Some radiative solutions of the Proca equations

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Due to the constantly changing angular relataionships of light cone solutions, the trajectory of an unaccelerated particle is perceived as being curved. The particle appears to move along a straight line, but there is a virtual accelertion term parallel to the retarded velocity, which is a form of curvature. The propagation delay causes the perceived angular and translational velocites to become coupled. This relationship is included in the Liénard-Wiechert retardation equations, but there are relativistic corrections for it that are not included.

I. OVERVIEW

This paper is not finished. It will be refined and expanded in the coming months.

Most calculations are shown in more detail in the Supplemental Online Material (SOM). Due to the length of the expressions, some of the calculations are only available in the SOM.

This paper is archived at
`http://vixra.org/abs/1707.0344` and
`https://figshare.com/articles/An_approximate_non-quantum_calculation_of_the_Aharonov-Bohm_effect/5477056`. The preferred download site is `www.s-4.com/cone`. The version shown there may be more current, and the supplemental data is easier to access. There is no supplemental data at the vixra.org site.

The supplemental data at the figshare.com site is a single .zip file containing many HTML files. The file must be downloaded and decompressed for viewing. The top level file is index.html, and all files are accessible with links from the entry page. There are no subdirectories. Most browsers have the capability of navigating files on a hard disc in the same way that they are viewed on the internet. The files can be stored in any directory on any drive. The supplemental files at `www.s-4.com/cone` are easier to access if they are still available. They can be navigated like any other web page.

II. INTRODUCTION

If the protons and conduction electrons are retarded separately, as must be done, the Liénard–Wiechert[8] (LW) retardation solution for a rotating current loop is the same as for a stationary loop, even when the rim velocity is near c. It seems that there should be relativistic corrections for the magnetic field. The magnetic field is of order v^1 and relativistic corrections are of order $(v/c)^2$, so the missing terms should be of order v^3.

The solutions to the LW equations are always solutions to the Maxwell equations. Unless the Maxwell equations are more general than the LW equations, they have the same shortcoming.

In our frame of reference, similar equations apply to the constantly changing doppler shifted acoustic tone of a passing high speed vehicle. A changing pitch could be due to the transverse velocity of the vehicle, or it could mean that the vehicle is accelerating. It is not possible to quickly determine which case is which. The time and space coordinates of an event are difficult to separate with delayed observational data.

A closely related relationship is that the angular acoustic location of a low flying high speed aircraft lags noticably behind its visual location. It is not possilble to search ahead to find out where an electomagnetic signal came from, so the analogy is not complete, but it suggests that the retarded velocity and acceleration terms are not separable in the first frame of refernce.

Interplanetary spacecraft navigataon software incorporates corrections from the general theory of relativity. It has been demonstrated that the transverse doppler terms of those calculations do not need further consideration. The LW equations do not have transverse doppler terms.

III. THE LIENARD–WIECHERT EQUATIONS

In obtaining light cone solutions, there can be one observer and many particles, or one particle and many observers. When there is one particle and many observers, the time at the field point is not an independent variable. It is an independent variable in the LW equations. They are[8]

$$\begin{aligned} \boldsymbol{A} &= \frac{\mu_0 q}{4\pi R}\,\frac{\boldsymbol{v}}{1+\hat{\boldsymbol{R}}{\cdot}\boldsymbol{v}/c} \\ \psi &= \frac{q}{4\pi\epsilon_0 R}\,\frac{1}{1+\hat{\boldsymbol{R}}{\cdot}\boldsymbol{v}/c}. \end{aligned} \tag{1}$$

$\boldsymbol{A}$ is the vector potential and ψ is the scalar potential. $\boldsymbol{R}$ is the retarded location of the particle; $\boldsymbol{v}$ is the retarded velocity. $\boldsymbol{R}$ points from the field point to the source with this sign choice. The identities $\mu_0 = 1/(\epsilon_0 c^2)$ and $\epsilon_0 = 1/(\mu_0 c^2)$ can be applied to the solutions as appropriate. The potential equations do not contain accelertion terms, but accelerations terms do occur after the velocity terms

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are differentiated to obtain the fields. The LW equations are known to work well for accelerated particles.

