers. It then moves to the right with the velociy vx for the time $[(\Delta x)^2 + (\Delta y)^2]^{1/2}/c$.

If the observers assume that the simultensous point has an absolute significance, then neither observer can see that their solution for the retarded point is not the same as that obtained by the other observer. Each observer is certain of their solution, but the two observers are disconnected from each other. Each exists in their own special world.

The relationships of space and time are not independent, so if one observer keeps prolonged records of angular relationships, then one observer counts as several observers.

Since the location of the simulteneous point is relative to the observer, equations are idosyntractic if they are relative to the simultaneous point. They are not necessarily wrong, but they are difficult for other observers to interpret.

In 3+1 space, the solutions at both ends of drv have to be simultaneouls when computing the space derivatives, so the two solutons map into differnt points on the trajectory.

When working the retardation problem, the particle emits a single photon when it is at the retarded point. The photon propogates to the field point with the velocity c. In the meantime, the particle continues its journey to the simulteneous point. It arrives at the simultaneous point at the same instant the photon arrives at the field point.

The Lorentz transform does not exhibit absolute simultaniety[1], so there is some ambiguity in what is meant by the same instant.

Except for a power of c, the 4-potential transforms in the same way as the coordinates, implying the the coordinates can be viewed as a potental representaion. The loop integral of a potential is zero.

As expected, the loop integral of a potential actually is zero.

The false acceleration term would need consideration if the doppler equaion were to be integrated, but the calculations of this paper are for the retarded potentials.

The LW equaions do work in the same way at different times. They are form invariant for translations in time, which has about the same meaning as not containing a coordinate dependency.

However, they assume that the assistant's reports are instantly available. An observer at the field point would rather use a nearby clock for a time standard, which leads back to the problem with Eq –.

The partice can be viewed as moving along a marked course. There is a synchronzed clock at each grid intersction. An assistant near the trajecoty can report the time that the particle passes by each at-rest clock. Eqs – are the appropriate equatins for this model.

At low velocites, there are no significalnt quadradic terms in the equations and simple vecotor equations are a good approximatin. But when the angle between the simulteneios and retarded points is large, the location of the simultenous point depends strongly on where the obsrvers is, relative to the particle. It becomes relative to the observer. It ceases to have a meaning for any other observer.

One way of measureing the magnetice field is by the deflection of a moving charged particle. For an obsevee in the frame of reference of rhe particle, the moving conductin electrons and stataioury protons must be retarded separately. From the perspective of rhe other observer, the magnetic field is still a transformed E field, even though there is no external E field in the laboratory frame of reference. The meaning of the magnetic field is not a concrete as it seems to be.

For the same reason, the Coulomb equation is not the rigth equatin for compting the trajectory of a charged partcile is a static E field, although it is a good approximation.

The observer either has to wait for the assistant's reports to reach the field point or use a nearby clock for a time standard. This solutin assmes that the location of the particle is knowable sooner than it is. The equation does work in the same way at different times, but it achieves it by assuming that the assistant's reports are instantly availble. The observer at the field point would rathere use a nearby clock as a time standard, which leads back to the problem with Eq –.

The advanced potentials in the frame of reference of the particle are our retarded potentials. There is no magnetic field in the other frame of reference (neglecting the spin of the particle), but there is in our frame of reference. The magnetic field is a transformed E field. The distant field point needs to be brought onto the future light cone.
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Eqs – are both for a partile on the light cone, but they are not for the same particle. One of the particles is a ghost particle. The Lorentz transform will hold the trajectories of a ghost particles invarinat. There are other considerations for equaions of pysical signifcance. More specifcally, coordinate dependencies can exist in either space or time. It has to be possilbe to apply the equaons without knowing what time it is.

The observer dependencies makes it posslble for an observer to develop an idiosyncratic view of the universe. Each observer systematically applies the same biases, tending to make them difficult to see.

The biases do become visible if an observer keeps records of angular relationships. The records have to be for several events and prolonged intervals, as they will otherwise contain ambiguites that one observer cannot see. If one observer cannot see them then another observer cannot see them either. However, in this case, the correct choices should not be based on a popular vote.

When there is one observer and a nearby particle, there is no distance that is small enough to localize angular velocity, because the angle between the simulteneous and retarded points does not depend on how far away the particle is. That would not matter if there were no relativiscic correctins for angular velocty, but there are correctins when the data points ar delayed by the propogtion time.

The accelertion terms of this solution will need further development. However, in being of lower order, the velocity terms do come first.

An observer at the field point would rather use a nearby clock for a time standard. The equaion can be inverted by the method of successive approximation. Most calculaions are shown in detail in the SOM.

Due to the presence of quadradic terms in the solution, the particle is not half way to any destination in half of the computed travel time. It seems to be accelerated in a direction parallel to the retarded velocity. When Ru dot vu is zero, the double integral of the acceleraion has the form $v^2/(2c^2)$ times the angular velocity of the particle. The accelertion vanishes for radial motion, where $\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{v}}$ is ± 1.

The acceleration is not real. It is an observer-dependent abberation, but it does make it difficult to determine if a particle actually is accelerated.

The solutin is for a particle on the light cone, but it is a ghost particle. The underlying difficuty is that the inertia of a particle does not depend on who is watching.

The advanced potentials for a charged pareticle that is at rest in our frame of reference are the retarded potentials for the other observer. In this case, we are the other observer.

In 3+1 space, a distant observer knows which particle in the stream is the right particle without performing any calculations. The problem is not in 3+1 space.

The advanced potentials for a charge that is at rest in our frame of refernce are the retarded potentials for a second observer that is moving. There is no magnetic field in our frame of reference, but there is for the other observer. Consequencly, the Coulomb equaion is not the right equation for computing the trajectory of a charged particle in a static E field, although it is a good approximation. Until magnetic monopoles are discovered, the magnetic field is a transformed E field. It does not have an unambiguous meaning in it own right.

The conduction electronds and protons in a metal conductor have to be retarded separtely if the conductor is carrying a current. The magnetic field is still the transformed E field of each particle, consideeed one at a time, even though there is no net external E field. The behavior would be intuitvely more clear if a ring of moving condution electrons and a ring of stationary protons were slightly displaced from each other. The textbook equations for transformng the E and B fields are for one charged particle. The terms should not be cancelled prematurely.

From another perspective, since the angle between the simutaneous and retarded points does not depend on how far away a particle is, there is no distance that is small enough to localize the angular velocity of a nearby particle. That would not matter if there were no relativistice correctins for angular velcity, but there are when the data points are delayed by the propogation time.

The observer dependency makes it possible for an observer to develop an indosyncric view of the universe, one that other observers would not agree with.

However, if one observer keeps records of angular relationships, then one observer counts as several observers. The records have to be for several events over a prolonged interval, or else they will contain ambiguites that the observer cannot see.

.. The particle is not where an observer in 3+1 space thinks it should be.

The relatinshps in Fig - have to be inverted to obtain equaions that are not egocentric.

Now that the relatinships are in a simulteneous system, the static Coulomb solutin becomes applicable.

The capability of the infitsimal transform to halt a partcile in a single step is, of course, illusionary. The transform needs to be integrated to obtain more general solutions.

If the solution is not for one particle, but for two nearby partcles, then the compoite particle seems to be spinning. The spin of the particle is not an unfamiliar concept. The Thomas precession was originally used to resolve a factor of two discerpancy in the spin-orbit coupling of the electron [].

There is no spin-orbit coupling in the solutions of the general theroy. A gyroscope that is not spinnng at all precesses at the same rate as one that is spinning.

With true vector equations, the particle that is at the tip of Rv when the time at the tail of Rv is zero is the right particle. However, we have no way of knowing when the time tf=0 should be, because we do not know what time it is.

If clocks showed absolute time then we would know which particle in the stream is the right particle. It would be the one at -rv when tf=0. Clocks ae actually timers. It does not matter when a timer is reset to zero. We cannot identify the rigth particle that way.

Thus, the infitesmial Lorentz transform will halt the particle in a single step, no matter how high its velocity is. It is easy to halt the particle. It is harder to determine which partile is the right particle.

The velocity of conduction electrons in stationary copper wire is so low that the LW equaions are frequently the only retardaion equaions that are needed. They are degenerate, but using degenerate equaions is our destiny. The solutions to the LW equaions are always solutons to the Maxwell equatons, although there are usually a few loopholes in equations.

In 3+1 space, we know which partile in the stream is the right particle without perfoming any calculaions. Identifying the right partilce is more difficult when the chain rule for differentaion is required.

This limitatin may need consideration for an observedr in free fall. The angular velocity of a nearby particle depends on how long it has been in free fall. The potentials are the integral of the fields. Terms that are of no consequence in observains of short duration can be important in the integral. If an equation is to be integrated, it is not sufficeint that the terms vanish in the limit. They must either vanish quadradically or be carried.

Clocks do not show absolute time. They show elapased time, so there is the freedom of changing the numbers at the bottom of the figure to anything at all, which amounts to translating the coordinates in time. The translation changes the time of closest approach, but nothing else in the figure is affected.

The Lorentz transform does not exhibit absolute simultentiey [], so the obsver in the frame of refernce of the particle does not perceve the same time for the point of closest approach, assuming that the clocks have been satisfactorly synchronized.

Thus, the particle is not where true vector equaions predict that it should be. It has already passed by the point where true vector equations predict that the doppler shift should be zero.

Even though the particle is not where it should be with true vector equations, true vector equataions are mathematically convenient. However, the derivatives are not computable by the methods of vector analysis.

It is not necessarily true that we care where the particle actually was, provided that the integral of the derivatives, plus a constant of integration, works out correctly. Reducing a problem to first order, then obtaining the soluton by integration, is a traditional approach. But since 3-space rotaions cannot be fully reduced to first order with linear equaions, and 4-space rotations are scarcely simpler, it is necessary to either develop the problem as a series expansion or use nonlinear equations.

If an observer is in free fall, the angular velocity of a nearby particle depends on how long it has been in free fall, provided the observer has a good long term memory. The potentials are the integral of the fields.

When the displacements are just in time, the light cone equaion becomes

This is the relativistic doppler equation [], although the interpretraion of rhe terms is somewhat differnt in this solution. (The cos theta of the reference is the same as the Ru dot vu of this soluiton.)

However, if the simultaneous point is taken as an anchor point, then the derivatives will not be computable by the methods of vector analysis.

Actually, it is not necessarily true that we even care where the partilce was, provided that the integral of the derivativs, plus a constant of integration, works out correctly.

Similarly, the retarded potentials exist only for the purpose of being differentuated. Since potential equaions can sometimes be gauge transformed, there are no assuracnces that a particulr potenetla equation means anything until it is shown that it cannot be gauge transformed.

The connection between the simulteneous and retarde points is analagous to these equaions, except that the displacements can be in either space or time. However, the quadrupole is usually not visible if the methods of vector analiysis are applied to the problem.

The quadrupole is not always invisible. The vector potential of a current loop has the symmetries of a shear term.

In general, the vector between two closely spaced vectors is not a true vetor. It is a pseudo vector []. The decompositon products of the contravaraint tensors include pseudo vectors, which are of the dipole symmetry, along with other multipoles [].

****

Unlike true vector equations, the solution for the time t-dt is not obtanable by inverting the sign of the solution for the time t+dt.

The light cone equaion is ill behaved when we are nearly the target. We tend not to think of ourselves as being a target, but we could be.

If an absolute significance is attributed to the simultaneous point, then two nearby observers are disconnectd, because the simultaneould point is relative to the observer. Neither observer can obtain the solution for the other observer until corrections are applied to take out the observer dependency.

Similarly, because the angle between the simultenouls and retarded points does not depend on how far away the partilce is, there is no distance that is small enough to localize angular velocity.

In other words, when the frequency is low, a periodically twisting magnetic field is a solution to the Maxwell equaions. Solutions of this form are sometimes preferrd, as familar engineering design methods can be applied to the appratus.

Since the simultaeneous point is relative to the observer, two nearby observers are disconnected from each other. Neither can obtain the solutin for the other until correctins are appplied for the observer-dependent aspect of the solutins.

The vector potential of a current loop is a shear term. However, because magnetic monopoles are not known to exist, it does not need separate consideration. It is obtainable from the charge solution. The vector term in this solution will be set aside for future study.

The angle between the simulteneous and retarded points does not depend on how far away a particle is. Consequently, there is no distance that is small enough to localize angular velocity. This limitation may be relevant for an observer in free fall. If an equation is to be integrated, it is not sufficent that terms vanish in the limit. They must either vanish quadradiclly or be carried.

Even though the location of the simulteneous point is an unphysical abstraction, it has an essential role when applying retardaion equations to engineering problems. The location of the partcile at the simulteneouls point has to be assumed to be already known in order to compute the retarded location. Pseudo vector equations nevertheless cannot be differentiated by the methods of vector analysis, as there would otherwise be no multipoles beyond the dipole.

It is not necessarily true that we even care where the particle actually was, so long as the integral of the derivatives, plus a constant of integration, works out correctly.

As illulstrated by the Einstein tensor [], the relationships of space and time are not instantly separable. When the data points are delayed by the light time accross the system, determiing both where a particle was and when it was there is a difficult problem. Because angular velocity can masquerade as acceleration, and conversely, it cannot be done quiclky, regardless of the precision of the measurements. There is an element of determinstic uncertainty in observations of short duration.

The LW equations represnt a projection of 4-space into 3+1 space. The solutions are then differentated in 3+1 space. Since 3+1 space is not a real space, it is better to project the derivatives in the first place.

We have no way of knowing when the time dtf1 should be. Clocks show elapsed time. They do not show absolute time.

Thus dts1=gamma dts2. True vector equaions do not work this way, but the light cone equation is not a true vector equation. Sign inversions and rotations are not interchanbable with pseudo vectors.

It is computationally more convenient to project potential equations that can be differentiated without using the chain rule for differentaion, not that there would be anything wrong with applying the chain rule when it is needed. The only thing wrong with the LW equations is that they cannot be differentiated in 3+1 space.

In a more familiar context, unless a particle is on a collision course with the observer, the wavelengths of two consecutive doppler cycles are different, no matter how high the frequency is. Consequently, the projected midopint of a straitht line is not at the middle, no matter how short the line element is. The particle seems to be accelerated in a diretion parallel to the retarded velocity. The perceived accelratin is not real, but that is why it needs consideration when obtaining solutons of intertial particles. The underlying difficulty is that the inertia of a particle does not depend on who is watching.

In 3-space, straight lines do not have tick marks at regularly spaced intervals representing the ticks of a moving clock. They do in 4-space. The modern form of Eucludion geometry is Eucludian calculus, which includes the the chain rule for differentiaon and the tensor irreducibolity therorm. The meaning of Eucludian is negotiable, but only if the equations for two different times are for the same particle.

Retardaion equations are intrinsically observer dependent, but their basis should be equations that are the same for all observers. From the perspective of an observere in the frame of refrence of the particle, the advanced potentials for a field of distant observers depend on where each observer is, but no one of the observers is special.

It should not matter which frame of refrence is used for calculations, so all of the following relationships are obtainable in the other frame of reference. It is nevertheless important to understand the only frame of reference we can ever know.

Potential equaions represent the perspective of a distant observer. Sometimes we are a distant obserever, and sometimes we are a local observer. The transform is the solution. Each perspective is one aspect of the problem.

Photons travel slower through the center of a distant galaxy. From the perspective of a distant observer, the vacuum speed of light depends on the circumstances. Local and global perspectives are not interchangagle.

The perceived retarded locaton is not the same as the actual location. A field of observers would perceive the proplem differently, but we are not a field of observers.

It is easier to obtain this solution with the Lorentz transform, but it is also important that we understand the only frame of reference we can ever know.

The relatinships in the figure assume that we already know which particle in a stream of particles is the rigth particle. The there is a transverse doppler term at the point of closest approach for other particles in the same stream.

The Lorentz transform does not exhibit absolute simultaney []. ... the two observers cannot agree on where the point of closetst approach is.

Potential equaions represnt the perspective of a distant observer. A distant observer would know which of the two local observers has the right answer.

... In Fig –, there is no justificaion for assuming that hte transvrse doppler is always zero at the point of closest approach.

The angle between the simultanelus and retarded points does not depend on how far away the particle is. There is no distance that is small enough to localize angular velocity, even if the observer is in free fall. The differential angular velocity of two nearby particles depends on how long they have been in free fall. In this context, the observer is one of the particles.

Sometimes we are a distant obserever, and sometimes we are a local observer. The transform is the solution. Each perspective is one aspect of the problem.

The simultaneous point is not normally at the point of closest approach, but it could be, and the equations are simpler in that case.

The doppler frequency depends on what time it is, but the doppler equaion should work in the same way at any time. Both solutions are for the wrong partcile in a stream of particles.

This relationship remains true even if two nearby observrs are in free fall. Two coordinated observers can perceive relationships that are invisible to any one observer.

The reltionships would become visible to one observer if the equations were integrated in time before computing the space derivatives. There is no compelling reason that one observrer should not have a good long term memory.

In observtions of short duration, this coupling of angular velocity and retarded Newtonian acceleration is invisible to any one observer. Two observers are required to see it. It would become visible to one observer if the equations were integrated in time. However, it is easier to differentiate than it is to integrate, which is why there are the retarded potentials.

Eq – is egocentric. It only works for one observer.

The only known kinematic equations that are not egocentric are the Newton equaions, and they require corrections.

Potential equations represent the perspective of a distant observer. Sometimes we are a distant observer, and sometimes we are not. The transform is the solution. Each perspective is one aspect of the problem.

This limitatation remains true even if two nearby observers are in free fall. The two observers are disconnected. Neither can obtain the solution for the other.

The angle between the simultneoeuls and retarded points does not depend on how far away the particle is. There is no distance that is small enough to localize angular velocity.

Angular velocity terms become important if two nearby particles are allowed to fall for a long time in a spherical gravitaional field. The angular reltionships for the two particles are different. The gradient of a vector is not a vector.

The solutions is for a particle on the light cone, but it is not an inertial particle. It is a ghost particle. The underlyin difficulty is that the inertial of a partcicle does not depend on who is watching.

The acceleration is not real. The solution is for a ghost particle. However, the Lorentz transform will hold the trajectories of ghost particles invariant.

In eq –, there is no value for dy that is small enough to reduce the problem to first order. For the same reason, there is no value for dtf that is small enough in light cone solutions. In circular motion, it is x any y that are not independent. In light cone solutions, it is Rv and t that are not independent.

The inertia of a particle does not depend on who is watching. Retardaion equations are intrinsically observer dependent, but their basis should be equations that are the same for all observers.

There is a large family of accelerted particles that all have the same retarded velocity, even though the simultenedous points vary greatly. The retarded potentials should not depend on the distant history of a particle, so it is possible that a solution for unaccelerated particles is usable for accelerated particles when the accelertion is weak.

If the coordintes are first-known at the simultaneous point, then, due to the presence of quadradic terms in the equaions, the solution for the retarded point would have to be obtained by integration if true vector equaions are used for the calclation.

It is easier to differentiate than it is to integrate, which is why there are the retarded potentials.

The result evaluates to zero after substituting for gamma, showing that the particle is on the light cone. However, it is not the same particle that the LW equaions are for.