The vector equation is equivalent to three copies of the scalar equation in three orthogonal directions, multiplied by the scalar velocity. That is the right connection between a scalar and a vector in 3+1 space. Propogation delay can be projected into 3+1 space, but it is not intrinsic to the space.

The zeroth order retardation equation for charge is the Coulomb solution. According to the LW equations, the magnetic field is a transformed E field, which is why the Coulomb solutions do not have a magnetic field.

The Lorentz transform in vector form is

$$\begin{aligned} \boldsymbol{r}' &= \gamma(\boldsymbol{r} - \boldsymbol{v}t) - (\gamma - 1)(\boldsymbol{r} - \boldsymbol{v}\boldsymbol{v}\cdot\boldsymbol{r}/v^2) \\ t' &= \gamma(t - \boldsymbol{v}\cdot\boldsymbol{r}/c^2), \end{aligned}$$

with $\gamma = (1 - v^2/c^2)^{-1/2}$. Except for a power of c, the 4-potential transforms in the same way as the coordinates.

The quantity $\boldsymbol{r}\cdot\boldsymbol{r} - c^2t^2$ is invariant with the Lorentz transform. The invariant quantiy is zero for light cone solutions. They are special cases of more general solutions.

Exact solutions for the light cone equation are usually but not always obtainable, because there is no equation for the fifth root of a polynomial. Series calculaions are sometimes more convenient. To order v^3, the Lorentz transform in series form is

$$\begin{aligned} \boldsymbol{r}' &= \boldsymbol{r} - \boldsymbol{v}t(1 + \frac{v^2}{2c^2}) + \frac{1}{2c^2}\boldsymbol{v}\boldsymbol{v}\cdot\boldsymbol{r} \\ t' &= t(1 + \frac{v^2}{2c^2}) - \frac{1}{c^2}\boldsymbol{v}\cdot\boldsymbol{r} - \frac{v^2}{2c^4}\boldsymbol{v}\cdot\boldsymbol{r} \end{aligned}$$

As illustrated by the calculaions of the Thomas precession[6], two consecutive Lorentz transforms are not representable with a single Lorentz transform. Two consectutive transforms are equivalent to a single transform followed by a space rotation. The Lorentz transform is therefore not the most general invariant coordinate transform. The equations of the Lorentz group[5,9] are still more general.

IV. AN ALTERNATIVE FORMULATION OF THE LW EQUATIONS

The light cone constraint is

$$t_s = t_f - \frac{1}{c}(\boldsymbol{R}\cdot\boldsymbol{R})^{1/2},$$

where t_s is the time at the source and t_f is the time at the field point. Differentiating with respect to t_f

$$dt_s/dt_f = 1 - \frac{1}{c}(\boldsymbol{R}\cdot\dot{\boldsymbol{R}})/(\boldsymbol{R}\cdot\boldsymbol{R})^{1/2}$$

It can be seen by inspection that

$$\hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}} = \dot{R} = \frac{\boldsymbol{R}}{R}\cdot\dot{\boldsymbol{R}} \tag{2}$$

Substituting $\boldsymbol{R}\cdot\boldsymbol{R} = R^2$ and applying the idendities in Eq 2

$$dt_s/dt_f = 1 - \hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}}/c = 1 - \dot{R}/c \tag{3}$$

and

$$dt_f/dt_s = 1/(1 - \hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}}/c) = 1/(1 - \dot{R}/c). \tag{4}$$

The retarded velocity is $d\boldsymbol{R}/dt_{\rm s} = d\boldsymbol{R}/dt_{\rm f}\ dt_{\rm f}/dt_{\rm s}$. Substituting for $dt_{\rm f}/dt_{\rm s}$ from Eq 4

$$\boldsymbol{v} = \dot{\boldsymbol{R}}/(1 - \hat{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}}/c) = \dot{\boldsymbol{R}}/(1 - \dot{R}/c). \tag{5}$$

After substituging for $\boldsymbol{v}$ from Eq (5) into Eqs. (1) and simplifying, the LW equations become

$$\begin{aligned} \boldsymbol{A} &= \frac{\mu_0 q}{4\pi}\frac{\dot{\boldsymbol{R}}}{R} \\ \psi &= \frac{q}{4\pi\epsilon_0}\frac{1 - \dot{R}/c}{R}. \end{aligned} \tag{6}$$

$\dot{R}/c$ is less than +1 for receding particles. It is unbounded for approaching particles. This asymmetry is the basis of the superluminal jets in some astrophysical objects.