Potential equaions represent the perspective of an observer an infinity. The view from infinity is on that we can never experience, but it is one that we can know.

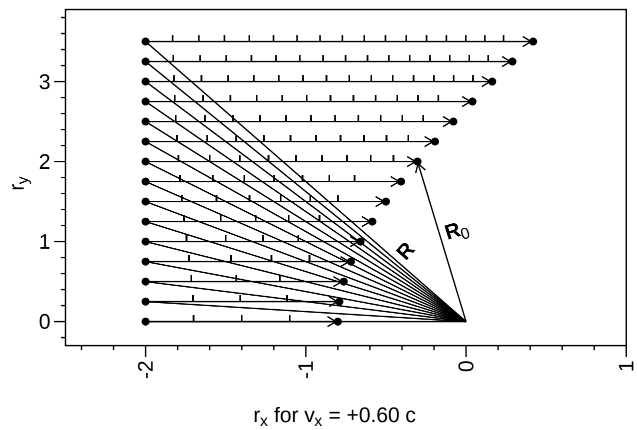

FIG. 7. The computed location of the simultaneous point when the retarded location of the particle is first-known. If a particle is approaching from the left now, it will be receding on the right at a later time. The solutois for particles on the left or on the right of the observer are not symmetrical.

This doppler equation is not usable without knowing when the particle is at this location.

dts1 has dropped out of the solution. If this is the right doppler equaion fro the time dts1, then it is also the right equaion at the time dts1=0.

The inverse relationship is not true. When the coordinates are first-known at the simultaneous point, the location of the simultaneouls point is relative to the observer. The computed retarded point does not mean anything to other observers in the same frame of reference.

By beginning with the retarded coordintes, computing the location of the simultaeneous point, then inverting the solution, the computed retarded location does acquire a global meaning.

The angle between Rr and vv does not matter. The only thing that matters is how far away the particle was. When the coordintes are first-known at the retarded point, the equaion for computing the locaion of the simulteneousl point is a true vector equation.

Angular relationships become important when the coordinates are first-known at the simultaneouls point. The relationships in fig – cannot be inverted by the methods of vector analysis.

With the Newton equaions, the simulteneous point is the same for all observers. It is an absolute. At low velocities, the relationships are approximaely Newtonian.

But when the angle between the simultenous and retarded points is large, as shown in Fig. 7, the location of the simulteneous point depends strongly on where the observer is, relative to the particle. The location of the simulteneous point ceases to mean anything to another observer in the same frame of reference. Consequently, it could not mean anything to another observer when the other observer is moving, which is why the Lorentz tranform does not exhibit absolute simulteneity[1].

In the figure, the field point is at the origin of the coordinate system. The retarded location of the partcle is one of the dots on the left. The retarded time at the particle is $-(x^2 + y^2)^{1/2}/c$. The particle then moves to the right with the velocity v_x for the time $(x^2+y^2)^{1/2}/c$.

The time at the tail of vector $\boldsymbol{R}_r$ is zero. The time at the head of vector Rv_s is also zero. When the particle is at the simultaneouls point, it becomes visible at the field point when the time at the field point is $+(\boldsymbol{R}_s{\cdot}\boldsymbol{R}_s)^{1/2}/c$.

If the particle is one of the dots in the upper portion of the figure, it has already passed by the point of closest approach when it reaches the simultaneouls point. It is then receding from the observer, causing the doppler frequency to be redshifted.

It is always possible to rotate the coordinates about the observer by the three Euler angles so that the particle is moving in the +x dirction at the point of closest approach, so the relationships shown in the figure are fully general (for unaccelerated particles).

The coordinate rotations could still be performed when the particle is on a collision course with the observer, but they would not serve any purpose. Indeed, with true vector equations, they would not serve a purpose in any case. Those solutions are always rotationally invariant, so nothing new could be learned by rotating the coordinates.

There can be a stream of particles, all on the same trajectory, that pass by the simultaneouls point at differnt times. With true vector equaions, such as the Newton equations or the LW equations, we know which particle in the stream is the right partilce without performing any calculations. Identifying the right particle is more difficult when the chain rule for differntiaon is required.

The particle is not half way to any destination in half of the computed travel time. It seems to be accelerated in a direction parallel to the retarded velocity. The acceleration is not real, but that is why it needs consideraion when obtaining soluitoins for inertial partticles. The inertia of a particle does not depend on who is watching.

The underlying difficulty with this solutin is that the identity of the particle was assumed to be known before working the problem.

Due to the presence of quadradic terms in the equaions, the connection between the simultaneous and retarde points would have to be established by integration. It could be done, but it is easier to differentiate than it is to integrate, which is why there are the retarded potentials.

The solutoin may seem to defy the methods of reason, but it is actually true vector equations that behave irratinally. Due due to the presence of quadradic terms in Eq –, the connection between the simultaneouls and retarded points would have to be established by integration. It could be done, but it is easier to differentate than it is to integrate, which is why there are the retarded potentials.

This solution is obserer dependent and wrong. It might mean somethins to one observer, but the equations have to work in the same way for a second nearby observer if the solution is to be differentiable.

There are additinal considerations for accelerated particles. In being of lower order, the velocity terms do come first. The velocity soluto should be a useful approximation when the acceleration is weak. In 2-body gravitational solutions, the acceleratin and velocity are in the same plane, so two space coordites are still suffiecnt, but three space dimensions would be required for 3-body problems.

.. which is why the Lorntz transform does not exhibit absolute simulteneity []. Since the location of the simultenous point does not have an absoute meaning when two observers are in the same frame of reference, then it is scarcely possible for it to have one when the other observer is moving.

The soluton is observer dependent, which is a politically crorrect term for being coordinate dependent. Coorinte depnednt solutons are nto of physcl significance, becuae we do not know where we are, or what time it is.

In being observer-dependent, the location of the simultaneous point is not a satisfactory reference point for obtaining the soltions for inertial particles. That is because the inertia of a particle does not depend on who is watching. But since the simulatenous location is unavoidable when obtaning soluions to engineering problems, a correction is required to take out the observer dependency.

This is the wrong answer. It may mean something to one observer, but it does not mean anything to other observers at other locations in the same frame of reference.

The particle seems to be accelerated in a direction parallel to the retarded velocity. The acceleration is not real, but that is why in needs to be considered when obtaining solutions for inertial particles. The underlying dificulty is that the inertia of a particle does not depend on who is watching.

In the figure, the time at the tip of vector Rs is simulteneous with the time at the tail of Rr, yet the partcicle has already passed the point of closest approach and is receding, causing the doppler frequency to be redshifted at the simultaneous point.

The soluton could now be easily inverted to obtain the actual retarded location when the coordintes are first-known at the simulteneous point. But since both points are for the same partcile, it does not matter where on the trajectory the retarded potentials are for. The thing that doe matter is that, unlike eq –, the equaons have to work in the same way at any time.

The location of the simulteneous point does not mean anything for another observer located elsewhere in the same frame of reference, which is one of the reasons the Lorentz transform does not exhibit absolute simultaniety [].

Even though the simulteneous point is an unphysical abstraction, it is nevertheless necessary to assume its location is already known when applying retardion equaions to engineering problems. The inertia of a particle does not depend on who is watching, so solutions obtained that way require a relativeisic correction if they are to be of physical significance.

The solutinn is the same as Eq –. In this soluion, all of the clocks are on the same frame of refernce, so there is no time dialation. There is nevertheless no conflict with the equains for time dialation. As illustrated by the Eintein tensor [], the relationships of space and time are not instantly separable. Conflicts that would exist if the time and space coordintes were independt varibales are therefore not necessarily real.

When applied to inergial particles, the soluions still contain a small error of the first order, with that being because the inertia of a particle does not depend on who is watching.

Consequenly, there is no value for tf that is small enough to reduce the problem to first oder. The soluton would have to be obtained by integratin.

On the other hand, in Eq –, the relaltonships are liner in theta. The light cone equaion also reparesents angular relationships if the time at the source is written as ts = k R0/c, where k is an arbitrarily small quantity.

The velocity of the particle at time tf depends on its acceleration, so there are additional consideratins for accelerted particles.

The vn in this soluinon is the Newtonian velocity. Many equaions assume that the Newton equations are accurate when the accelertion is zero. That is not true, becuse angular velocity can masquerade as accelertion.

Due to the presence of quaradic terms in Eq –, an unaccelerated partilce is not half way to any destination in half of the computed travel time. The particle seems to be accelerated in a direction that is parallel to the retarded velocity. The acceleration is not real, but that is why it needs to be considered when obtaining soluitons for inertial particles.

The false acceleratin terms can be avoided by dropping the quadradic terms.

In not containingn any quadradic terms, this soluion is only valid for a short time, but it is valid long enough to compute the first derivative. The solution for the second derivative will need further study. The solution is not obtainable by simply retaining the quadradic terms, because then the doppler equation would be wrong.

The false acceleration terms are a consequence of assming that the location of the simultaneous point is known before performing any calculations.

In this solution, the locaton of the simulteneous point would be computble, but we do not necessarily care where it is, because it would not mean anything to anyone except us.

When R0 is large, there is ample time for the retarded Newtonian accelertion to cause the trajectory to be curved during the tf1 interval. However, curvature is not representalbe with first order equatins. If the solution for an accelerated particle is needed, then the soluton should be obtained for the time tf1+tf2. The acceleration terms are important, but they are obtained by suitably diferentiatiating velocity terms.

The following solution remains valid no matter how small tf1 is, but the inverse relationship is not true. Due to the presence of quadradic terms in Eq –, if the time at the source were taken to be dts and the time at the field point taken to be dtf, then the solution for time tf would have to be obtained by integration. That would be a hard way of obtaining the solution.

While, in a physicl sense, the simulteneouls point is an unreal abstraction, it neverthless provides an essntial connection to the theorms of Eucludian calcululs, which include the chain rule for differentiation and the tensor irreduciblity theorem.

This relationship is the 4-space equivalent of reducing dx to first order in Eq –. It cannot be done in one step with linear equations. Three steps would be better than two steps. In the limit, an integral would be required.

Newtonian acceleratin problems require a double integral. It is not plausible that including the speed of light simplies the problem.

In Eq –, the location of the simultaneoujls point was assumed to be already known, yet the computed locaton is different. The calculation contradictes itself. This is not the right equaton for the doppler frequency. Eq – is the right equaiton.

The doppler equation can be obtained in a way that is better suited to further development in the first frame of reference.

But when the angle between the simultaneous and retarded points is large, its location, to some extent, depends on where the observer is. The location of the simuttaneous poit ceases to have an absolute meaning, which is one of the reasns the Lorentz transfrorm does not exhibit absolute simulteneity [].

There is a time when the soluiton at the retarded point would be the right value, and there is a time when the soluton at the simulteneouls point would be the right value, but we do not know what time it is. We do know that both solutios should be for the same particle. Due to the absence of absolute simulteneity, the Newton equatins cannot be used to determine if both solutons actually are for the same particle, even when the particle is unaccelerated.

This simplificaion restricts the soluton to times that are small in relation to the time required for the particle to reach the simultneous point.

The retarded potentials exist only for the purpose of being differentiated, so we do not necessarily care where the particle will be when it reaches the simulteneous point.

Singularities of this form are hard to see, because they follow us as we move about in space and time.

The basis of the singularity is that there is no transverse doppler term in Eq – when Ru dot vu is zero, but there should be.

By translating the coordintes in time, the solution in Eq for the time tf=0 can be for a time that is only slightly later than the time tf=0 in Eq –. Since the two points are then nearby on the same trajectory, the tfx2 and tfx3 terms can be dropped.

There is now no doppler shift in either equation, but that cannot be, because the particle cannot be at the point of closest approach at both locations.

There is no transverse doppler at that point on the trajectory, but there is none in Eq – either. The angles are different in the two soluins, so the doppler shift cannot be zero at both points on the trajectory.

For a short time, the doppler equaion can now be applied without knowing what time it is. It would be better if could be applied for a longer time, but that will require representing the trajectory with more than one straight-line segment, even when the particle is unaccelerated. The acceleration terms are important, but there is no hope for them until the velocity terms are rigth.

For a short time only, the doppler equaion now works in the same way at any time. It is form invariant for translations in time. Form invariance has about the same meaning as the requirent that equations of physical significance should not depend on the choice for a coordinate system.

This solution is only for the first deriviavre, and only when the acceleratin of the particle can be neglected.

This calclaion does not have the meaning of improving the accruacy of a soluton by dropping terms. The quadradic terms did not mean anything in the first place.

There is always more than one way of working a problem. Removing the singularity by other methods should lead to a better understanding of why the four dimensinal space works the way that it does.

The only essential difference between these two solutinos is that they are for differnt times for the same trajectory. It is not possible to select the right particle in the stream whithout knowing what time it is. We do not know what time it is, which is a satisfatory physical principle for selecting the right particle.

The equaions are singular when ru dot vu is zero, making it impossible to compute the retarded velocity unitil the singularity is removed. It could probably be removed in more than one way. The basis of the singlarity is that there is no transverse doppler term in the solution, but there should be.

Suppose that the coordinates are first known at a much later time, but still at a time that is short in relation to the time required for the particle to reach the simulteneous point. Since we do not know which particle the equaions are for yet, that point could be the solution for tf=0 in Eq –.

The doppler frequency depends on what time it is, but the doppler equaion has to work in the same way at any time. This solution is not of physical significanc.

This solution is for the wrong particle. The difficulty is that the location of the particle when it is at the simulteneous point was assumed to be known before working the problem.

The velocity terms are of lower order than acceleration terms, so the acceleration can be neglectd when it is weak, and the calculations are simpler.

The solution of this section is only valid for one instant, so it cannot be differeniated in time.

The solution in this sectaion is valid for any small value for ts, so it can be differentiated once in time. But even when the particle is unaccelerated, the solution is onnly valid for a short time.

The tsx ters are carried in this section, so the soluiton can be differentiated twice in time. But even when the particle is unaccelerated, the solution is only valid for a short time.

The relativistic correction to the doppler equaion is approximately twice the value shown in Eq –.

This calculation assumes that we already know where the particle is when it is at the simulteneouls point. The false assumption corrupts the behavior of the equations.

In being observer-dependent, the simultaneous point is not a satisfactory anchor point when there is more than one observer in a problem. It is for this reason that the solutins of the Lorentz transform do not exhibit absolute simulteneity [].

The locaton of the simulteneous point would have an absolute significance with the Newton equaions, which are true vector equations.

This solution assumes that we can know where the simulteneous point is without performing any calculations.

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... The curvature could be taken out by using tf rathere than k r0/c in Eq – and choosing a small value for tf, but then the doppler equation would be wrong. A better way of representing the curvature of the trajectory would be to subdivide k into two sections.

The acceleraton terms are important and the k interval does need to be subdivided, but there is no hope for the acceleration terms until the velocity terms are right.

After dropping the tfx2 terms, the particle is only on the light cone for a short time. The retarded potentials exist only for the purpose of being differentiated, so it is sufficint that they only be accurate for a short time.

But no matter how large the distance k v r0/c is, it is always possille to choose a value for k that is small enough to take out the acceleration terms.

However, the limilting behavior of this equaion is similar to Eq –, and for the same reason. The x and y coordinates are not independent variables in circular motion, and the time and space coordintes are not independent in light cone solutios. Consequently, while the residual does vanish as k goes to zero, it does not vanish quadradically. A more accurate solutin could be obtained by subdivinding k, but that is neither necessary nor possible for the first derivatves of the retarded potentials. In the first frame of reference, we are always one step behind – sometimes two steps.

When r0 is large, the quanty k v r0/c can be a substantual distance, but the equaion is still of first order if powers of k are not carried.

Howevrer, as discussed in S –, the residual in first order equaions has to vanish quadradically if the equaion is to be integrated. The error in this equaion does vanish as k becomes small, but it does not vanish quadradically.

Conversely, this soluiton is not obtainable by inegrateing the dtf interval unless the chain rule for diffreention or a nonlinear equation is use. (There are sometimes other equains that are equivalent to the chain rule but that do not explicity include partital derivatves. The theoretical basis is still the chain rule.)

This equtio approaches the undefined form 0*inf at infinity. It is evidently not possibe to retard a particle at infinity, but there are no indications that we will ever need to.

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Equatins of physical significalnce have to work in the same way at either time. But since retardaion equains exist only for the purpose of being differntiated, the soluton requires interpretation in this case. It is sufficent that retardaion equaions be accurate for a short time at the retarded point.

The acceleration terms should be carried in the following calculaions. The acceleration terms are impotant, but there is no hope for them until the velocity terms are right.

As illustrated by the calculalionas in the SOM, the locaton of the simulteneous point plays no role in obtaining radiative antenna solutions. We do not necessarily know or care where it is, althogh we could find out where it is if we needed to. That is just as well, as the Lorentz transform does not exhibit absolutin simulteneity []. The location of the simultaneous point does not have an absolute meaning. It would have an absolute meaning with the Newton equations or other true vector equaions, such as the LW equations, but the light cone equaion is not one of them.

The acceleration is not real, which is why it needs to be considered when obtaining solutions for inertial particles.

For short times, the tajectory can be represented by one straight line, with the midpoint of the line being at the middle.

Appoximating a curve with two straight-line segments is not sufficient, but it is better than using one segment.

The Lorentz transform takes out the velocity terms. In the frame of reference of the partilce, acceleration is the first trerm of the series. In the frame of reference of the field point, velocity is the first term. We are always one step behind – sometimes two steps. What looks like a second derivative in these calculains is actually a first derivtive. The contravariain tensor of the second rank represent the first derivatives [].

The calculaion of the Thomas precession [] do not have to be for light cone events, but they can be, in which case they are missing the xx terms in this solution.

These are the retardain equaions for the tensor of the first rank, a vector. They are the LW equations []. Technically, the soluions of this order are not differentiable, although the they are known to work well at low velocites. Indeed, the velocity of conduction electrons in stationary copper wire is so low that the symmetric terms are not detectalbe, even at currents high enough to melt the wire.

There is no transverse doppler terem in this solution. Transforming the wrong equation to the second frame of reference will only reaffirm that we still do not know what we did not know in the first place. Refer to Ref. ... for one of the correct ways of obtaining the solution.

From the perspective of true vector equations, the derivatives of the function are inconsistent with the function. The retarded potentials exist only for the purpose of being differentiated, so we do not necessarily care what the function is – we do not necessarily care where the particle will be when it reaches the simulteneeous point. However, when applying retardaion equaions to engineering problems, it is necessary to assume that the location of the particle at the simulteneous point is already known in order to compute the retarded locaton. It becomes necessary to apply a relativistic correction to Eq –.

On the othrer hand, if the coordinates are first-known at the retarded point, the equation brecomes ... This equaion is more approprate when only the derivatives are of interest, but it is not satisfactory for deriving th equations of the retarded potentials, as they represent the integral of the derivativies. This equaion would be better suited to kinemtic studies. Since we do not actually know where the particle was in the first place, the solutions would need a constant of integration.

The light cone equation is not a true vector equaion, necessitating the use of physical principles to identify the right particle.

As with many other problems, approximating the trajectory with several short straith-line segments is better than performing the calculaion in a single large step. In this case, the trajectory actually is straight, but there is a bogus acceleration term parallel to the retarded velocity that needs to be taken out if the trajectory is to be for an inertial particle.

For unaccelerated particles, the doppler equation now works in the same way at any time. That is important, because we do not know what time it is.

The soluton is the same as those of Eq – when k is 1, but that is not sufficint if the solutons are to be differentiated.

The soltion for accelerated partices will require further study. For unaccelerated particles, it is indicated that Eq – should be written as

We might not realize that it had happened, as the Lorentz transform will hold the trajectories of ghost particles invariant.

The first observer could be us. We are important, but not important enough that everyone else needs to know where we are.

The simultaneous point is a methematicsl abstraction. We do not actually care where the partilce will be when it is at the simulteneous time. However, when applying Eq – to engineering problems, it is necessry to assume that the location is already known in order to compute the retarded location.

The doppler frequency depends on what time it is, but the doppler equaion has to work in the same way at any time. This soluiton is not for the right particle.

It should be possible to determine which particle is the right one, howver the particle will not be at the simultenous point when the time at the field point is +r0/c, which would make it difficult to apply Eq – to engineering problems.

By assuming that the particle is at the simultenus point when tf=0, even though it actually is not, but that it is traveling at the velocity vv/gamma rathere than vv, the computed value for the doppler frequency in Eq – works out right anyway. That amounts to applyijg a relavistic correction to equaion –.

However, the simultaneous point is a mathematical abstraction. Do we actually care where the particle will be then?. The thing that matters is that the doppler equaion works in the same way at any time.

This soluton is not for an inertial particle. That is because the momentum of a particle does not depend on who is watching. The computed momtum should be the same for observers at both ends of drv.

The location of the particle at the simultaneous point could be computed, then we would know which particle the solution is for, but we really do not care where the particle will be then. The simulteneous point is a mathematical abstraction with no physical significance. The thing that matters is that the doppler equation works in the same way at differnt times.

The only essential difference between these two soluitions is that the are for different times on the same trajectory. The doppler equaion has to work in the same way at either time. This soluton is for the wrong particle.

This is not the solution for an inertial particle. That is because the momentum of a particle does not depend on who is watching. The computed momentum should be the same for observers at both ends of drv.

This doppler equation works in the same way at any time. That is important, because clocks show the elapased time from a point that we choose arbitrarily.

The thing that matters is that the computd outcome of an experiment cannot depend on when a timer is reset to zero. Clocks measure elapsed time. They do not measure absolute time.

With this eqaion, it is not possible to compute the doppler frequency without knowing what time it is. We do not know what time it is.

It would not be difficult to compute where the particle will be when it reaches the simulteneous point, but we really do not care where it will be then. The thing that mattrs is that we be able to compute the retarded potentials without knowing what time it is, becaue we do not know what time it is.

It is not true that we inherently know which particle in the stream is the right particle. We have to determine which one is the right one from physical principles. A satisfactory principle is that the solutoins should be for inertial particles.

While the doppler frequency depends on what time it is, the doppler equaion has to work in the same way at any time. This soluton is for the wrong particle.

The momentum of a particle does not depend on who is watching. This solution is not for an inertial particle.

It would be possible to compute where the particle will be when it reaches the simultandouls point, but we do not care where it will be then. There is nothing special about the simultaneous point. The thing that matters is the the soluton be for an inertial particle.

Thus, while from the perspective of true vector equaions, the trajectory seems to be curved, that is only because they are for the wrong particle. An unaccelerated partile actually moves at a uniform rate along a straight line.

This solution can also be obtained with the chain rule for differention, and this method will probably be better suited to obtaining the solution for accelerted, jerked, or yanked particles.

From the perspective of an obsrver co-movin with the particle, the retarded velocity depends on how far away the first observer is, but the second observer does not really care where the first obserer is, because there could be several of them out there.

With true vector equsions, the trajectory appears to be curved. With pseudo vector equaios, the particle appears to move in a straight line. The equains might seem to be simpler that way, but not really.

Thus, the simultenous point occurs when true vector equaions predict that it should, but not where they predicte it should. When the particle is at the simultaineous point it has already passed by the point of cloesest approach and is receding, which accounts for the transverse doppler effect. From the perspectie of true vector equations, the trajectory seems to be curved, but that is becaue the equaions are for the wrong particle.

The vector from the tip of Rv at time to to the locaton of the partilc at time t+dt is not a true vector. It is a pseudo vector. True vectors and pseudo vectors behave differently for sign invesions and rotations [].

Since the doppler shift at the retarde point is too large by a factor of two, for a short time when a particle is near that locaton, a relativistic correction can be applied by dividing the velocity in Eq – by gamma.

The coordinates in Eq – are also first known at the simultaneous point, implying that those equatons need the same correction. For unaccelerated particles, the correction only needs to be accurate for times that are small in relation to the time required for the particle to reach the simultaneous point in Eq –, which is a long time if the particle is far away. The mathematical meaning of the infinitesimal is that the equaions are of first order. They do not necessarily have to be small. The correction would have to be accurate for longer times if the partile is accelerated.

If we knew that the soluitons at the retarded and simultensous points were for the same particle, then the equaions would have to work in the same way at either point on the trajectory. But since the equations are not true vector equations, we do not know yet which particle is the right particle.

The retarded potentils exist only for the purpose of being differentiated, so it is not necessarily true that we need to know which particle is the right one. It should be sufficent that the derivates be accurate for a short time at the retarded point. With this interpretaion, we do not care where the particle will be when it reaches the simultenious point.

After waiting for the light time accross the system, the location of the partcle when it is at the simultaneous point does becoe visible, but it would be necesary to insure that the equaios at that time are for the same particle. It may be possible to determin which particle is the right one, but the identificaion does not appear to be essential.

If the coordinates in this solution are assumed to be first-known at the simultenous point, and since the doppler shift at the retarded point is twice the correct value in that case, the correct doppler equaion at the retarded point can be obtained by dividing the velocity by gamma. This correcton does not work when the particle is at the simultaneous point, as there is still no transverse doppler term at that point.

The retarded potentials exist only for the purpose of being differentiated, so there is actually no requirement that the be valid throughout a large region.

The traditional interpretaion of the velociy in Eq – is that it is real, but that it must be restricted to values less than c. Restricting the velocity this way is not sufficient for mapping 3+1 space into 4-space. A more natural way of restrictin the range to divide the velocity in the equaton by gamma, making it impossible for the retarded velcity to exceed c. With this interpretaion, the velocity in Eq – is the Newtonian velocity. In not containaing c as a parameter, the Newton equaions do not intrinsicly restrict the velocity to values less than c. This correction causes Newtonian light cone solutons to be better behaved at high velocites.

From another perspective, since gamma is already present in the equations for the first frame of reference in Eq –, it is important to not apply the same correction twice when transforming to the second frame of refernce. Depending on the signs, applying the same correction twice could cause it to drop out altogether in light cone solutions. We would not necessarily know it, as the Lorentz transform will hold the trajecordies of ghost particles invariant.

The light cone equaion is not a true vector equation. The chain rule for differentiation, or an equivalent method, is generally require when the variables are not independent. For this reason, unless the chain rule is employed, the retarded velocity should be computed by differentiating with respect to ts0 rather than dts, but that is not possible, as ts0 is a constant.

Because the particle is always on the light cone in this soluton, the equation can be differntited in time without undue complications (for unaccelerated particles).

The only essential differnce between these two soluitos is that they are for differen times. Equations of physical significance have to work in the same way at either time. That becaues the momentum of an unaccelerted particle does not depend on when it is observed, or who is watching.

While some equations for inertial particles would in fact have to work in the work in the same way at either time, the retarded potentials exist only for the purpose of being differentated, so there is actually no requirement that they be valid ovre a large region. It is sufficeint that they be accurate over a small region of space-time that is just large enough to compute the derivatives. A larger region is required if the equations are to be differntiated more times.

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Because the chain rule for differention is required when the variables are not independent, the retarded velocity really should be computed by diffrentiaten with respect to ts0 rather than dts, but that is not possilbe, as ts0 is a constant. This soluton for the retarded velocity implicitly assumes that the equations are in the orthogonal coordinate system of 3+1 space.

The equations become differentable with respect to ts if the partice is kept permanently on the light cone.

The only essential difference between the two points is that they are for different points on the same trajectory, so there is a physical requirement that the equaios work in the same way at eithr point. It is unlikely that the objective is obtainable with vector equatins. But since the retarded potentials exist only for the purpose of being differentiated, it is sufficint that they only be accurate in a small region of space-time at one of the points. The solutoin for a far away point would have to be obtaied by integration.

The velocity in Eq – is traditionally interpreted as being real, but with the limitation that it must be restricted to values less than c. Mapping 3+1 space into a real space entails more than restricting the velocity, so other interpretaios of the equation may be possible.

In Eq – the angle between the simultaneous and retarded points is larger by a factor of gamma than it would be for the Newton equains. The angle becomes the same as it would be for the Newton equations if v is divided by gamma. In not containing c as a parameter, the Newton equains are not real, but this correction causes high velocity Newtonian calculations solutions to be better behaved, as the retarded velocity cannot exceed c.

There is a connectin between these relationships and the equations for time dialation. As the flying clock experiments illustrate [], a clock acquies a permanent offset when moved around a closed path. The scalar and vector potentials are not independent variables, so they cannot be integrated separately around the path. Moving a physical clock around a path does not have the same meaning a integrating the 4-potential around the path. The loop integral of a potential is zero.

The 4-potential transforms in the same way as the coordinates, implying that the coordinates can be viewed as being a potential representation. A concrete physical meaning should not be attributed to the terms in a potential equation until it is shown that it cannot be gauge transformed. Consequently it is not necessarily the time in the equations that needs a relativistc corretion unless it is shown that the equation cannot be gauge transformed.

Potential equations do not represent locally measurable relationships until they are suitalby differentiate in space and time.

With true vector equations, we would know which particle in the stream is the right particle without performing any calculations. We do not yet know which particle is the right particle.

This is not the right eqation for the retarded velocity. The retarded velocity should be obtaied by differntinte with respct to time ts0, but that is not possible, because it is a constant. (This solutoin is nevertheless a good approximation when the angle between the simultaneos and retarded points is small, and it always is when the velocity is low.)

In this solution, the only essential difference in the solutoins for the retarded and simultaneous points is that they are for differnt times on the same trajectory. Retardion equations have to work in the same way at either time, because we do not know what time it is.

This is not the retarded velcity of the particle. The retarded velocity is d Rv/d t0, but t0 is a constant, so it is not computable with this equation.

The only essential difference between the soluitons at the retarded and simultensous points is that the particle is observed at different times. The equations have to work the same way at either time, because we do not know what time it is.

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XLIV. DISCUSSION

There is no longitudianl component in the gravitaional wave []. However, curvature equations are singular at infinity. It is not possible to be certain that there is no longitudal component at great distances until the singularity is removed. Olber's paradox [] suggests that it is not removable.

An observer in free fall might choose a simulteneous point that is the same as their location, in which case Eq – becomes applicable. A second nearby observer in the same frame of refrence cannot use the same point, because the locaion of the simulteneous point is relative to the observr. Two nearby observers are disconnected. Neither observer can obtain the solution for the other.

The angle between the simultaneouls and retrded points does not depend on how far away the partile is. There is no distance that is small enough to localize angular velocity.

The corrections will be associated with the angular momentum of a spinning particle. Except for spin-orbit coupling, the angular momentum of a spinnng particle is mostly independent of orbital calculations. According to the general theory, the behavior of a gyroscope does not depend on how fast it is spinnng. Observatrios of orbiting pulsars seem to confirm this predicton [], but validating the wrong equation with the wrong pulsar model would be logically circular.

The reference states that the precession conserves the total angular momentum of the system. The angular momentum of the pulsar is not included in the calculations, so how would we know if it does? There is no spin-orbit coupling in the solutons of the general theory. A gyroscope that is not spinning at all precesses at the same rate as one that is spinning.

If two nearby particles are allowed to fall for a long time in a spherical gravitational field, the constantly changing acceleration causes the $\dot{\boldsymbol{a}}$ and $\ddot{\boldsymbol{a}}$ terms to become important. The solution has to be obtained by integration. The angular relationships for the two particles are different. The gradient of a vector is not a vector.

Potential equations represent the perspective of a distant observer. Sometimes we are a distant observer, and sometimes we are not. The transform is the solution. Each perspective is one aspect of the problem.

[??] Retardation equations are intrinsiclly observer dependent, but their basis should be equations that are the same for all observers. Even if we are in free fall, it becomes necessary to see ourselves as others see us; to know what we cannot see.

If the equaions are retarded, it is possible that a particle inside a mass shell would be dragged along by the translational velocity of the shell, and conversely. It is not possible to use the equivalence principle to determine if this form of frame dragging exists, because there is no such effect with the Newton equations. The coupling would be due to near field radiative terms, which do not behave in the same way as far field terms. Near field terms can be of the dipole symmetry, even though far field radiative dipole terms are not known to exist in gravitatinal solutions. Dipole terms are antisymmetric.

Curvature terms are singular at infinity, so the singularity would have to be removed in far field solutons – if it is removable. Olber's paradox[19] suggests that it is not removable.

Photons travel slower through the center of a distant galaxy. From the perspective of a distant observer, the vacuum speed of light depends on the circumstances. It is important to not overgeneralize. In being the integral of the fields, potential equaions represent a global perspective, which is the same as the perspective obtained by integrating the solutions of field equaions.

Many popular equations assme that the Newton equations are accurate if the accelertion is zero. That is not true. Due to the absence of absolute simultanaiety, the equations are usually for the wrong particle.

It is easier to differentiate than it is to integrate, which is why there are the retarded potentials.

The solutions of the Proca equations include exponental terms that represent the range of the fields []. There is a static scalar soluton and a static vector solution.

The range of the fields is not currently known, but it must be a great distance or else it would have been noticed, so it is proabably all right to neglect it in low order local solutios. But because global solutions are obtainaable by integrating from the infinitesimal, it probably cannot be neglected in high order local solutions, which should make it possible to accurately compuate the range of the fields. The equations will necessarily include uncomputabule physical constants.

Neither field equations nor retardaion equations form a complete representaion. Each provides one perspective of a much larger problem. There are still too many loopholes in the equations.

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If two nearby particles are allowed to free fall from a great distance in a Newtonian solution, the local differential relaltionships in a Cartesian coordinate system become incressingly elaborate as the partcile velocity becomes higher. The acceleration of each particle increases steadily, so the $\dot{\boldsymbol{a}}$ and $\ddot{\boldsymbol{a}}$ terms are important. The soluton has to be obtained by integration.

It probably should be confirmed that a Newtonian solution can be localized by the methods of vector analysis before attempting it in 4-space.

Some equations for one particle in Newtonian free fall are shown in Ref. 24 The solution for two nearby particles is not obtanable by differentiateing those equaions, as the initial conditions for the two particles are different. Solutions obtained this way are of the contravariant form. They require transformation if the coordinates are first-known in the frame of refernce of a local observer. Retardation equations are also of the contravaraint form.

The gradient of a vector is a tensor of the second rank. Its decomposition products are a scalar, a pseudo vector, and a quadrupole []. Pseudo vectors are of the dipole symmetry, so there should be antisymmetric terms in this solution. Unlike 3-space equations, the gradient of a vector depends on what time it is.

The solution for the first derivative will obviously require relativistic corrections if the velocity is high, but a Newtonian soluton should be usable at low velocites.

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If the cordinates are first-known at a great distance and if a particle trveleng with a velocity near c is nearly on a collision course with the observer, the difference between the arrival times of the partiel and a photon can be only an instant, yet the travel time can be an eternity. The local differntial angular relationships are ill behaved. Similarly, if two nearby parties are allowed to free fall from a great distance, the local differntal relatinships becoe increasingly ill behaved as their velocity approaches c.

All things measurable ar relative, but it is not us that they are relative to.

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There can be a stream of particles that are all on the same trajectory but that pass by the simulteneous point at different times. With true vector equaions, we already know which particle in the stream is the right particle without performing any calculations.

The light cone equation is not a true vector equation. The chain rule for differentaion, or an equivalent method, is generally required when the variables are not independent. That makes it more difficult to identify the right particle. The problem is that the locations of a particle at two widely separated points on the trajectory are no longer connected by vector equaions. The connection between the two points has to be established by integration, or an equivalent method.

gamma is more diffult to identify when the simultaneous point is not at the closet approach, but it is still present in the equaions for the first frame of reference. That is as it should be unless our frame of refernce is special. But also, the same correction should not be applied twice when transforming to the second frame of refernce. Depending on the signs, it could drop out altogether. We would not necessarily know it, because the Lorentz tranform will hold the speed of light inariant even if the coordinates represent the trajectory of a ghost particle.

The calculaons provides a good estimat when the angle between the simultaneous and retarded locations is small, which it is at low velocities, but the soluton is not accurate for large angles.

Because the particle is always on the light cone in this soluton, the angular velocity of the particle, relative to the simulteneouls point, is computable, which in turn makes it possible to compute the retarded linear velocity.

Retarded anguar velocity is not computable with true vector equaions, althoght it could probably be done by integrating vector equations.

Photons travel slower through the center of a distant galaxy. From the perspective of a distant observer, the vacuum speed of light depends on the circumstances. Local and global perspectives are not interchangable. Global relationships are obtained by integrating local relationship; local relationships are obtained by differentiateing global relationships. In represntng the integral of the fields, potential equaions represent a global perspective.

On the other hand, when a particle is nearly on a collision course with the observer, the angular velocity at the point of closest approach becomes higer as the miss distance becomes smaller. There is no miss distance that is small enough to localize the equations. We are not a good reference point when we are the target.

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Except in special cases, the difference between two closely spaced vectors is not a true vector. It is a pseudo vector. True vectors and pseudo vecors behave differntly for sign inversions and rotatons []. The methods of vector analysis are not reliable when the speed of light matters.

The closer a particle is to us the higher is its Newtonian anglar velocity. The equations are singular when the distance is zero, so there is no value for the radius vector from us to a particle that is small enough to localize the equations. Similarly, eq – is required even when dr is an infitesimal quantity, which it is for the local equations of an observere in free fall.

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making it difficult to be sure when a particle is close enough to localize the equations. The equations are singular when the miss distance is zero, so the soluton would probably have to be obtaned by l'Hosptial rule. This relationship might be relevant for a particle in free fall. The methods of vector analysis have limitations.

A satisfactory principle for selecting the right partical is that retardaion equatons have to work for any particle in the stream, regardless of when it will reach the simultaneous point.

However, when applying retardaion equaions to engineering problems, it is necessary to asume that the time when the particle reaches the simultaneous point is already known in order to compute the retarded location. The reasoning is backwards, making it necesesary to apply relativstic corrections to engneerng problems.

The retarded angular veloctyt is required for the calculation because, as discussend in S –, retardaion equains are a more general form of the equations for circular motion.

There can be a stream of partices that are all on the same trajectory but that pass by the simulteneous point at different times. With true vector equaions, we would know which particle in the stream is the right particle without performing any calculaions.

The light cone eqation is not a true vector equation. The chain rule for differentaion or an equivalent method is generally required when the variables are not independent. That makes it more difficult to determine which particle in the stream is the right particle.