The effort required to obtain solutions to practical problems with this parameterization is about the same as with the Newtonian parameterizaton. The solutions are exactly the same either way.

V. THE THOMAS PRECESSION

An equation can be reduced to a system of several first order equations[2], but not in one step. For example, two equations are required to integrate numerically around a circle. The sine and cosine functions can be extrapolated with the Taylor theorm, but it is not possible to extrapolate exactly to the $\pi/2$ point on a circle with Tayor theorem for one variable. Extrapolating with the Taylor theorm is equivalent to integrating. The calculations of the Thomas precession[6] attempt to integrate around a circle with one equation. That is not possible unless the equation contains π. ($e^{i\theta}$ terms implicitly contain π.)

Carrying more terms in the equation would help, but the error would nevertheless grow rapidly. Integrating numerically around a circle in Cartesian coordinates could be done, and it would be one way of computing π.

The pivot point for the Thomas rotation is at the origin of the coordinate system, which is not necessarily at the particle. For equations of physical significance, invariance is a necessary but not sufficient condition.

The tensor irreduciblity theorm[5] represents the differential angular relationships between the tips of two or more closely spaced vectors. Many of the theorms of Eucludian calculus are for one independent variable, in which case the chain rule for differentiaon may be needed first, especially for problems involving angular relationships.

VI. AN INTRODUCTION TO EUCLUDIAN GEOMETRY

The modern form of Eucludian geometry is Eucludian calculus. Euclid did not know about the chain rule for differentiaon.

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The vector from the field point to the particle rotates as the particle moves. During the full traverse of an unacclerted particle, the angle ranges from from $-\pi/2$ to $+\pi/2$. $\hat{\boldsymbol{R}}$ rotates though half of a circle. It depends on where the particle was, not where it is. The equation is Euclidian, but it is not a true vector equation.

This example is for one velocity, but in other problems the light cone solution for the sum of two velocity vectors is generally not in the same direction as the vector sum of the velocites. When a particle is accelerated, $\dot{\boldsymbol{v}}\ dt$ is a second velocity vector.

The contravariant tensors represent the differential angular relationships between the tips of two or more closely spaced vectors[2]. The tensor of each rank is irreducible[5], as manefested by the multipole order of the solutions increasing with the rank of the tensor. The highst order multipole of the tensor of the fourth rank is a hexadecapole (4 lobes). The decomposition products of the fourth rank tensor include six vectors. The tensor of the first rank is a vector, but tensors of any rank can be the basis of equatons that look like vector equations. As exemplified by the the Lorentz condition, $\boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, the equations include symmetric terms. The gradient of the Lorentz condition is another vector, and its divergence is one of the scalars represented by the tensor of the fourth rank. The highest order multipole of the LW equations is a dipole. (The sum of two dipole solutions is sometimes referred to as a quadrupole, but it is reducible to two dipoles.).

There are vectors and there are pseudo-vectors. They are not locally distinguishable, but their behavior with sign inversions is different.

The tensor irreducility theorm only applies to linear equations. Retardation equations are linear equations. The eqations of the general theory of relativey are not linear equations[7].

Retardaion equations represent the perspective of a distant observer. They are not usable in an accelerted frame of reference, except that the earth's gravity is weak enough that it can usually be neglected.

VII. THE LIGHT CONE EQUATION

The light cone equation, $\boldsymbol{R}\boldsymbol{\cdot}\boldsymbol{R} - c^2t^2 = 0$, can be solved by the method of successive approximation or by selecting one of the roots of a polynomial. Due to the presence of quadradic terms in the equation, the solution for the sum of two velocity vectors is generally not in the same direction as the vector sum of the velocities. $\dot{\boldsymbol{v}}\ dt$ is a second velocity, so the cross terms between a velocity vector and its derivatives are present in light cone solutions.