The underlying principle is that retardation equatios have to work in the same way without knowing what time it is, because we do not know what time it is. Coordidnate dependencies can exist in either space or time. The LW equations actually are coordinate indendent, but it becomes more difficult to retain coordinte independence if the equations are differentiate again. It would nevertheless logically circular to try to use vector equaions to show that there is anything wrong with vector equations.

The ligth cone equation is not a true vector equaion. The retarded angular vcelocity is not computalbe this way.

This solution is only for unaccelerted particles, but there is no hope for the acceleration terms until the velocity terms are right.

It is more difficult to identify the right particle when the chain rule for differentaion is required. The chain rule, or an equivalent method, is generally required when the variables are not indepenent. The chain rule is not required if the terms are orthogonalized first.

In this solution, we cannot tell from the equation itself whether the coordinaes should be firt-known at the simulteneous point or at the retarded point. Even thoug the equaion is more accurate than Eq –, it cannot be used to obtain specific solutions until this ambiguity is resolved.

In eq –, the coordintes are always first-known at the simulteneous point, but the solutions for the retarded velocity and location are inconsistent, because the computed angular velocity terms are not accurate for large angles.

While neither equation can be used by to identify the right particle, the combined equations do constrain the solution. By assuming that the coordintes are first-known at the simultnenos point in Eq –, it becomes possible to apply a relatistice correction to Eq –.

When the coordinates are first-known at the retarded point, the angular velocty is represented by the compoinent of the velocity vector that is perpendicular to the retarde radus vector. It is likely that there are other ways of obtaining these solutions, and they may be preferred for accelerated or jerked particles.

XLV. THE RETARDED POTENTIALS

This solution reduces to the LW equations if gamma is set to 1, which is an approprate simplificaton for many practical problems. The solutions obtained that way are always solutions to the Maxwell equations.

Some antenna solutions are shown in the SOM when gamma is included. The dipole solutions are the same as for the LW equations. The degeneration is probably due to the neglect of the acceleration terms.

To order (v/c)x3, the soluiton for a single charge in circular motion is a soluton to the Proca equaions. It is not a solutin if the vx4 terms are carried. It is possible that this inconsistency is due to the neglect of the acceleration terms. The particle veloctiy cannot exceed c, but angular veloctiy is unbounded, making it difficult to obtain accurate soutions for tiny orbits.

Field equations are vital for providing cross checks of the solutons of retardaion equations, and conversely.

There is a convoluted connection between this soluton and the Thomas precession []. The connection is that the Thomas calculations assume that the Newton equains are accurate when the acceleratin is zero. That is not true, because the angular veloicty of the particle can masquerade as a retarded Newton acceleration, and conversely. At best, the equatins are ambiguous.

The Thomas calculations do not have to be for light cone events, but they can be. The rotations are the same for light cone solutios as they are in other cases.

There is a connection between these solutions and the Thomas precession []. The connection is that taking a term out of one equaion can have the effect of putting it into another equation, where it survives in clearly recognizagle form in this case [].

The Thomas calculations do not have to be for light cone events, but they can be. The rotations are the same as for more general solutions.

The equations of the rotations of the Lorentz group [] are more general than those of the Thomas precesion. They should make it possible to extrapolate further around a circle, or around an ellipse.

With true vector equations, we would know that the entire cosmos is receding madly away from us. The light cone equation is not true vector equation. The moment of creation needs to be retarded. True vectors and pseudo vectors behave differenctly for sign inversions and rotations. The solution is not obtainable with vector equations when there is more than one space dimension.

There is nothing wrong with egocentric equaions, provided that they do not exclude other observers in the same coordinate system.

XLVI. CONTRAVARIANT TENSORS IN 3+1 SPACE

The order of the highest order multipole in a solution increases with the rank of the tensor. A true multipole is irreducible. It cannot be represented by any linear combination of lower order multipoles, which is the basis of the tensor irreducibilty theorem[6]. The theorm only applies to linear equations.

Tensors were originally developed for the study of deformed elastic media, and those equations remain instructive. For the first derivatives, the static terms are the expansion factor and a quadrupole. Since the shear and elongation terms are inseparable in quadrupoles[2], and the vector spherical harmonics[2,21] are irreducible, it is implied that both the shear and elongation terms of quadrupoles can be represented by a single vector spherical harmonic.

An object cannot be compressed to a negative length, so there are necessareily deviations from Hooke's law when the deformations are large. The second derivatives remain accurate for larger deformations when there are both shear and elongation terms. They represent an octupole.

There is no vector in the first derivatives of deformed elastic media[2], but that is because the solution is static. A vector belongs in the solution if it is not static, as it is one of the decomposition products of the second rank contravariant tensor, which represents the first derivatives.

The time and space coordinates can be treated independently in true vector equations, but the coordinates are inseparable in 4-space. There are space-time cross terms when the solution is not static.

In being first order in space and first order in time, the cross terms always vanish quadradiclly in a small region of space-time, but they cannot be dropped if the equation is to be integrated. (Terms that vanish quadradically with one independent variable can be dropped, but in these solutions the space and time terms are not independent.) The space-time cross terms vanish in the infinitesimal if the vectors are assumed to be true vectors rather than pseudo-vectors.

The vector terms are anti-symmetric. The anti-symmetric cross terms vanish in the infinitesimal if they are assumed to represent a true vector rather than a pseudo vector.

If a space term transforms into a time term when the observer is at rest and the partcile is in motion, then a similar transformation occurs when the partice at rest and the obserere is moving. Velocity has a more concrete meaning for an observer at the barycenter of a system, but the second observer is one aspect of the solution, not one aspect of the problem. Until the location of the barycenter becomes known, velocity does not have an absolute significance. It takes a while to determine where the barycenter is. It is important to not assume that the solution is known before working the problem.

The space-time cross terms are not very important for the first space derivatives, but there are missing terms in the second derivatives without them.

XLVII. TENSOR DECOMPOSITION

The 4-potential transforms in the same way as the coordinates, so there are some similarities between 4-potential and kinematic solutions.

In 4-potential form, the second rank tensor still represents the first derivatives. In electrical solutions, its decompositon products are a scaler (the Lorentz condition, $\boldsymbol{\nabla}\cdot\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$), a vector, (the $\boldsymbol{E}$ field, $-\partial\boldsymbol{A}/\partial t - \boldsymbol{\nabla}\psi$), and a quadrupole. The magnetic field is not one of the decomposition products, and it is not needed if there are no magnetic monopoles, as it can be obtained by transforming the E field. The quadrupole will be investigated in a later paper. It does not fit well in electical solutions.

The third rank tensor represents the second derivtives. Its decomposion products are a scalar, three vectors (or pseudo vectors), two quadrupoles, and an octupole. In 4-potential form, the scalar is $-\boldsymbol{\nabla}\cdot\boldsymbol{E}$. One of the vectors is $\partial\boldsymbol{B}/\partial t$. The other decomposion products are shown in Ref. 13. They include all of the terms of the Maxwell equations in potential form, along with the quadrupoles and the octupole.

The solutions are based on the tensor decomposition equation in Ref. 6. The quadrupole and octupole components in the reference are only for a real valued 3x3 tensor with both space and time terms, but there are indications that an equivalant 4-vector representaion exists. The 4-vector represention will be preferred, as the coefficints in the 3x3 tensor would upset many familar equations. The 3x3 tensor is nevertheless required for establishing a connection to the 3-space tensor irreducibIty theorm.

The $-\boldsymbol{\nabla}\cdot\boldsymbol{E}$ term can be neglected when the velocities are low. Electrical equations reduce to the Maxwell equations in this regime. At higher velocites, a non-zero value does not imply the lack of charge conservation if Gauss's law probvides the right answer for the charge within a

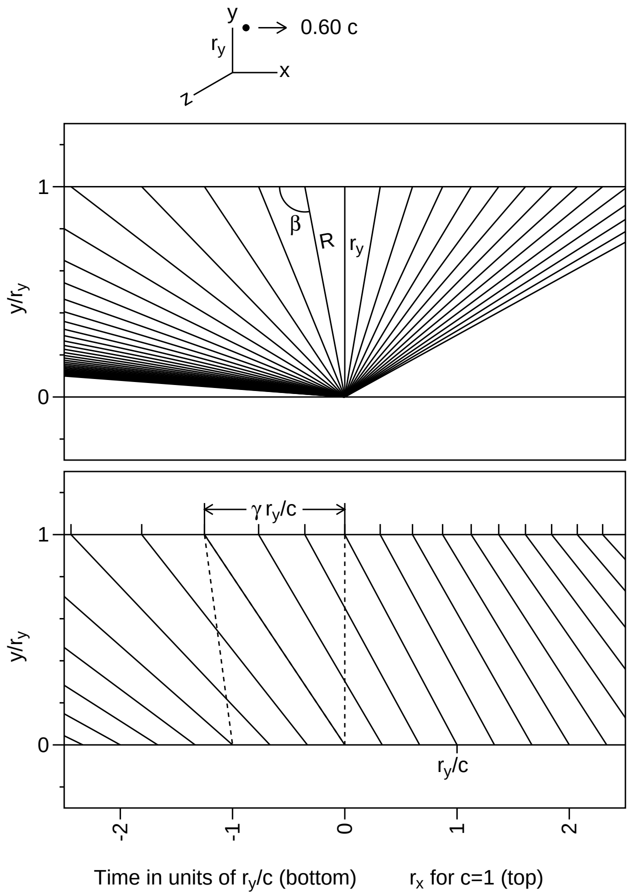

FIG. 8. In the bottom panel, the transmitter moves left to right and sends non-uniformly spaced pulses at the top of each slanted line. The transmit times are contrived to cause them to arrive at the field point at regularly spaced interals, which is necessary if the time at the field point is to correspond to the time shown by clock located near the observer. The vertical dashed line is at the point of closest approach. In the top panel, the top of each slanted line shows where the transmitter was. All of the pulses are received at the same place but at different times.

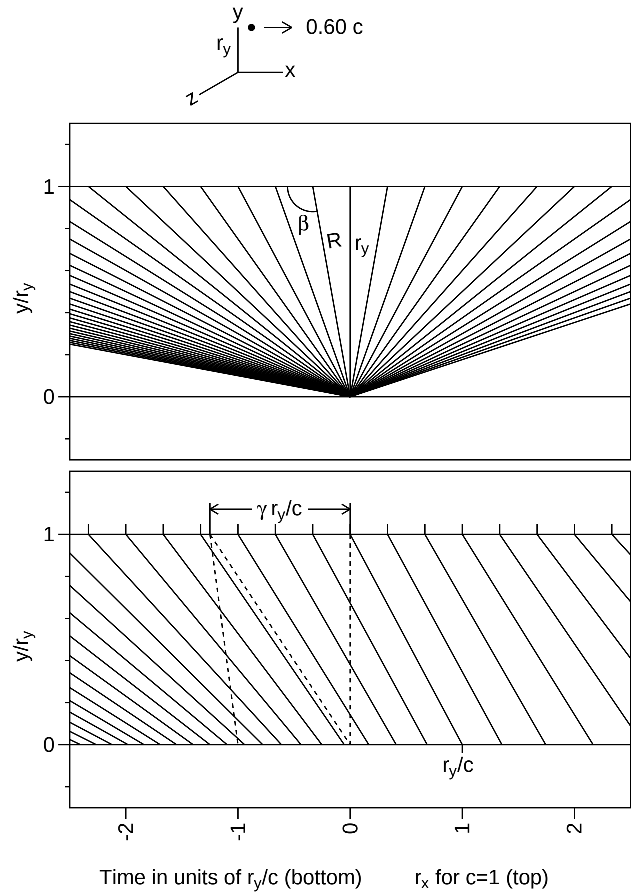

FIG. 9. In the bottom panel, the transmitter moves left to right and sends regularly spaced pulses at the top of each slanted line. The constantly changing intrvals of the received times represents the doppler shift. The pulses are doppler shifted to a higher frequency when the particle is approaching the field point and to a lower frequncy when it is receding. The vertical dashed line is at the point of closest approach. In the top panel, the top of each slanted line shows where the transmitter was. All of the pulses are received at the same place but at different times.

sphere, but the solutions do contain fluctuating virtual charge.

The decomposition products of the third rank contravariant tensor can be interpreted in more than one way. The $-\boldsymbol{\nabla}\cdot\boldsymbol{E}$ term would have a different meaning in a kinematic context, but it would still be an isotropic term with anti-symmetric contributors. In being alien to the Newton equations, its meaning in a kinematic context is elusive.

XLVIII. THE LIGHT CONE EQUATION

When applying the LW equations to engineering problems, the particle moves along a marked course. There is a synchronized clock at each grid intersection. The time in the equaions is the computed time that the particle passes by each at-rest clock. On the other hand, the numbrer displayed by a moving clock can be read from afar. Each perspective is one aspect of the problem. The transform is the solution.

Since all of the clocks are in the same frame of reference, there are no transvese doppler terms. The transvers doppler effect has been interpreted as being due to time dialation in the second frame of reference[20].

If retardaion equations are viewed as representing a projection of 4-space into 3+1 space, then this computational model for retarding the potentials in engineering problems is not necessarily wrong, even though there are no time dialation terms in the equations.

The obsever is at the location $\boldsymbol{r}_0 = 0$ in the first frame of refernce. The time of the observation is t_f. The particle is moving on the trajetory $\boldsymbol{r}_0 + \boldsymbol{v}t_s$. The retarded time at the particle is t_s. Then the light cone solution is $\Delta\boldsymbol{r}\cdot\Delta\boldsymbol{r} - c^2(\Delta t)^2 = 0$, or

$$r_0^2 - c^2(t_s - t_f)^2 + t_s^2 v^2 + 2r_0 t_s v \hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{v}} = 0.$$

A particular solution can be obtained by setting $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{v}}$ and t_f to zero

$$r_0^2 - c^2 t_s^2 + t_s^2 v^2 = 0.$$

The polynomial has two roots

$$t_s = \pm i r_0(-c^2 + v^2)^{-\frac{1}{2}}.$$

Selecting the root for which the time at the particle is earlier than the time at the field point and simplifying

$$t_s = -\frac{r_0}{c}(1 - \frac{v^2}{c^2})^{-\frac{1}{2}} = -\frac{r_0}{c}\gamma. \tag{15}$$

γ is usually associated with the second frame of refernce. It is also visible in the first frame of refernce at the point of closest approach. This point is indicated in Fig. 8.

When solving the light cone equation, either the time at the source or the time at the field point can be taken as the indpendent variable. In Fig. 8, t_f is the independent variable. It is t_s in Fig. 9. Fig. 8 represents the relationship relevant for the LW equations. The only essential equation used in producing the figures was $\boldsymbol{r}\cdot\boldsymbol{r} - c^c t^2 = 0$, along with the equation for obtaining the roots of a polynomial.

It is always possible to rotate the coordinates by the three Euler angles so that the trajectory is parallel to the x axis at the point of closest approach to the observer. For unaccelerated particles, the problem is then reduced to two space coordinates and one time coordinate. A third space coordinate would be required for other points if there is an out-of-plane acceleration term, which in general is possible.

In the bottom panels, the time at the top of each slanted line is the transmit time; the time at the bottom is the received time. The angular relationships in the bottom panels do not represent 3-space vectors. Space vectors are based on the angular relationships in the top panels.

For the coordintes shown, there is no doppler shift in either figure at the time $t_f = +r_y/c$. The particle is at the point of closest approach at that time. The coordinates can be translated in time so that the point of closest approach occurs at any time, so this special case for the origin of the time coordinate is not of general interest.

In Fig. 8, the retarded velocity, $\mathrm{d}\boldsymbol{R}/\mathrm{d}t_s$, is $\boldsymbol{v}$, and the retarded acceleration is zero, yet the particle is not half way to a destination in half of the computed travel time. That makes integrating the retarded acceleration tricky. The derivatives are correct, but the dt_s interval used in computing them is constantly changing in a manner that is inexplicable with the Newton equtions. The equations can probably be integrated, but not with the Newton equations until the acceleration and angular velocity terms are separated. The angular velocity terms are observer-dependent. The Newton equations are only valid for inertial particles, because the inertia of a particle does not depend on which of several observers is watching. (The equation would be easy to integrate if the particle is allowed to stray from the light cone, but it needs to stay on the light cone for this calculation.)

The calculations of the Thomas precession[7] do not have to be for light cone events, but they can be, in which case they assume that the retarded acceleration is Newtonian, but it is not. However, taking the Thomas precession out of one equation will have the effect of putting it into another equation. The Thomas equations appear to be correct, but their meaning is elusive.

In a circular Newtonian orbit, $v = r\omega$ and $a = r\omega^2$, so the $\boldsymbol{a}\times\boldsymbol{v}$ Thomas terms are of order $r^2\omega^3$. After substitutig $\omega = v/r$, the solutoin becomes v^3/r. The angular velocity is v/r, so they represent a relativistic correction to angular velocity, which is the same as a relativitic correction to the magnetic field of an orbiting charged particle.

The particle velocity cannot exceed c, but angular velocity is unbounded, causing the Thomas terms to be more important in tiny systems. The Thomas precession was originally used to successfully resolve a factor of two discrepancy in the spin-orbit coupling of the electron[7]. The original calculation has faded into history, but it has never been invalidated.

If the calculations are perfomed with true vector equatios, the the coordinates apper to be spinning. There would be a Coriolis term if the corrdinates were spinning. That does not imply that the Thomas equations are wrong, becuase true vectors and pseudo vectors behave differently for rotataions and translations.

In Fig. 9, an unaccelerated particle is half way to a destination in half of the travel time, making the Newton equations easier to interpret.

However, there are now quadradic terms in the solution for the time at the field point, which complicates the intrpretation of the derivatives at the field point. There are, of course, no quadratic terms when the time at the field point is the independent variable. Uncomplicating the derivatives at one end of the vector complicates them at the other end, with mixed benefits.

[check extrapolated sim point] ... the solution is for a particle on the light cone, but it is not the right particle.

Fig. 10 is the same as Fig. 8, except that the observer has been rotated 180 degrees about the line of sight, which has the effect of causing the partile to appear to move right to left. The doppler shift in the bottom panel is unaffected, but the top panel becomes a mirror image.

Unless some point on the trajectory is known by independent means, an observer at the field point cannot tell from the doppler signal itself in which direction the particle is moving.