As shown in the SOM, two of the cross terms are

$$\dot{\boldsymbol{R}} = -\frac{1}{c}(\boldsymbol{v}\hat{\boldsymbol{R}}\boldsymbol{\cdot}\dot{\boldsymbol{v}} + 2\dot{\boldsymbol{v}}\hat{\boldsymbol{R}}\boldsymbol{\cdot}\boldsymbol{v})\ dt$$

There are other terms in the full solution that are not cross terms. Despite the nonlinearity of the light cone equation, there are no cross terms between the components of one velocity vector. A single velocity vector is rotationally invariant, allowing the light cone equation to behave like a vector equation in more ways than it is.

There would be more cross terms if the $\ddot{\boldsymbol{v}}$ terms were carried. Unlike the Newton equations, the solution depends on the angles between the derivatives. The contravarient tensors represent the differential angular relationsips between the tips of two or more closely spaced vectors[2,5]. The difference between two closely spaced vectors is itself a vector, but in general the two vectors are connected by a tensor of the second rank.

The decomposition products of the tensor of each rank include vectors[5]. The tensor of the first rank is a vector, but tensors of any rank can be the basis of equations that look like vector equations. As exemplified by the the Lorentz condition, $\boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, the equations include symmetric terms.

There are vectors and there are pseudo-vectors. They are not locally distinguishable, but their behavior with sign inversions is different.

VIII. SOME LIGHT CONE SOLUTIONS

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the solutions are not affected by translating the coordinates in either space or time.

In this solution the particle emits a signal at the time $r_0/c + t_f = 0$. The signal arrives at the field point at the time $t_f = 0$, with the total time being $+r_0/c$. In the meantime, the particle proceeds on to the simultaneous point, still on the light cone. It arrives at the simultaneous point at the same time the signal arrives at the field point. If it emits a second signal when at the simultaneous point it will be received at the field point at the time $+r_0/c$. The events are shown in Fig. 1 for the case where the motion is radial.

The slanted lines in the figure could represent periodic pulses transmitted by the source. The time interval is longer at the source than at the field point, so the pulses are compressed in time and doppler shifted to a higher frequency. The period of all the pulses is the same, so the received frequency is constant. The transfer function is linear in time.

2+1 space light cone solution when the transverse velocity is low. The field point is at the location $z = r_z$. The particle moves left to right along the y axis. $c = 1$. The top of each slanted line is the scalar distance from the field point to the particle when it emitted the signal.

The figure assumes that the particle is at the tip of the r_0 vector when $t_f = r_0/c$, but it is difficult to be sure

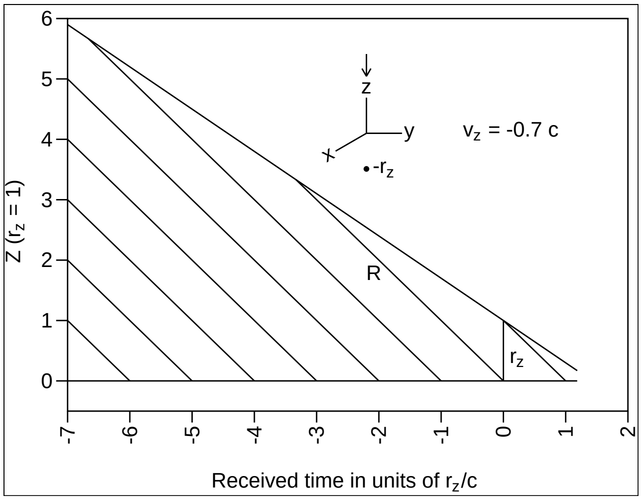

FIG. 1. 1+1 space light cone solution for retarded radial motion. The top line is the trajectory of a particle approaching the field point along the z axis. The slanted lines from the trajectory are light rays propagating downward to the field point. The field point is at the location $z = r_z$, with $r_x = r_y = 0$. $c = 1$. All calculations shown in the figures are based on an exact root of a polynomial. The series calculaions used elsewhere are not accurate enough.

of where the particle actually is then without a nearby assistant to report the time of the conincidence.

The LW solution is unaffected by translating the Eq – coordinates in time by the amount $-r_0/c$. The equations can also be parameterized by the value of the radius vector at the time $t_f = -r_0/c$, and it makes no difference in the final solution. It also does not matter whether the light cone equaion is solved in the first or second frame of reference. The LW equations are the same either way. There are endless ways of deriving the LW equations.

The coupling of angular and translational velocites is unique to the four dimensinal space. Doppler shifted acoustic signals are also in 4-space, so the light cone equation would work equally well for them, with c being the speed of sound rather than the speed of light. The coupling is a consequence of the propogation delay, which has no meaning in 3-space. Straight lines in 3-space do not have tick marks equally spaced along their length, so we cannot tell when they have been stretched. In 4-space, straight lines are curved when the stretch is not uniform along their length. Some of the stretch is due to the cosmological expansion factor, which exists within a mass shell. The expansion factor affects the time and space coordinates equally, so the locally measured speed of light is not affected].

The coupling of the angular and translational velocites can be neglected when the velocity is low. But when the relativistic corrections are already of order $(v/c)^2$ times the momentum of the particle, the coupling is in the same order as the corrections, in small systems.

The equations are parametersized by rv1 instead of rf0, but they look same at either time. They are form invariant for translations in time. Form invariance has about the same meaning as the requirement that the equations should not contain a coordinate dependency. Coordinate dependencies can exist in either space or time.

When there is more than one observer in a problem, each observer is not free to choose their own coordinate system. Both observers must use the same system. This is what the solution looks like when the coordinates are first-known at time $t_{off} = -r_0/(2c)$ by the other observer.

The retarded Newtonian acceleration is zero in the figure, but, for observations of short duation, the solution would look the same if the particle had a different retarded velocity and a non-zero retarded acceleration parallel to the retarded velocity. One observer cannot quickly tell the difference between Newtonian acceleration and angular velocity. Two observers would know the difference. One observer evaluating a system at two widely separated times counts as two observers.

In being of order v^3, the midpoint shift depends so strongly on velocity that the LW equations are usable even at moderately high velocities. The v^3 terms also occur with acoustic signals. The propogation delay causes the acoustic location of a low flying high speed aircraft to lag behind its visual location. The doppler shifted tone of a passing vehicle is constantly changing. At low velocities, the rate of change of the tone is at a maximum at the closest approach. As illustrated by the top panel of Fig.**??**, the v^3 terms cause the maximum to occur earlier in the trajectory when the velocity is high.

In the figure, Rvdot is shown aligned with the time at the source, but the effects of Rvdot are not perceived at the field point until one light time later. In Eq (), the solution for Rvdot is not delayed. The LW equations include only the radial aspect of the propagtion delay. The consequences of the propagation delay on the transverse component are not included.

An isolated and unaccelerated inertial particle will reach half way to the destination in half of the travel time. The LW equations assume that light cone solutions behave the same way. They usually do not. The LW equations are not for inertial particles.

This calculation was not performed the right way, so it is difficult to be sure of what it means. It does indicate that the vx2 terms of the scalar equation and the vx3 terms of the vector equation are not meaningful.

The basis of the inconsistency is that the LW equatons assume that an unaccelerated particle reaches half way to the destination in half of the travel time. Isolated and unacclerated inertial particles do behave that way. Light cone solutions usually do not.

The basis of the inconsistency is that the LW equations are not form invariant when there is a transverse velocity component.

The equations are still form invariant if the v^2 terms are dropped from the scalar equaion and the v^3 terms from the vector equaton. The vector equation is already of order v^1, so a $(v/c)^2$ correction to the LW equaions