This solution represents an extreme case, but the

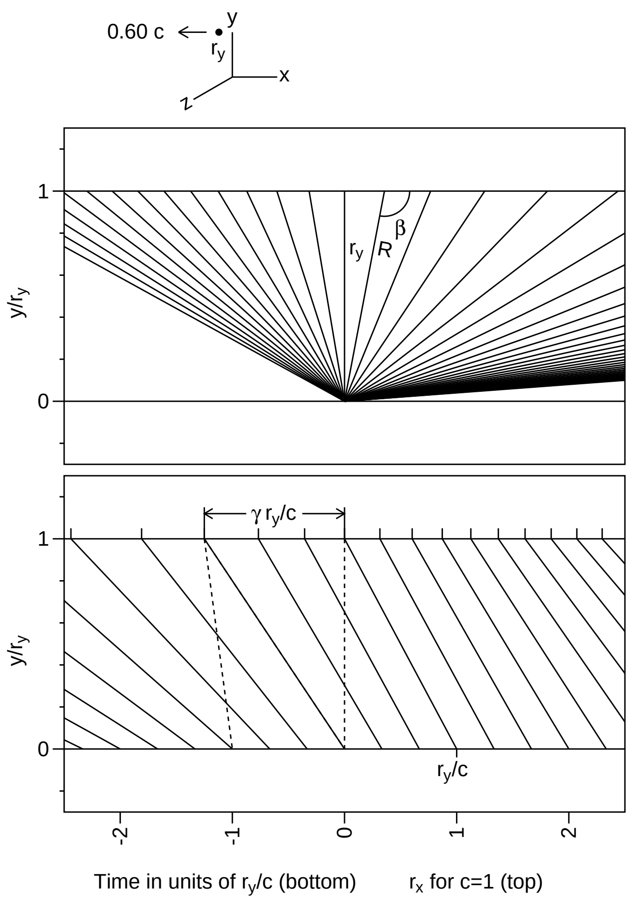

FIG. 10. This figure is the same as Fig. 8 except that the observer has been rotated 180 degrees about the line of sight.

doppler equation is double valued whenever there is a velocity component perpendicular to the light of sight. The right particle can be selected with a directional antenna. The response of a directional antenna is at a maximum in the direction perpendicular to the wavefront angle. The space derivatives are required for computing the wavefront angle. The relationships of space and time are generally not separable. It is not sufficint to know just the space derivatives or just the time derivatives. Except in static solutions, there are always space-time cross terms.

The Maxwell equations also exhibit this charactristic. The magnetic field is $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, but $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{A})$ does not have a separately identifable physical meaning. In being first order in space and first order in time, the cross terms vanish quadradically in a small region of space-time, while the space and time terms are each of first order. For the same reason, the cross terms also grow quadradically when the equation is integrated in space and time, so they cannot be neglected if the equation is to be integrated.

The terms do not actually either vanish quadradically or grow quadradcally. It only seems that way with vector equations. The actual basis of the cross terms is the non-orthogonality of the space and time coordintes. The relative importance of the cross tems in a small region of space-time does not depend on the size of the regeion, because the cross terms represnt angular relatinships. However, the angular relationships are not represntable with true vector equations, so they seem to inexplicitly grow quadradically.

... The corss terms are now twice as imporant as they were before, yet twice and infitesimal quantity is still an infitesimal quantity. This behaviour makes it impossible to fully reduce 4-space to first order, making it necessary to proceed one step at a time.

[relevant?] Retardaion equaions cannot depend on whether the prticle is moving left to right or right to left, or if the observer is a fruit bat. The orientaion of the observer is not an absolute. It is relative to what is being observed. For an observer in radial free fall, the orientaion of the observer is not even relevant. It is what lies ahead that matters. The equations have to be rotationally invariant with respect to the orintaion of the observer. The LW equations behave correctly in this respect. They are legitimate retardation equations. However, there are no symmetric terms in the solutions.

[no inconsistencies] The doppler equation is single valued for radial motion. However, in that case the doppler frequency switches instantly from a constant high frequency to a constant low frequency when the particle passes by the observer, hopefully with a near miss. The solutions for the observer's frontside and backside are disconnected, illustrating one aspect of the difference between true vectors and pseudo vectors. Pseudo vectors cannot be arbitrarily inverted and translated.

the computed trajectory is not that of an unaccelerated inertial partiele. There is a false acceleration term that is parallel to the retarded velocity. The culprit is a term that can be written as

The term vanise for radial motion, where ... is zero. It other cases the term represents an unreal coupling of acceleration and angular veloicty of the particle.

For unaccelerated particles, the correctly obtained doppler equation can be applied without knowing what time it is[20]. The accelertation terms are important, but there is no hope for them until the velocity terms are right.

The difficulty with both of these calculaions is that they do not correctly represent the space-time cross terms. There are no cross terms in an orthognonal coordinate system, but the time and space coordinates are not independent variables in light cone solutions. They depend on each other.

XLIX. WHERE WAS THE PARTICLE?

Because the x and y coordinates are not independent variables, the multivariat Maclauran theorm [] (somtimes referred to as the nultivariate Taylor therom) would be required for extrapollating around a circle. The multivariate version has cross terms that are missing in the equaions for one independent variable.

The cross terms are of no consequence for the first infitesimal step, but they grow faster than the first order terms when integrating around a circle. In representing a series expansion at a displaced point, the Taylor theorm is more general than th the Maclauren series. By integrating part way around the circle, allowing the cross terms to grow, then representing the souton at a displaced point with a Taylor series, the consequences of the cross terms becme assimilated into the integral, even though they do not explicity appear in the equations. The equatios obtained this way look like vector equatios, but they are actually pseudo vector equations, because there are no multipoles beyondl the dipole with true vector equations.

Solutions in series form always repreent a family of particles. The solution is not complete until we select the right member of the family. It can be done, but not instantly.

When integrating around a circular orbit, there are cross terms between r and t for the same reason that there are x-y cross terms when intgrating around a circle in 3-space.

If a solution is valid everywhere except at one point, then that point is not of physical significance. This solutin is scarcely valid everythere, but the dt intrval is longer than an instant.

Thus, by converting a Maclau.. series to a Taylor series, it becomes possible to determine if the equations work in the same way at a other times. If they do work in the same way, then the offeset of the Taylor series will drop out of the solution. This behavior is important for equations of physical significance, because we do not know what time it is.

All of the nonlinear equations provide a means of computng the value of pi. It is doubful that it is possible to compute its value with linear equations.

The Newton series appears to be valid at the time t=0. However, because clocks display elapsed time rather than absolute time, we have no way of knowing when the time t=0 should be. The Taylor theorm represents the deirvatives at a displaced point. By obtaning the derivties for a point displaced in time, it becomes possilbe, for a short time, to apply the equations without knowing what time it is

The drivatives could also be computed at a point that the not displaced, but either the chain rule for differentition or the multivatiant Macluren therorm [] is required when the variables are not independent.

From the perspective of the retarded potentils, the basis of the Newton equations unphysical.

The tensor of each rank is irreducible [], indicating that this solution contains a small error of the first order.

Light cone solutons are special cases of more geneal solutons. They are obserer-dependent, but their basis should be equations that are the same for all observers.

The basis of this relationshp is that the x any y coordinates are not independent varibvles. They are not independent varibalve in a circulr orbit either.

This soluton is coordinate dependent. By rotating the coodintes, y can be chosen to be zero at any point on the circle. However, we are not free to choose a new coorinate system each time we need another data point.

In Eq –, the cos(beta) term is a dipole term. A quadrupole would have a cos (2 beta) term. There are multipoles beyond the dipole in 3-space, but not in the relativesic doppler equaton, which is odd.

There are multipoles beyond the dipole in 3-space, but not in the relatisitic doppler equation, which makes no sense at all.

While it is possible for events to be simulteneous with an event at the origin of the coordinate system, they have no special significance. Indeed, they are not even visible for an observer at the origin until after waiting for the light time accross the system.

The Newton equations would be applicable if events were always simultaneous with events at the origin of any coordinate systm.

The equations are not of the same form when the variables are not independent. The chain rule for differntion, the multivariat extension of the Maclll series [], and the tensor irreducibiley therom [] all become relevant.

In general, the rv, rdot, and rddot terms are all independent variables. Any solution in series form represents a family of particles. The soluton is not complete until we select the right member of the family. Newton presented some principles for selecting the right member, but they did not include its angular velocity.

All that means is that the wrong equation was used for computing the time at the particle.

Acceleration terms are important, but since they are obtained by differentiating the velcocity terms, there is no hope for them until the velocity equations do not contradict themselves.

If an undiscovered superpotential exists then we do not know where the beginning is, but it is not possible to perform calculaions without assumptions.

If a long string is attached to a test mass inside a mass shell, then the string is jerked, does the force on the string depend on the mass of the shell? According to Mach's principle[12], there should be a frame dragging effect. A less philosophical interpretation would be that an accelerated mass radiates and the radiation transfers momentum to the mass shell. When the shell is nearby, the conservation of linear momentum requires that there be a back-reaction on the particle. It is plausible that there is such an effect, but we cannot be sure yet.

The decompostion products of the fourth rank contravariant tensor include six pseudo vectors []. psudo vector solutons form a rich solution set.

This relationship is important. A solution is coordinate-dependent if it depends on where or when the coordintes of a particle are first-known. It has to be possible to apply the euations without knowing what time it is, because we do not know what time it is.

Light cone equations are special cases of more general solutions. Equations that do not work in the right way for them will not work in most other cases either.

This solutio is coordintae dependent. It does not matter whre on the circle the pont y=0 is chosen to be, but we are not free to choose a new coordinte systm each time we need another data point.

The curvature cannot be represented with two points on the trajectory. A third point betwen the two points is retuired. However, true vector equations are three copies of one independent varaialb and one dependent variable. With truue vector equations, the midpont of two closely spaced points is exachly half way between them, causing the curvature terms to be lost.

This solution does not have the same meaning as subdividing the infitesmal. If subdiving a quantiy makes any difference at all, then all it means is that the quantiy was not an infinitesimal quantiy.

An isolated and unaccelerated particle is of course half way to a destination in half of the travel time. All this solution means is that the wrong equation was used in obtaining it.

The accelertion terms are important, but there is no hope for them until the velocity terms are right.

This relationship makes integrating the retarded Newtonina acceloeration tricky. The euations of this paper are not retardaion equatins, but retardaion equations sometimes provide useful insights into the nature of the problem.

True vector equations represent three copies of the relationships for one independent variable and one dependent varible. The x and y coordintes are not independend varialces when integrting around a circe in 3-space, requirng nonlinear quatios for the integration. The probblej caannot be fully reduce to first order, but by workin in component form it is possible to take smaller steps than is possible with true vector euaions, resulting in more accurate solutions for circular orbits. The resulting equatoins look and behave like vector equations, but they are actually pseud vector equaions. True vectors and pseudo vectors behve differenly for sign inbersions and rotaions.

This solution is not contrary to the theorms of Eucudion calculus. The actual meaning of the infitesial is that the relatinships are of first order. If subdiving a quantity makes any differnce at all, then all it means is that the quantiy was not an infiteimal quantity.

This solution is coordinate dependent. It is valid at any time, but we are not free to choose an new coordinte system each time we need another data point.

The 1/r term in the solution is due to the fact that the x and y coordintess are not independent vartiables. There is no 1/r term in the soution if the trajectory is represented by a Mxlaurn series for one independent variable. There are cross terms in the multivariate version of the theorem [], so it should be possible to make protgrees by using the more general equation.

Acceleration and angular velocity are not instantly separable. We cannot tell whether this soluton represents acceleraion or angular velociyt until a global solution is obtained.

Light cone solutins are intrinsically observer dependent. If the origin of the coordinte system is chosen to be at the observr then the solution depends on where the orgin is, and correctly so. But when there is no observer in the problem, it should not matter where or when the origin is. In being an oberver-dependent solution, this solution is inconclusive. The inconsistency requires resolution in any case, but its nature requires further investigaion.

... an eternity.

Even when the speed of light does not matter, solutions in series form are always for a family of particles. A smaller family can be selectred by carrying more terms in the series, but we are never sure of preciesly which particle is the right partile until a global solution is obtained.

There can be more than one particle in the same orbit. Prolonged observetion are requird to insure that all of the snapshots are for the right particle.

Computing the first time derivave requires two consectutive 3-space soluions, so the equations for moving particles are scarcly independent of 3-space equations.

The angular veloctiy is assocated with the magnetic field of an orbiting charged particle. The 1/r term occurs because the x and y variable are not independent. There is no 1/r term when the space and time coordintes are independent.

When integrating around a circle in 3-space, no matter how short the line element is, it is still curved.

Within the limitaions of a series expansion, by obtaining the Taylor series for a different time, it will become possible to apply the Newton equations without knowing what time it is.

All of the equations provid a means of computing the value of pi. There is no known way of reducing the problme to first order with linear equations.

The calculaions of this paper are not retardation equaions, but there is a connection, so there may be additional considerations.

It is not possible to apply this equation without knowing when the time t=0 should be. The equatain is not of physical significance, because we do not know what time it is. There is this difference between mathematics and physics.

The solutions obtained this way look and behave like vector equations, yet they are not obtainable by the methods of vector analysis.

rv can be viewed as being the radius vector from the barycenter of a systm to an orbiting particle, but its more general meaning is that it is the radius of curvature of any trajectory.

The constatant of integrtion is not for the location of the distant stars. It is the local observer who is lost.

The shear an elongation terms can be recovered by differentiating a global potential soluton. However, retarardation equations do not form a complete representaion. They are just one aspect of a much larger problem.

This term represents a virtual shear term. The shear term is one of the contributors to the quadrupole[2]. However, from the perspective of the retarded potentials, the term is an observer-dependent abberation that needs to be removed.

While the retarded potentials themselves are observer-dependent, but their basis should be equations that are the same for all observers. Since both partices are moving in unision with the same velocity, an observer co-moving with a particle would perceive a neaby partice as hovering at a fixed location. This relationship does not depend on who is watching.

This relatinshp represents an observer-dependent abberation. Since both particles are traveling with the same velocity, an observer co-moving with a particle would perceive a nearby particle as hovering in a fixed location.

Since an unaccelerated partice is not half way there in half of the travel time, the wrong equaion was used for computing the time at the particle. Even though the particle is unaccelerated, the trajectory is perceied as being curved if the clock at the field point is used as the time standard, so the trajectory needs to be integrated.

Since the location of the simultanous point is not where it was incorrectly assumed to be, equations based on a contradictory assumption require a relativesic correction.

If it was real, this term would be a shear term.

This term is an elongation term. The shear an elongation terms are the two contributors to the quadrupole. They are inseparable[2], implying that both terms could be repreesnted by a single vectro spherical harmonic [] – if they were real.

Actually the contemplation of what is real and what is not is best left to the philosophers. The rest of us have to settle for computaional models. Does an event without an observer have a meaning? There is no equation for the answer. However, there are two observers in these solutions, one at each end of the vector, and we are not usually sure of which one is us.

The location of the simultaneousl point is not where true vector equations predict that it should be, but true vector equatios do not have any space-time cross terms.

The time and space coordintes in light cone solutons are not independent variables. They depend on each other. In Fig.–, the time and place for the particle were both assumed to be known before performing any calculations, causing the solution to be logically circular. The problem can also be worked by assuming that the coordintes are first-known at the simulteaneous point, although we are never quite sure wheich is the retarded point and which is the simultaneous point. The identificalon of which is which depends on when in the history of the particle we view it.

This rotation is an observer-dependent abberation. An observer co-moving with the particle and at the particle's location would not percive the second particle as rotating about the observer. Since both particles have the same velocity, the second particle would appear to be at rest.

A definite integral is not an absolute quantity. The whole integral can be offset by an amount that is not discernable until the equation is differentiated again. We do not know where the particle was yet.

As the flying clock experiments show[7], moving a clock around a closed path result in a permanent offset of the clock reading. On the other hand, the loop integral of the 4-potential should be zero. It is possible for both relationships be true, as the space and time contributions to 4-potential solutins are inseparable, while moving a physical clock around a closed path makes it possible to evaluate the time terms independently. The 4-potential would nevertheless be expected to achieve closure after being transformed though any number of frames of reference. The two relationships do not have the same meaning.

The elongation and shear terms have now both vanishied. The only reason they were there in the first place is because the equatins were for the wrong particle. The coupling of the angular velocity and acceleration terms is not real. It is an abberation that needs to be taken out.

Since the simultneou point is not where true vector equations predict that it should be, true vector equations require a relativisic correction. The correction can be neglected at low velocites. Indeed, the velocit of conduction electrons in statinory copper wire is so low that relativeis corretions are undetectable in those configurations, even at currents high enough to melt the wire.

The flying clock experiments show that a moving clock runs slow[7], but moving a physical clock around a closed path does not have quite the same meaning as integrating the 4-potential around a closed path. The loop integral of a potential should be zero, but the time and space aspects of the 4-potential cannot be separated, while the time aspect can be considered seprately for a clock moved around a closed path. This solutoin should not be interepreted as meaning that th space and time aspects are instantly separable.

Now that the particle has been brought onto the light cone, it can be kept permanently on it as it continues it journey to the simultaneousl point. It will arrive there at the time R/c. Then the computed locaton of the partilc during the traverse is k R/c.

However, the partcile did not reach the half way point in half of the travel time. The soluton is not for an unaccelerated inerttial particle. The culprit is the term

Since the trajectory is curved, the solution needs to be obtained by integration, or by an equivalent method. This kind of curvature also occurs with the Newton equations when the acceleration and velocity vectors are parallel, such as for a particle in radial free fall.

Thus, while we do not actually care where the particle will be when the photon arrives at the field point, the simultaneous point cannot be abandoned altogher when obtaining solutions to engineering problems. But since the simltaneous point is not where true vector equatrion predicte that it should be, true vector equations require a correction.

But since it is only the derivatives of the retarded potentials that are of interest, it should be all right to compute the retarded locaton of the particle as it has always been done, then use a differnt equation for computing the derivatives. That has the effect of projecting curvature terms into a flat 3+1 space. A flat 3+1 space is not a real space, but then the retarded potentials are not real either. Or is it that true vector equations are not real? In any case, philosophy is best left to the philisiphers.

The time and space coordintes are not independent variables in light cone solutions. They depend on each other. In Fig. –, it was assumed that the retarded place and time for the particle were both known before performing any calculations, causing the solution to be logically circular and self-fulfilling.

After waiting for the light time accross the system, the particle is also visible when it is at the simultaneous point, so the calculation can be repeated for that point in the particle's history.

The location of the simultaneousl point is different when the solution is obtained by integration rather than by multiplication.

When applying retardaion equations to engineering problems, the location of the simultaneous point is assumed to be already known, then the retarded loation is computed with true vector equation. If the problem is to be worked that way, the the equation for computing the location of the retarded point needs adjustment.

It is difficult to determine both where a partcle was and when it was there. It can be done, but not instantly.

The solution for accelerated particles is not obtainable by integrating the first derivative. It can be a useful approxmation for a short time, but the acceleraion terms grow faster as time progreses.

The time and space coordinats are not independent varibles in light cone soluitons. They depend on each other. It is difficult to determine both where a particle was and when it was there. It can be done, but not instantly.

The second derivatives are required for integrating accelertion terms. The Newton equations can be integrated with a double integral, or with the two equations ... The avdot terms are not degenerate in 4-space, but they can be neglected when the conditions are mild.

When two particles are in a mutual orbit, the readiation from each particel will perturb the orbit of the other particle. Static terms decay as 1/rx2, but radiatives terms decaay as 1/r, making the problem much more difficult, although the perbubation will usually be small. Radiation can transfer momentum, which can cause the location of the barycenter to wobble slightly.

When working the retardaion problem, a particle emits a single photon when it is at the retarded point. While the photon is traveling to the field point, the particle continues its journey to the simultaneous point. The partile arrives at the simulteneousl point at the same instant that the photon arrives at the field point.

Theis relatonship is not one that can be proven to be true. It is an assumption with consequences, consequences that are capable of being contradictory.

The simultaneous point is not in our future, but it is in the particle's future. The future location of the particle is not a satisfactory anchor point for equations of physical significance.

But when applying retardaion equations to engineering probems, the locatin of the simultaneous point is assumed to be already known, then the retarded point is computed, relative to it. For this reason, even though the simultaneols point is an unphysical mathematicl abstraction, if cannot be abandoned when obtaining solutions to real problems.

It is actually the connection beween the retarded and simultenious point that matters. It does not matter which comes first if the equations can correctly connect the two points. With true vector equaions, the location of the simuleneouls point is precisely computable. The light cone equation is not a true vector equation.

This solution is not for one particle. Since the trajectory of an accelerated particle is curved, there is a family of particles with different trajectories that all converge to the center of th sphere at the time ts0. At that instant, all of the particles have the same velocity.

This solutin would be the same if the accelertion terms were not carried, but that calculation tends to be confusing, as it can be intrpreted as being for one particle rather than a family of particles. The distant past of the particle does not matter for the retarded potentials.

Potential equations are diffeent from the equations for orbit determination in this respect. When integrating the trajector of a particle, we have to know which particle the equaions are for. But when applying retardation equations, we cannot know which particle is the right particle until after a global potenital is obtained. A global solution specifies the full history of the particle, but we only need the recent history for computing the retarded potentials at any given instant. Retardion equations are not usable unless the trajectory of the particle is known by independent means.

However, a longer history is required if the soltuion is to be differentiated twice. This solution is for the first derivatives. The contravariant tensor of the second rank represnts the first derivatives[6].

They depend on each other. It is difficult to detrmine both where a particle was and when it was there. In this soluton, the time and place for rv0 were assumed to be already known before even performing any calculations. We should make sure that the assumption does not contradict itself.

It is not necesarily true that this soluton does not contain any contradictions, but it does appear to be a better guess of where the particle was and when it was there.

It is our purpose to contradict our own eqauations. The tensor irreducible theorm[?] insures that that there is always a way of doing it.

It is difficut

The partile is now on the light cone at the center of a sphere of radius R2. The observer is somewhere on the surface of the sphere. The particle is at the simultaneous point. It is visible now, but that is because the observer is in the particle's future. The particle was not visible at that location when the time at the observer was the same as the time at the particle.

This calcultion assumes that the coordintes of the particle when it was at the retarded point were were known before working the problem. The reasoning is backwards. The coordinates assumed for Rv0 were ficticous. They exist only in the mind.

The soluint is not for one partiele. It is for a family of partiacles. Since the trajectroy of an accelerated paticle is curved, there are manu paraticles with diffent histoites that are at the same place and have the same velcity at the center of the sphere. If the accelertion and velocity are both known from future observations then the solution is for a smaller family of partices, but the av-dot term would still be a free parameter. By continuing furthr, while we can never know precisely which particle the soluton is for, a small enough family of parcitls will behave in about the same way as a single particle for a short time near the center of the sphere, and for observers anywhere on the surface of the sphere. In othere words, real partilcles seem to be fuzzy, but they become less fuzzy in observations of longer duration, where more terms of the Taylor expansion of the trajectory become determinable.

Since the particle is visible when the observer is on the future light cone, the history of the particle is obtainable by extrapolating the trajectory backwards in time.

No matter how many terms are carried, the particle is slightly fuzzy. But for a short time near the center of the sphere, a tiny fuzzy particle will behave in about the same way as a single particle.

These coordinates can be translated in space or time, or both, and the translation will have no effect on the final solution.

The transmitted frequency could be capacitvive coupled to the turntable. Mechanical imperfections would result in small variations in the received phase that depend on the angle of the turntable. A phase locked loop with a long time constant could be used to extrct the average phase. Other methods of transferreing the reference frequency may be better.

In some ways, it would be better to take rv0 as pointing from the center of the sphere to the observer, but this sign choice is better when there is more than one particle in the problem, which is always the case when integrating around a current loop. The "right" direction for an observer co-moving with the partilee is the wrong direction for the observer at the field point, and conversely. The direction that rv0 points does not matter if the usage is consitent.

A phase shifter would not be exact, but if the transmit frequency is digitally synthesized to be slightly different than the received frequency then the phase will drift steadly at a precisely computable rate. Phase pulling effects can also introdce false harmoncs, when two frequencies interact, but the problem is easily managed with digital freuency synthesis and physical isolation.

In this order, the retardaion equaions for massless charged particles are the LW equations []. In beng true vector equations, the solutions do not contain any multipoles beyond the dipole. Massless charged partiesls are not known to exist. The LW equations are nevertheless a legtimate term of the retardeion series. They are frequencly the only retardaon equations needed.

At the point of closes approach to the observer, ... is zero in Eq –, so the solutio of this oder doees not have a transvers doppler term.

rv0 should actually be taken as pointing from the center the sphere to the observer, but most obsrvers prefer the other direction, which is not a problem unless the vector is rotated about its tip. Furthermore, this sign choice is better when there is more than one particle in the problem. It is nevertheless a choice, and it does not matter which direction a vector points if the usage is consistent.

The observational data for orbiting pulsars[4] would work just as well, but this experiment provides a means of evaluating the equivalence principle in greater detail for yanked particles.

In the beginning, there is a charged particle at rest at the center of a sphere. The obserere is anywhere on the surface of the sphere. In this order, the retardation equaton for charged particles is the Coulomb equation, which is usable in quasi-static solutions if the velocity of the particle is low enouh that the magnetic field can be neglected. This solution is for the tensor of the zeroth rank, a scalar.

The solutio assumes that an undiscovered superpotentials does not exist. If one does exist, then the scalar potential is a derived quantity, in which case we do not know where the beginning is.

The folowing calculations assme that the retarded potentials for a particle tht is at rest in our frame of reference, but accelerated, are the same as for an unaccelerted particle. That is not necessarily true, but there is no solution without assumptions. Contradictions will arise at some point in the progression if the assumption is wrong. While the acceleration terms are important, there is no hope for then until the velocity terms are right.

There would be a Corilous term in our frrame of reference if the coordinates were spinning. That does not mean that the Thomas equations are wrong. It means that pseudo vectors and true vectoes do not behave in the same way for rotations and translations.

When working the retardaion problem, a particle emits a single photon when it is at the retarded location. While the photon is travelling to the field point, the particle continues its journey to the simultaneous point. The particle arrives at the simultaneous point at the same instant the photon arrives at the field point.

Even though, in a physical sense, we do not care where the particle will be when the photon arrives at the field point, the location of the simulteneouls point provides a vital connection to the theorms of Eucludian calculus, which include the chain rule for differentiation and the tensor irreducibity theorm[6]. The modern form of Eucludian geometry is much more advanced than it was for the ancient geometricians, but not to the extent that the location of the simultaneous point can be abandonded.

However, the simultaneousl point is a mathematical abstraction that has to be computed. It is not a satisfactory anchor point for equations of physical significance, because we cannot know where a particle is now until after waiting for the light time accross the system. Many equations of the special theorey[1] do attribute an absoute signifance to the location of the simultaneous point, but they should not. The simulteneoujus point is not in our future, but it is in the particle's future. We can never be completely sure of what the future of the particle will be, although we can make reasonable extrapolations for a short time into the future.

In only beeng accurate to order vx1, there are no transverse doppler terms in the soluion of this order.

If the particle is being jerked then it was nowhere near the center of the sphere at the time ts=0, but the distant past of the partciel does not matter for the doppler equaton.

If the particl is aceelerated then it was not at the center of the spahere at the tim ts=0, but the doppler equaion is sill valid for a short time.

But the additonal terms do not improve the accuracy of the soluion. It is only accurate to order vx1. Even when ther are no acceleration terms, because of the couplling of the anglar veocitu and accelertion terms, this solution is only valid for a short time.

If true vector requains are used for the calculations, then the coordinates appeara to be spinning, but that is because accelertion is not representible with true vector equaions. That does not mean that the Thomas euations are wrong. It means that rotations and translations do not work in the same for pseudo vectors as they do for true vectors.

Ther is, of course, no doppler shift in the frame of refrence of the partice because the partile is at rest in that frame of reference.

Under these circumsancnes, no matter how small dtf is, it is not small enough to reduce the problem to first order. From the consderations of §–, this behavour is not contrry to the theorms of Eucludian calculus unless the equation at the field point is known to represent the total differential.

The time and space coordintates are not independent variables in light cone solutions. They depend on each other, making it difficult to determine both where a particle was and when it was there. It can be done, but not instantly.

In this solution, the pseudo vector pointing from a partiel at the location y to a nearby particle at y+dy appears to rotate as the two particles move in unision. If the clock at the field point is used as a time standard, then the chain rule for differentiation would be required for connecting the trajectories of the two particles. The angle of rotation does not depend on the magnitude of dy, so it cannot be negelcted by simply choosing a smaller value for dy. This term is a shear term.

... This term represents an elonggation. The shear and elongation terms both contribute to the quadrupole of the second rank tensor, but they are inseparable[2].

The shear and elongation terms have now both vanished. They were only there in the first place becauses the equatios were for the wrong partice. However, these calculations represent the perspective of a distant and detached observer. The equatons for a local observer would be different. There would not even be any quadruple terms if the particle is unaccelerated, although there would be if it is accelerated.

From a different perspective, the radius of curvature is infinite at infinity. The singularity at infnity might be removable.

The solution reduces to the relatvisic doppler equaion[20] if th $\cos 2\beta$ terms are dropped.

ts and beta are disconneced in this soution. beta can be any value when ts is zero. If ts is zero and beta is pi/2 at the point of closest approach, then the the sinx2 terms vanish. The transverse doppler terms have been experimentally verified, but the sinx2 terms have not been noticed. It is unlikely that they would have been, as detectign them would require very precies mecanicl motion. On the other hand, detecting the second harmonic of a minerature radio transmiter in a cirular obit would be relatively easy, as phase coherent detection methods are effective in extracting weak periodic signals from noise. Multipath effects from nearby reflectors could be minimized by adding a variable phase delay to the transmitter, then correcting for it in the receiver. There would be other harmonics if the ts^2 terms were carried.

This solution seems not to contain gamma but, as shown in the SOM, repeating the calculation with more poweres of velocity shows that it does.

ts is zero when the partcile is at the simultaneousl point. The simultaneousl point can be almost anywhere when the partice velocity is high. We cannot know where it is until a global solutin is obtained. If we already know a global solution, then we do not need to know how to inegrate the doppler equation. Unless the solution is known before working the problem, it has to be possible to apply the doppler equation without knowing what time it is.

The fourer trasform is effective for separating weak periodic signals, so the other terms may be experieantlly accesible for a transmitre in a circualr orbit. Howver, the doppler equation for accelerted particles should be obtained befor attempting the experiment. The exeriment is not actually necessary, as the orbitng pulsar data is already available[4], although constructing a minerature model of an orbiting pulsar in the laboratory would be fun.

This soution reduces to the relativistic doppler equation[20] if the $\sin^2 \beta$ terms are dropped. The calculations shown the reference take some shortcuts that should not be taken. There is no general way of abbreviating the Lorentz transform.

The textbook version of the relativistic doppler equations appears to contain an error, with the basis of the error being the use of an ovrsimplified version of the Lorentz transform.

The transvers doppler terms eqatioon have been experimanealy verified, but evaluating the .. terms would be experimentally difficut, as the doppler shift is large when they are large. The mesurment would have to detect a small discrepancy in a large value, which would require very precise mechanical motion. On the other hand, experiental evaluation of the harmonics of circular motion may be feasible. However, the doppler equaion in the first frame of referne for accelerated partices should be obtained before attemting the experiment. It is not true that equaions for the second frame of refence automatially apply to our frame of refernce, especially for radiative solutions.

It is not possible to exrapolate past the pi/2 point of a circular orbtit with the Taylor theorm, so the location of the simultenous point is not in general computable with the Tayor theorm for one variable. The simultenoss point is a mathematicl abstraction. The location of the particle at that point is not observable until after waiting for the light time accros the systm. There are nevertheless essential mathematicl relationships between the siultenous and retarded points. The simultaneouls point cannot be abandoned altogther without abandoning the methods of Eucludin calculus.

ts has dropped out of the solution. It is now independent of the selected coordinate system for time just long enough to compute the first derivative.

Actally, the solutions can be differentiated more than once, but derivaves beyond the first are degenarate. Degenration does not have the same meaning as being wrong, but not much should be expected of them.

The coupling is not real. It is an abberation that has to be corrected, but it makes integrating the retarded Newtonian acceleration tricky.

The doppler frequency changes with time as the partile moves, but the doppler equation has to work the same way at any time, because we do not know what time it is.

ts does not drop out of eq – if terms quadradic in ts are carried, indicating that the the solution is coordinate-dependendt for observations of longer duration. Since Eq – was not derived with the Lorentz transform, its meaning is not necessarily quite the same as relative doppler equation, even though the solution is the same for observerons of short duration. The light cone equation apperars to need some more work, because we do not know what time it is.

The solution for an observer in radial free fall is not static, so there are anitsymmetric space-time cross terms in the solution. Even though they vanish in the infinitesimal, they grow faster than the first order terms when integrating in space and time, so they probably should not be neglected. This paper is for the retarded potentials.

dr1 has dropped out of the solution, meaning that it can be differentiated once in space. Actually it can be differntiated more than once, but the derivatvies will be degenerate. Being degenerate does not have the same meaning as being wrong, but not too much should be exected of them.

Thus, the particle was not where the first observer thought is was. The retarded time and space corrdinast are not independent variables. It is extremely difficult to determine both where a particle was and when it was there. Assuming that the solution is known befor working the problem is not appropriate, although it is necessary to start somewhere. The LW equations are a good starting point for a long journey.

The location of the particle at simultaneous point is not where the observer in the first frame of reference thought it should be. But how could the observer have known where it should be? The location of the particle at that point is not visible until after waiting for the light time accross the system. On the other hand, the particle is at rest in the second frame of refernce. The other observer knows where the particle is.

The distant observer could enlist an assistant to report the time that the particle passes by a synchronized clock that is at rest in the first frame of refernce, but that would not allow the observer to find out where it was any sooner.

Thus, the prticle was not where the observer in the first frame of reference thought it was. The time and space coordinates are not indpendent variables in light cone solutions. They depend on each other. It is all right to assume that either one is already known. It is not all right to assume that both are already known before working the problem. It is difficult to determine both where a particle was and when it was there. It cannot be done instantly.

We do not know where the particle was yet, but it is only the derivavives of the retarded potentials that are of interest. However, a longer history of the particle would be required if the potential soluion is to be diffrentiated more than once. The Maxwell equations are in terms of the second derivatives. Perhaps one day we will know where the particle was, but not soon, for the tensor of each rank to at least the fifth is irreducible[6].

The contravarian tensor of the second rank represnents the first infinitesimal step[6]. However, the tensor of rank n+1 is the gradient of the tensor of rank n, and potential solutons have to be suitably differentiated to obtain the fields. The tenosr of the third rank is therefore more relevant for contravariant field equations. There have been some historical misunderstandings in this respect.

two inconsistencies

It is this term that is responsible for the perceived unphysical behavoir of the particle. The terms is zero for radial motion, where is pm 1. It is at a maximum for a transverse velocity, where .. is zero. These terms result in a coupling of the angular veloicy and retarded accelration. The couplintg is not real. It is an abberation that has to be corrected before the dopple frequency can be integrated.

The doppler frequency changes as the particle moves, but the doppler equaion has to work in the same way at any time, because we do not know what time it is. This solution depends on when the time dts=0 (or dtf=0) is chosen to be. This solution is unphysical. It is also the wrong equation for the doppler wavelength [].

It is not sufficient that equainsns of physical signifaicne be invariant. The must also not depend on the choice of a coordiante system. Coordinate depencencies can exist in either space or time.

Interpl...

After substituting $\beta =$ into Eq. –, its reciprocal becomes

τ ranges from $-\pi/2$ to $+\pi/2$. It is negative for approaching partices, and zero at the point of closest approach.

The solution is singular when the velocity is radial, in whcih case the partile collides with the observer. The solutions are restricted to cases where the particle misses the observer by a small amount. At the point of closest approach, the angular velocity of the partcie goes to infinity as the miss distance goes to zero, so the equation are ill behaved in that region.

A third space coordnte would be required to represent the out-of-plane terms of accelerated partices, but two space coordinates are sufficint for unaccelerated particles.

A solution was obtained by the method of undetermined coefficients. The solutoin was first expanded in a series in v0, then the coeficents for each powere of velocity were discovered, one power at a time, working upward. After several terms are obtaned this way, it is not too difficult to guess the closed form solution.

The singularity could be removed with L'H's rule [], except that the rule only applies to pure-number quanites. The numerator is a dimensioned quantity in this case, which is probably managable complication, but there is anothre way of obtaining the solution.

The rule only applies to pure-number quantites. The units would be wrong in this solution if only some of the terms were differentited.

The time and space coordintes are not independent variables in light cone soluins. They depend on each other. But now that ts has dropped out, Rx0 can be chosen independenlty. It is no longer necessary to treat it as a constant.

gamma is usually associted with the second frame of reference. It usually goes unnoticed in the first system, but it occasionally occurs in those equations too.

The singularity can be removed with L'h's rule [], except that the rule only applies to pure-number quantites. It can be used with dimensioned quantites if the units of the numerator and denomator are both the same, which can be accomplished by dividing through by Rx0. But since Rx0 is a constant at this point, dividing through by it does not affect the derivatives.

The singularity could be removed with L'H..'s rule []. However, the doppler equation xhas to work in the same way at any time, not just at the time of closest approach, so the procedure would not be useful.

tau is pm pi/2 for radial motion, so removint the singularity at the closeet approach creates a new one for radial motion. The particle is on a collision course with the obeserver in that case, but a near miss is more likely.

Removing the singularity has no effect on the doppler equation, but it does affect the solution for the retarded potentials.

In general, there are out-of-plane terms when the particle is accelerated, so the soluton for two space coordinates and one time coordinate is not applicble in that case. But for a short time, the angular relationships at the retarded point do not change significaly, so the solution for unaccelerated particles should be a useful approximation, albeit of too low an order to be of much practial interest.

From the perspective of a distant observer when integratin in the inbound direcction, this surpuflus degreee of freedom does not look like a contant of integration. But from the perspective of a local observer when integating in the outbound direction, the surpufullus degeree of freedom is nearly invisible, requirinvve a constant of integration if the equatios are to behave correctly at great distances. The oubbound integral represnts the perspective of field equations. It is easier for a disant observer to evaluate the constant. However, retardaion equaions do not form a complete representation. The are just one aspect of a much larger problem.

This solution could also be obtaned by taking two consecuves steps in time rather than one in space and one in time.

This reparameterization has the effecte of detrmining a constant of integration. From the perspecive of a distant observer, it does not look like a constant of integration when integrating in the inbound direction. But from the perspectives of a local observer integrating in the outbound direction, the starting point of a definite integral would have to be offset in order to obtain solutions that are commensurate with those of the distant observer.

The other perspecteve represents the integral of field equaions, which would require a constant of integration in global solutions.

In being first order in space and first order in time, cross terms vanish quadradiacally in a small region of space-time. For the same reason, the cross terms grow quadradially when integratiing in space and time. Consequenly, even though the cross terms are not very important for local obsevers, they cannot be neglected if the solutions are to behave properly at great distances. The cross terms cannot be completly neglected in the local region either.

The singularity at the Sch... radius is an example of the offset that is required when intgrating in the outbound direction, while the offset is vanishingly small for a distant observer when integrating in the inbound direction. From the persepce of the distant observer, it does not even look like a constant of integration, but it is in a global solution.

With other methods of soluion, a constant of integration would probabhly be required to take out this surpruflous degree of freedom if the the fields are to behave properly at great distances. The constant of integraion could take the form of a dispacement in the starting point of a definite integral.

From the perspectved of a local obesrver, the starting point of a definite integral can be assumed to be zero when integrating in the outbound direction. A distant observer would not necessarily agree with the consequences of the assumption.

We would not know what to do with the equations of the retarded potentials if we had to know what time it is before applying them, because we do not know what time it is. The dts0 terms have to be taken out. For a short time only, the equations will then be usable.

The doppler shift depends on what time it is, but the doppler equation should work in the same way at any time. This equation depends on when the time ts=0 is chosen to be. It contanins a coordintate dependicy. Coordinate depencies can exist in eithe space or time.

A possible approach would be to integrate the first time derivave of the doppler shift, which would requires at least two steps in time. There are mathematcl similarites between two steps in time and one step in time followed by one step in space. The advantage of this approach is that the space and time terms are each of first order, although there is always more than one way of working a problem.

There is the possibilty that this equation is not the only solution that can be discovered. The behavior of the equation is shown in detail in the SOM.

The singularity is presumably rempvable, perhaps with L'Hs... rule or by other methods, but since its basis is a misunderstanding, the equation will not be developed further at this time.

We cannot know where a particle is now until waiting for the light time accross the system. The simultaineouls point is not in our future, but it is in particl's future. The future location of a particle is not a satisfactory anchor point for equations of physicl significance.

The tensor of the zeroth rank is a scalar. The tensor of rank $n + 1$ is the gradient of the tensor of rank n. If the solution for the tensor of the zeroth rank is complete then nothing else is required. It is doubtful that complete solutions are obtainable, but it is usually best to begin at the beginning and not skip steps.

The following calculations assume that the retarded potentials for a charged particle that is at rest in our frame of reference but accelerted are the same as for an unaccelerted particle. That is not necessrily true, but there is no solution without assumptions.

The calculaions also assume that an undiscovered superpotential does not exist. If one does exist, then the scalar potential of these solutions is a derived quantity.

As shown in Ref. 11, the soluions of the Proca equations contain exponential terms that represent the range of the fields. The range of the fields is assumed to be infite in these calculations. Due to Olber's paradox[19] and the singularity of curvature terms at infinity, where the radius of curvature is also infinite, it is likely that the range of the fields cannot be neglected in high order radiative solutions, but it proabably can be neglected in low order solutions.

As mentioned in §–, this solution is for two space coordinates and one time coordinte. There will probably be additional terms in the soluton for accelerated particles. But since the LW equations are known to be usable for accelerted particles, it is likely that this soluton is also.

Thus, no matter how small dtf is, it is not small enough to reduce the problem to first order. The dts interval might or might not be small enough, but in any case is is much smaller at high velocites.

It is possible to begin a new experiment at this time, in which case the distant past of the particle is no longer of interest.

This solution is therefore incomplete. In missing a cosmologiacl term, it is probably incompltete in more than one way.

A potential equation has to be valid at the time t+dt if it is to be differentated once in time. Unless the equation is valid at that time then, while it might be valid in some sense, it is missing a constant of integration. It is the nature of retardation equations that the constant of integration varies from place to place and time to time, so the equation needs to be integrated if it is to be differentiated.

From the perspective of a local observer, the constant of integration takes the form of a displacement in the starting point of a definite integral when integrating in the outbound direction. The constant of integration is required if the equatons are to behave properly at great distances, even though it is of little consequence for local observers. From the perspective of a distant observer, when integrating in the inbound direction, the constant is zero, or nearly so, making it easier to evaluate, especially when the local observrs do not even know that they need one.

The chain rule is not needed when there is only one independt variable. Provided that the particle is not accelerated, there is only one independent varaiable in the calculations of this section. The solutions for unaccelerated particles are not of much practical interest, but the velocity terms do come first. Furthrmore, after the potential solutions are differentited to obtain the fields, acceleation terms do occur in the solutions.

The doppler shift, in terms of wavelengths, is

There is no transvers doppler term when ts and Ru1 dot vu are both zero. However, since ts did not drop out of the solution, we do not know when the partciel is at the point of closest approach. The soluton is not for one particle. It is for a family of particles. The v0x2 terms can be neglected at low velocities, in which case it is a small family.

The underlying difficulty with this calculation is that the chain rule for differentiaon is generally required when the variables are not independent. The time and space coodinates of light cone solutions are not independent. They depend on each other.

The 1/R1 terms result in a coupling of the angular velocy and acceleraion terms. The coupling is not real. It is an abberation that has to be corrected before the doppler equation can be integrated.

It is only the derivatives of the retarded potentials that are of interest, so this bogus curvature term is not necessarly a culprit in its own right, but there are consequences of it.

The light cone equation is not a true vector equation. It cannot be reduced to first order in the same way that true vector equations are.

There is always more than one way of working a problem, Possible alternatives in this case would be to apply the chain rule for differentiaon or to integrate over the dt interval. The following calculaions repreesent a differnt way, which might or might not be the preferred way.

Discrete difference equaions are widely used in digital signal processing. The discrete Fourier transform is one of these algorithms. The solutions approach analog solutions when n is large.

When the interval is arbitrarily small, the first differnce is equivalent to the first derivatve. But the second difference is only equivalent to the second derivatve when n is large. However, for the light cone equaion, n does not have to be large for the seond difference to be zero, simplifying the equations.

Similarly, the third difference would be zero for accelerted particles, and the fourth difference would be zero if the avddot terms were carried. Consequency only a few points are required to reduce the equations to first order. The light cone equaion is special in this respect, and only when the time at the source is the independent variale. Most equations do not behave this way. For most equations, the second difference at one point is not an accurate substitute of the second derivatve unless n is large.

This extrapolation equaion for the second differene is not accurae uless the third difference is zero. For most equaions, the third difference is is not zero.

The simultaneous point is a mathematical abstraction. It is not a satisfactory anchor point for equations of physical significance.

The solution cannot be integrated in this form, because the doppler shift is not for the time dtf=0. The transverse doppler term vanishes at that time.

Extrapolateing the first difference is equivalent to extrapolating the first derivatie.

This equaton for extrapolating the third differnce is not accurate unless the fourth differene is zero. It is not zero for most equations.

L. AN INFINITY OF INFINITESIMAL STEPS

While the dtf interval at the field point seems like an infitesimal quantity, the dRv at the other end of the vector can be arbitrarily large when the velocity is high. The other end of the vector needs to be integrated over the dtf intrval.

Thus, the simulteous point is not where true vector equations predict that it should be. Except in special cases, the difference between two closely spaced vectors is not a true vector. It is a pseudo vector. The decomposion products of the fourth rank contravariant tensor include six pseudo vectors[6]. The order of an equation cannot be easily judged by its appearance.

Whenever the data points are delayed by the light time accross the system, many terms in a Tayor expansion would be required to extrapolte the trajectory to the simulteneoous point. The $1/6\dot{\boldsymbol{a}}t^2$ and $1/24avddott^3$ terms would make a large contribution when the simulteneouls point is far into the future. The solution for unaccelerated particles is much simpler.

All the absence of absolute simultaneity[1,16] means is the we did not know where the particle is in the first place.

In other words, the observer does not need to know what time it is in order to apply the retardin equations.

The observer at the field pont would like for this vector to be dR rather than delta Rv, but that is not possilbe. No matter how small the dtf interval is at the field point, delta ts can still be an eternity.

The time and space coordinates are not independent variables in light cone solutions. The chain rule for differentiaton is generally required when the variables are not independent. By formulating the problem this way, there is only one independent variable, avoiding the need for the chain rule, not that there would be anything wrong with applying the chain rule to the equations. There is always more than one way of working a problem.

The accelertion terms should be carried in these calculaions. The appropratie equation in that case is vn=v(n+1) - a(n+1) dts, but, for now, the acceleration terms will be neglected.

A computer program ends up in an endless loop, producing the same wrong solution on every pass. The location of the simultensouls point is not computable this way.

The acceleratio terms really should be carried in these calculaions, but the velocity terms do come first.

The solution would have powers of velocit in all orders if tf were the independent variable. From the perspective of an observer co-moving with the particle, that is only because the series expansion of 1/(1+v/c) has powers in all orders. From the perspective of the other observr, powers byond the first are not meaningful, although they have to be carried to obtain self-consistent solutions. But from the parspectevie of the othr observer, the solution is not nearly accurate enoght to predict where a high velocity particle will be with it arrives in the vicinity of the field point.

To order vx2, this soltion is the same as the doppler equaion in §–. The solution would be more accurate if more powwers of velocity were carried. However, it is rarely useful to carry very many terms in velocity equaions while neglecting the acceleration terms.

The acceleration terms will need further study. However, there is no hope for them until the velocity terms are right. But also, it rarely useful to carry very many terms in velocity solutons while negelecting the acceleration terms.

In the figure, we have no interest in where the partice will be when the signal arrives at the field point. It is only the derivatives in the vicinity of the retarded point that are of practical interest.

The relationshps in Fig – are ill behaved when the partcile is on a collision course with the observer, or nearly so, with a velocity near c. In extreme cases, the difference between the arrival time of the photon and the particle can be only an instant, yet the travel time can be an eternity, making it difficult to compute precisely where the particle will be when it arrives in the vicinity of the field point. There is ample time for transvere accelertion terms to deflect the particle, yet accelertio terms are not representalbe with dt_f. The problem cannot be reduced to first order by simply choosing a small value for dt_f.

If the partile is far away, the equation is not nealy accurate enough to predict where it will be when it arrives to the neighborhood the field point.

There are still quadradic terms in the vicinity of the retarded point, but since is not necessary to extrapolate to a point near infinity from a nearby simultaneous point, the equations are simpler when it is not necessary to know where the simulteneous point is. We do not care where it is in the first place, so there would be no point in confounding the equatons with a singularity at infinity. We cannot see that far anyway.

The radius of curvature, relative to ourselves, is infinity at infinity. It may be unintuitve, but it is possilbe for equations to be singular at infinity, and indeed they frequencly are, as is illustrated by Olber's paradox[19]. A singularity at infinity is not necessarily removable.

For retardaion equations, the singular point is not far away. It is arbitrarily close to us if the photon and the particle arrive at about the same time. The singularity might be removabvle, or it might not be. It is possbile to find out if it is removable with L'Hôpital's rule[18].

As shown in the SOM, the method of successive approximation does not converege when attempting to compute the location of the simultaieouls point. All the absence of absolute simultaneity[16] means is that we did not know where the particle is in the first place. We cannot know precisely where the partilce is without a perfect ability to predict the future of the particle from delayed observations.

Thus, even though the particle was not where the equations of 3+1 space predict that it should have been, the equations are slippery enough to accomodate the alternative value. This relationship remains true if more powers of velocity are carried.

The acceleration terms will need more work. But since the LW equations are known to work well for accelerated particles, it is plausible that these equations do too, even though they are incomplete.

In Eq –, the additainoal velocity av dts acqured during the first infinitesimal step will cause gamma to be slightly different for the second step.

When working the retardaion problem for unaccelerated particles, the particle emits a single photon when it is at the retarded location.

While the partice is continuously visible, it is necessary to consider just one point at a time when developing the equations. From this perspectrie, the particle is only on the retared light cone for an instant, then it continues its journey to the simultenous point in darkness.

But since the particle actually is continuously visible, it is possible to write equations for that case too.

The coupling is not real. It is an observer-dependent abberation that has to be corrected before the doppler equation can be integrated.

needs to be corrected if the solution is to be for an inertial partilce. Measured data points for real particles are always delayed by the light time accross the system.

The coupling is not real. It is an abberation that has to be corrected before the retarded Newtonian acceleration can be integrated.

The couplling is not real. It is an abberetion that causes the integration of the retarded Newtonian accelertion to be tricky.

The simultaneous point and the point of closest approach to the observer are the same when ts and ru dot vu are both zero. In that case ... is gamma, which is the transvers doppler effect. The frequency is redshifted at that point.

ts can be chosen to be zero at any point on the trajectory. The equations contain a coordinate dependency if that is not possile. Equatios with a coordinate dependency are not of physical significance.

If we already know where the particle will be when it arrives at the simultaneous point, for any choice of ts, then we already have a global soluton and do not need to know how to integrate the doppler frequency. It other cases, it has to be possible to apply the equation without knowing what time it is, because we do not know what time it is. The relativistic doppler equation is slippery. It has more degrees of freedom than constraints.

Interplanetary spececraft navigation software incorporates corrections from the general theory of relativity when integrating the doppler frequency. The accuracy of the corrections has been demonstrated[22]. We do know how to integrate the doppler frequency, but the light cone equation needs some more work.

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The cos theta term of the relatistic doppler equaion is a dipole term. There are no multipoles beyond the dipole in the solution.

The calulations of this paper are for the retarded potentials. They represent the perspective of a distant and detached observer. The retarded potentials are not usable for orbit determination calculaions. On the other hand, retardaion equations are not obtainable without knowing the recent history of the particle, so the problems are not completly independent.

When two particles are in orbit about each other, the radiation from each particle perturbs the orbit of the other particle. Static terms decay as 1/rx2, while radiative terms decay as 1/r, causeing the full solution to be of formidable complexity, even when the partices are well separated. It may be tractivle by developing the ratiative terms as a pertubation of the non-radiative solution, but this relationship makes it difficult to detrmine the recent history of the particle.

The problem would be simpler if the location of the barycenter is assumed to be known before working the problem. The problem is harder if the barycenter wobbles, even though that is not possible with the Newton equations. The location of the barycenterr is one aspect of the soluton. It is mostly not a good idea to assume that a solution is known before working the problem, especially when there are radiative terms in the equations.

While the doppler shift depends on what time it is, the doppler equations itself should work in the same way at any time. That makes it possilbe to work the problem backwards by discovering a constant of integration for which the doppler equation can be applied without knowing what time it is.

may be possible to infer the full form of the doppler equation for accelerated particles from those solutions. Barycenter corrected solutions would be easier to interpret. The calculation would be relevant, but it would probably not help in reconciling the LW equations with the doppler equation.

It often seems that we should already know where the prticle is if it is not accelerated. It does seem that way, but intuiton is not a good guide in the four dimensinal space.

[later] The invrse relationship applies for a partilce in radial free fall. Even though the antisymmetric terms vanish in the infitesimal, they are nevertheless requied if the equation is to be either differentited or integrated.

Thus, when a clock near the observer is used as a time standard, the angular velocity and Newtonian acceleration of the partile are coupled. The coupleing makes it impossible to integrate the retared Newton accelertion until the angular velociy and acceleration terms are separated.

While the particle is continuously visible, it is necessary to consider just one point at a time when writing the equations. The prtile is on the light cone for an instant at the retared intersection, then its contunues its journey to the simulteneous point in darkness.

While the assistants can report the times that the particle passes by each clock, that does not allow the distat observer to find out where the partiel was any sooner. The distant observer cannot know when the particle reaches the simultenous point until waiting for the light time accross the system. The distant observer can nevertheless rely on the reports from the assistants for the purpose of refining the retaration equations.

The simultaenous point is not in our future, but it is in the particle's future. The future location of the particle is not a satisfactory anchor point for equations of physical significance. The future is not perfecly predictable. Even when there is only one particle, many terms in a Taylor expanson of the trajector would be required to accuatel extrapolate it far into the future. The extrapolation quickly become intractile if there are many particles. Extrapolatng the simutneous coordinats backwards in time to the location of the particle where it was visible would be equally difficult.

The simultaneous point is a mathematical abstraction. The future locatio of the particle is not a satisfactory anchor point of equations of physicl signifiance. There are nevertheless essential mathematical relationships between the retarded and simultaneous points.

Due to the presence of quadradic terms in the equation, if the coordinates are first-known at the simulteneouls point, the location of the retarded point would have to be obtained by integration, plus a constant of integration. Since the doppler equation has been experimentally confirmed, it is indicted that the constant of integration is not zero.

Similarly, if the coordinates are first-known at the retarde point, then the location of the simultaneous point would have to be obtained by integration, plus a constant of integration. Howevere, when derivng the equaions for the retarded potentials, we do not care where the particle will be when the signal arrives at the field point.

Since the simultaineols point is not a physical quantity, it is not a satisfactory anchor point for the integral from the simulltaneiouls point to the retarded point. The integral may requure a constant of integration to accoulnt for the unreal nature of simultenous point.

For equations of physical significance, the reasoning leading to figs – - – is backwards. The location of the particle at the simultaneous point is a mathematical abstrction. We do not care where the particle is when it is not visible.

Thus, the angular velocity terms masquerade as accelertion terms, distorting the connection between the retarded and simultaneouls points.

Figures ..,–, and – all assume that we already know where the particle is when it is at the simulaneous point. We cannot know where the particle is then until waiting for the light time accross the system. All three figures are unphysical.

the point of closest approach, where $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{v}}$ is zero, it also becomes visible in the first frame of reference.

The usual way of working the retardation problem is to assume that the location of the particle at the simultaneous point is already known, then compute its location at the retarded point. It is possilbe to invert the reasoning by assuming that the location is first-known at the retarded point, then computing the simultaneouls point.

In Fig. 7, each particle emits a single photon when it is at the location on the left side of the figure. The photons travel to the field point with the velcoity c. The photons are emitted sooner when the particle is further away from the observer. All of the photons arrive at the field point at the same instant. In the meantime, each particle continues its journey to the simultaneous point. The particles arrive at the simultaneous points at the same instant that the photons arrive at the field point.

The four coordinates are mutually orthogonal in 3+1 space, but not in 4-space. It may be possible to orthogoalize the equations by taking one step in space or one step in time, but never one step in space and time.

The initial particle location is r_x and r_y. The retarded time at the particle is $t_s = -(r_x^2 + r_y^2)^{1/2}/c$. The particle then moves to the right with the velocity $0.6c$ for the time $-t_s$.

In the figure, the time at the head of the vector $\boldsymbol{R}_0$ is 0. The particle becomes visible when it is at the simultaneous point when the time at the tail of the vector is $+(\boldsymbol{R}_0 \cdot \boldsymbol{R}_0)^{1/2}/c$, although by then it has moved on to a new location. The time at the tail of the vector $\boldsymbol{R}$ is zero.

Each projection of 4-space into two space dimensions represents just one slice of the problem. None of the projections, considered one at a time, is capable of representing the whole solution.

The simultaneous point is not in our future, but it is in the particle's future. The future location of the particle is not a satisfactorey anchor point for equations of physical significance. There are nevertheless essential mathematical relationshps between the retarded and simultaneous points. It does not matter which comes first if the equations are right, but true vector equations do not predct the right location.

The trajectory should to be extrapolated from current observations, which are always delayed by the light time accross the system. On the other hand, the location of the simultaneous point provides an important but indirect mathematical connection to the equations of Euclidian calculus.

The particle is not half way to the destination in half of the travel time. The solution is not for an unacceleraed inertial particle. The angular velocity terms masquerade as acceleraton terms, distorting the connection between the simultaineous and retarded points. Due to the presence of k^2 (and k^3) terms in the equation, the connection would have to be established by integration.

The bogus terms make it impossible to integrate the retarded Newtonian accelertion without further analysis.

The function and its derivtve are not consistent in this soltion. The time at the field point actually should be obtained by integration, and there would be a possibity of obtaining a consistent solution that way. But since it is only the derivatrvies of the retarded potentials that are of interest, there appears to a shortcut.

The solution for accelerted particles will need some more work, and some more contemplation. The incremental velociyt acquired by the particle by the equation dv=av dts will cause the value of gamma to be slightly different for the second infinitesimal step.

For retardaion equations, we do not actually care where the particle is. It is only the derivatives of the of the retarded potentials that are of intrest.

With some inspiration from the doppler equation in §–, it is not difficult to discovere the closed form equation with the same series solution. (More powers of velocity are shown in the SOM.)

LI. THE SECOND INFINITESIMAL STEP

The first step is always the hardest. There is nevertheless some uncharted teritory ahead.

LII. CHASING THE PARTICLE

LIII. WALKING THE VECTOR

LIV. THE CHARGE SOLUTION

There are no indications that this solution is the last term of the series.

The third space coordinate will need further consideration. However, the equations are not Newtonian.

Perhaps one day we will stop believing the the sun orbits about us, but probably not soon.

This residual is closely related to the Thomas precession. The connection is that some textbook interpretaions of the av cross vv terms do not distinguish between angular velociyt and Newtonina acceleration, even though they are globally very different.

The acceleration terms in the following calculaions will need furthr study. However, there is no hope for them until the velocity terms are right.

All powers of velocity are present in the series expansion for 1/(1+v/c), but there are no vx4 powers in the above calculins when dts is the independent variable, so there no possibility that the solutoion is complete.

There are no dt1 dt2 cross terms in the first frame of reference, making it impossilbe for an observer at the field point to determine that the soluton is not of first order, since subdiving a straight line does not accomplish anything. To the observer at the field point, dt1+dt2+dt3 is the same as dt. The capability of the observer for reducing the problem to first order is impaired, but there is no remedy for the limitation with vector equations.

A third space coordinte is not needed for obtaining the solution for an unaccelerated particle. Introducing a supurflous degree of freedom would result in the loss of all multipoles beyond the dipole.

These calculaions can be performed in the other frame of reference, but we sometimes neglect our own frame of reference more than we should. After all, it is the only one we will ever know.

The velociy vector has a component parallel to the line of sight at the retarded intersecton, making it possible for the light cone equaion to accurately constrain the time that the particle is at that location if it is known to be unaccelerated. The acceleration terms are important, but there is no hope for them until the velocity terms are right.

The solution for accelerated particles will need further study, but there is no hope for the acceleration terms until the velocity terms are right.

This solution is in the lowest order of interest.

Our orientation is not an absolute. We should not project it onto the cosmos, as was done in S –.

The acceleration terms will need further study, but there is no hope for them until the velocity terms are right.

These relationships will be developed further in later versions of this paper, but understanding why equations work the way they do is more important than knowing what the equations are.

rx0 has dropped out of the solution, yet the behavior of the equations is not comprehensible without it.

The simultaneous point is where the particle is now. It can be at the point of closest approach, but it does not have to be. The particle is not visible when it is at the simultaneous point. It is possible to find out where it is, but not until waiting for the light time accross the system.

The particle is visible when it is at the retarded intersection, so the equations are more real in a physical sense if the retarded location is taken as a reference point.

When retarding a current loop, if the conduction electrons are moving left to right on the nearside of the loop then they are moving right to left on the farside. Rtardation equations have to work in the same for either direction. It would be hard to write an equation that works in the same way for both Figs – and –.

Time has dropped out of the solution. That is good, because we do not know what time it is.

The handedness of the solution cannot matter, but it is nevertheless sometimes necessary to know what it is in order to choose the right sign in an equatin.

Ideally, retardaion equaions would be valid forever. A less ambitous goal is to obtain equations that are valid just long enough to compute the first derivative.

This solutiomn is in the lowest order of interest. There will be additional terms in the equations for more elaborate constructs.

Except for the point labeled $\gamma r_y/c$ in the figure, the bottom panel is the same for a particle traveling in either dirction, but the top panel would be mirror image of the one shown.

Since the two particles are moving in opposite directons along the same trajectory, their locations must be coincident at some point on the trajectory.

The location of the field point is taken as being first-known when applying retardaion equatins to engineering problems, so the location of the particle has to be adjusted.

An assistant near the trajectory could report the time that a particle passes by one of the at-rest clocks, but that would not allow the observer at the field point to find out where the particle was any sooner. When relying on stale observational data, it is difficult to be sure of precisely where the particle was.

The solutin is intrepable as meaning that the field point traveling backwards in time. If the other obeserver does not need this solutio then we do not either.

Inverting the sign of vv in this solution would be intrpretable as representing the trajectory of a particle moving backwards in time. The particle would not be on the light cone, so that solution is no longer available.

In this soluton, vu can be in any possile direction. But its sign cannot be inverted, because the solution for a particle traveling in the other direction is different.

There is no requirement that the equations for the first frame of refernce be vector equations. But if they are, then they have to work in the same way for either sign of the velocity vector, because we have no way of knowing what the right sign should be.

However, if the equations are not true vector equations, there is a possiblity that the right sign for the velocity vector could be selected anyway. The vector from the field point to the partice rotates as the particle moves. From the perspective of an observer in the frame of refernce of rhe particle, it rotates in the other direction. If one rotaion is left handed then the other is right handed. While it does not matter whether the rotaion is right handed or left handed, it is conceivable that the chareialy of the solution matters in the transform.

The following solution is therefore not necessrily unique. Until uniqueness is shown, it would not be acceptable to claim that we know where the particle actually was.

The following calculaions are for the tensor of the zeroth rank, a scalar. The tensor of rank $n+1$ is the gradient of the tensor of rank n, provided that the equation for the tensor of rank n is not degenerate, in which case there are no more terms in the series. It is possible to interpret degenration as representing completeness, but its actual meaning is nearly the opposite. On the other hand, being degenerate does not have the same meaning as being wrong.

Obtaining exact light cone solutions requires selecting the root of a polynomial. There is no known equation for the fifth root of a polynimial, suggesting that the solutions are either degenerate or divergent in some order, probably a low order.

dtf has dropped out of the solution, so if the equation is valid at the time tf+dtf then it must also be valid at the time tf+0. That is just long enough to compte the first derivatvie in a manner that does not depend on the choice for a coodrdiate system.

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This solution is obviously only valid for the first derivatives. The contravaraint tensor of the second rank repreesents the first derivatives? , but the tensor of the third rank is irreducible.

It no longer matters whether the partcle is moving to the left or to the right. The equaions will work in the same way for fruit bats as they do for us (not that they would care).

This solution is unphysical. The clock at the field point runs at a diffrent rate when the partcile is at the retarded point than it does when the particle is at the simultaneous point.

This solution makes no sense. The clock at the tail of Rv0 runs at a different rate than the clock at the tail of Rv, yet it is the same clock. The solution is for a partice on the light cone, but it is not the right particle.

The time in this equation is the computed time that a partille passes by an at-rest clock. It is not the same as the time read from the displayed number of a moving clock. It is important that the two ways of integrating not be used interchangeably.

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The solution for an accelerated particle will need further study. The solution would be expected to contain additional multipole terms as the observer rotates through 360 degrees about the line of sight.

Eq – indicates that this soution could also be obtained by integration. However, the soluton could still be incomplete if the constant of integration is not correctly determined. The constant of integration must not be neglected, because the loction of the simultaneous point is not a physical quantity. It is not a satisfactory anchor point.

Just as the symmtric cross terms vanish in the infitesimal in Eq –, the antisymmtric cross terms will also vanish if there are no anti-symmetric terms in the first place. The cross terms vanish in infinitessimal, but not in the integral. More precisely, the cross terms do not actually vanish at all, it is just that they are not representable with true vector equations.

The particle was evidently not where it was thought to be. 4-space can be projected into 3+1 space, but the reverse projections do not appear to be reliable if the calculations are based on true vector equations.

[later] As shown in the SOM, the exact solution (for unaccelerated particles) is

Sadly, the particle was not where we though it was. But then, determning where the particle was in the four dimensianl space is not easy, and it cannot be done quickly, because the angular velocty and retarded acceleration terms are coupled.

Actually, we do not care where the particle was. The thing that matters is that the derivatives integrate correctly. We cannot know where the particle was until a specific global solution is obtained. An observer near the trajectory can know where the particle was, but we cannot.

In an historical context, this solution can legitametly be interpreted as representing the Thomas precession. The equataions were correct. Understanding what they mean is our responsibility.

Similar equations for kinematic solutions should exist. There is the intriging possibilty that in some order, probably not in this order, the precession of the periastron becomes syncopated, with even and odd orbits being differnt. The calculaions have not been carried through, so the possibilty is purely speculative.

There can be several distant observers, and the time of closest approach for one observer has no meaning for the other observers. The observer in the frame of refence of the particle needs to cusomize the time of closest aproach for each of the distant observers.

The unreal nature of the simultanous point in Eq – is of no diretly measurable consequence, since it exists only in the mind. The equatins might have taken a different path if the doppler shift had been discovered earlier, but it wasn't. The vehicles of long ago were too slow to notice it.

The acceleration terms will need furthr study. Eq – implies that this soluton could also be obtaned by integration. The calculatons shown here are oriented to studying the form of the relatinships, and are not necessarily the best way of approaching the problem. There are probably several other ways. The chain rule for differentiaion, which can be appied recursively, is a likely candidate. Recurive applications are capable of converting first derivatives into derivaves of any order.

LV. X9

When applying the retardatin equations to engineering problems, the observer at the field point would believe the location of the partile to be given by Eq –.

In this frame of referce, the head of the vector is anchored and the tail is rotating, which does not appear to be appropriate behavior for a pseudo vector.

The tail of the vector is not anachored at the particle while the tip of the vector follows the future location of the field point. The retarded potentials in our frame of reference are the advanced potentials for an observer in the frame of reference of the particle.

Now translate the coordinates in space so that the tail of the vector is where we will be after the first infitesimal step.

there are dtx2 terms in the soluton, but the can be dropped if the first step is a small step.

The Newton series is usually better suited to kinematic calculations. It can also be used to parametrize the retarded potentials. It is the transform that matters, not which perspective is the right one.

Time marches on. dt1 has dropped out of the solution. There is no way back, not that we would want to go back.

This solution appers to be for the first derivatives of the potenenials. The tenosr of the second rank represents the first derivatives. The tensor of rank n+1 is the gradient of the tensor of rank n, so the derivatives of the retarded potentials would be expecte to correspond to the decomposion products of the third rank tensor in S –. It has not been confirmed that they do.

The actual problem is that velocity has the other sign in the second frame of reference. The field point is not actually traveling backwards in time, but it does look like it is. If the coordinates were first-known in the frame of refernce of the partcile, then the othere observer would not write the equations this way. The othre observer would choose the othr sign, so that the field point seems to be moving forward in time.

While it does not matter if a system is left handed or right handed, it is plausible that the transformation from one to the other entails more than an inversion of rhe sign of the velocity vector.

If one vector is rotating clockwise and the other is rotating counterclockwise, then they concide at one point. In this calculatin, the orgin of the time coordinate in the first frame of reference has been cntrived so that the point of coincidence is the same in both frames of reference.

The two solutions represnt the solutions for two different particles traveling in opposite directions, so they are very different, yet it should not matter which partcle the solution is for – if the equation is a true vector equation.

There is no requirement that the equaions be true vector equations in the first frame of reference. But if they are, then these solutions represent the solutions for two different particles traveling in opposite directions, and both solutions should be valid.

Thus, it does not matter which direction the particle is traveling. That is the way vector equations should work.

Thus, it could be that the light cone soluiton is for the wrong partticle when it at the point of closest approach, and we would not know it. We have access to a second opinion for the identity of the right particle.

The same equation can be interpreted in another way. If eq – represents a particle moving forward in time the eq – can mean that the particle is moving backwards in time. In the first frame of refernce, one observer cannot use the light cone equation tell the difference between the two solutions, yet they have different meanings.

The observer in the frame of reference of the particle can help in distinguishing between the two solutions.

It is not sufficent that retardaion equations work for partiles moving left to right. They must also work for partilels moving right to left.

For these particular equations, the difference between left and right and the difference between a partial moving forward or backwards in time is not always clear. While the difference between left and right does not matter, photons do not travel backwards in time.

The similarity to the doppler equation is apparent, but the two equaions should nevertheless not be directly compared, as the doppler equaion is missing the space derivatives. There is no possiblity that it could be used for navigational purposes without the space derivartives.

There is no point in transforming to the second frame of reference until the location of the particle is known in the first frame of reference. On the othere hand, unless our frame of reference is special or differnt, it should also be possilbe to work the problem in the second frame of reference.

Since the magnitude of R is at a quadradic minimum, the light cone equaion cannot precisely constrain the time of closest approach, although it will be a good approximation.

Consequently, it is possible for light cone solutions to be for the wrong particle if the time of closest approach is assumed to be already known. Furthermore, for any given uncertainty in the location of the particle at that point, the uncertainty in the time of closst approach becomes larger when the velocity of the particle becomes lower. No matter how low the velocity of the particle is, there is a systematic error in the time of the event.

The equations for computing the time and place of the closest approach are ill behaved. If the coordnates are first-known when the partcle is far away and the motion is neary radial, a minuscule error in computing the angle of the trajectory can result in a large error in the miss distance.

In this equation, a more general solution can be obtained by replacing y with $(y^2 + z^2)^{1/2}$, which can be arbiytrarily small, but not zero. Introducing a third space coordinte at the point of closest approach would introduce a surperfulous degree of freedom, allowing the constraining relationships to become sloppy and misleading.

These calculaions assume that the range of the fields is inifinte. We know that we cannot see to infinity, so there are additional refinements that are needed, but they can be deferred for a while. It is not necessarily true that the refinements will have no effect on local solutions, but it should be all right to neglect them in low order solutions.

However, if we knew how old the universe is, then we would know what time it is. Conversely, if we get the wrong answer for this calculaion, it might seem that we know more than we do.

In a universe where the range of the fields is infinite, the constant of integration for the doppler equation is

There should be another constant of integration in the equations if the first time derivative of the doppler equation is integrated. The dimensioned constants of physics are not computable. They have to be measured. Empirical pure number constants are not of proper form.

Remarkably, observers cannot tell which frame of refernce they are in. They all look the same.

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LVI. DISCUSSION

There is no gravitaional field inside a spherical mass shell, but the expansion factor in that region is not zero[8]. The gravitational redshift is a manifestation of the expansion factor. The expansion factor will influence the connection between the simultaneous and retarded points. But since the simultainous and retarded points are the same in static solutions, the expanson factor is not locally detectable in static solutions. It may nevertheless be measurabel in dynamic solutions, such as when a test particle is at rest and the mass shell is moving. When there are no acceleration terms, we have no way of knowing whether the shell or the particle is moving.

Similarly, the cosmologial expandion factor due to the average mass density of the cosmos will also influence the connection beteween the retarded and simulteneous points, although not in a way that is easily noticalbe, since we have never experienced what life in the laboratory would be like in an empty universe. All things measurable are relative, but sometimes we are not sure of what they are relative to. In any case, it is not us that they are relative to.

The LT effect does predict measurable relatioships when the shell is spinning[8]. There is no known measurabvle effect in the interior region for unaccelerated translational motion, perhaps because the equations are missing space-time cross terms that vanish in the infitesimal. Just as the symmetric cross terms in Eq– vanish quadradically, the anti-symmetric cross terms of dynamic symmetric solutoins will also vanish, but that does not necessarilly mean that the constant of integration is zero.

A back-reaction for an accelerated test mass that depends on the mass of the surrounding mass shell would be in accord with Mach's principle[12], although the equations for radiating particles and momentum conservation would be more concise. (The test mass could be jerked by a long string.)

With true vector equations, dR vanishes quadradially at the simultaneous point for transverse velocity. However, when the simultaneous point is not where it is thought to be, it is mathematically possible for $dR - R_0$ to be both anti-symmetric and of first order, although the offset would not be obervable in static solutions. The integral of space-time cross terms would be required for the terms to be observable.

It is possible for cross terms that vanish quadradically to also grow quadraciaclly when integrating in the outbound direction, requireing that the starting point of a defnite integral be displaced by a constant of integratin if the equations are to behave correctly at a great distances. But from the perspective of a distant observer, when integrating in the inbound directon, the constant of integration is zero, or nearly so, making it easier for the distant obeserver to evaluate the constant.

When the test mass is at rest and a surrounding mass shell is moving, the solution would represent a form of frame dragging. The role played by Mach's principle will be investigated in future papers, from the perspective of a distant and detached observer.

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The orientation of an observer is not an absolute. If the observer is in radial free fall, the observer's orientaion is not even relevant. It is the vector to what lies ahead that matters. Since the vector is unrelated to the observer's orientation, it should be rotationally invariant and single valued. The velocity of the observer is not an absolute either, so the calculation would be easier to perform if the observer is initially at rest and the particle is moving. Assuming that the location of the barycenter is known before obtaining a global solution is contrary to the relativity principle. The assumption attributes an absoluteness to either the observer or the particle that does not exist.

In a cosmologial context, it is not plausible that the constant of integrtion is computable – it would have to be measured. It is possible, even likely, that there are undiscovered geometrical relationships amongst the dimensioned physial constants, but in general their values are not computable. On the other hand, pure number empericl constants, such as the ratio of the electrostatic to gravitnal force for charged particle, are not of proper form.

This simplified integraon in space and time does not have the same meaning as moving a physical clock around a closed path. As the flying clock experimetns illustere[7], the loop integral of the clock rate in that case is not zero.

The dipole is one of the multipoles that can be lost if pesudo vectors are treated as true vectors. That is because there are no space-time cross terms in true vector equations. In being first order in space and first order in time, the cross terms vanish quadradically in a small region of space-time, but they cannot be dropped if the equation is to be integrated.

The magnetic field is $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, which looks like a pure space term, but, according to the Maxwell equations, $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{A})$ does not have a separately identifable physical meaning. In dynamic solutions, there are no pure space terms, and there are no pure time terms. There are always cross terms.

Refer to Ref. 11 for some useful background material for the Proca equations and the Thomas precession. The equations shown there need to be converted to SI units. As shown there, the Proca equations contain two static solutions. The eqations of this paper are only for the transformed scalar solution, and only for regions small in relation to the range of the fields.

It is likely that the range of the fields affects local solutions, but it should be all right to neglect those terms in low order solutions. But eventually, we should be able to infer the range of the fields from local laboratory measurements. We do not exist in isolation in an empty universe.

The decomposition products of the third rank contravariant tensor in electrical solutions are shown in Ref. 13. Those calculations need to be extended to the tensor of the fourth rank.

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