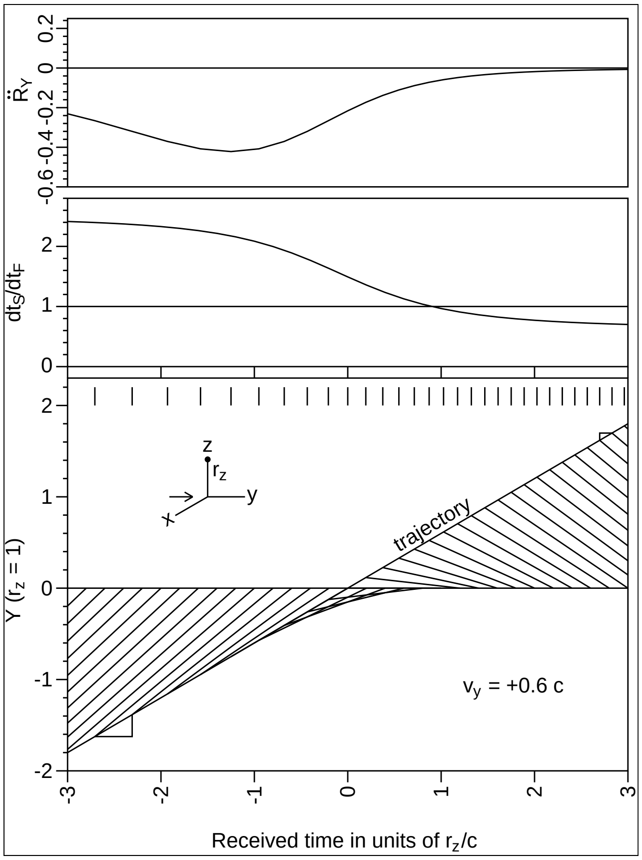

FIG. 2. One slice of the 2+1 space light cone geometry. The transmitter is moving left to right along the y axis. The receiver is at $x = y = 0, z = r_z$. The tick marks along the top of the bottom panel show when each pulse was transmitted. The pulses are not sent at regular intervals, but they are received at regular intervals. The dt_s/dt_t curve in the middle panel represents the doppler shift. The $\ddot{R}_y$ curve in the top panel is shown aligned with the time at the tip of $\boldsymbol{R}$. $\ddot{R}_x$ and $\ddot{R}_z$ are zero. $c = 1$.

is needed. That does not imply that there is anything wrong with the equations if they are applied within their range of validity. There is no possiblilty of detecting the shortcoming with currents in stationary wires.

The solution becomes form-invariant if the v^2 and v^3 terms are dropped. The velocity of conduction electrons in copper wire is so low that the LW equations are more than adequate in those applications. They are the valid second term of the retardation series. The static Coulomb solution is the first term of the series. Newtonian gravity is in the same order as the Coulomb solution. The Newton equations may seem to be more accurate, but that is mostly because the gravitational field is weak on the laboratory scale of things.

The time when $t_f = 0$ can be chosen arbitrarily. Once the time is selected, the equations cannot be readjusted as needed if there is, or could be, another observer in the problem. There could be another observer at the particle, and it could be us.

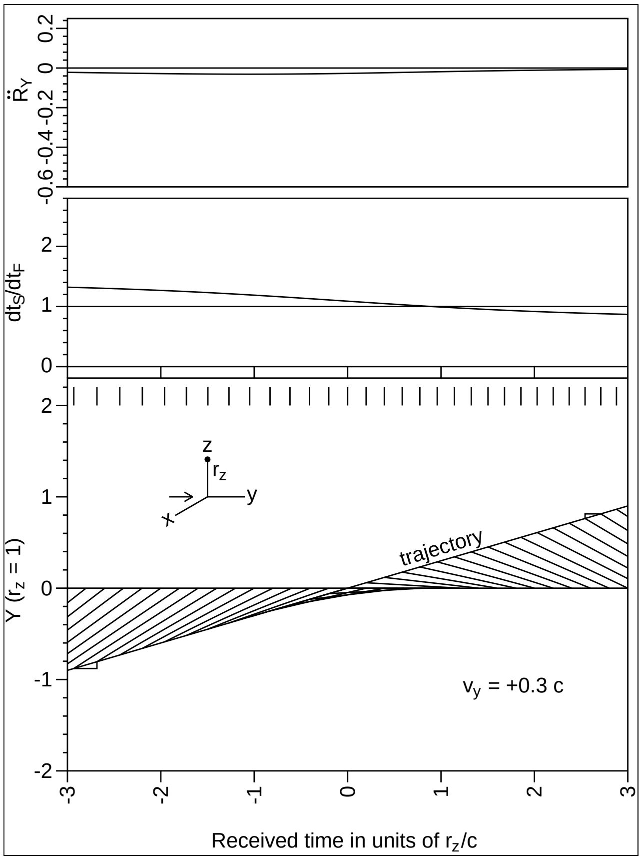

FIG. 3. The scales in this figure are the same as in Fig. 2, but the velocity is only half. The perceived acceleration depends so strongly on the velocity of the particle that it can usually be neglected.

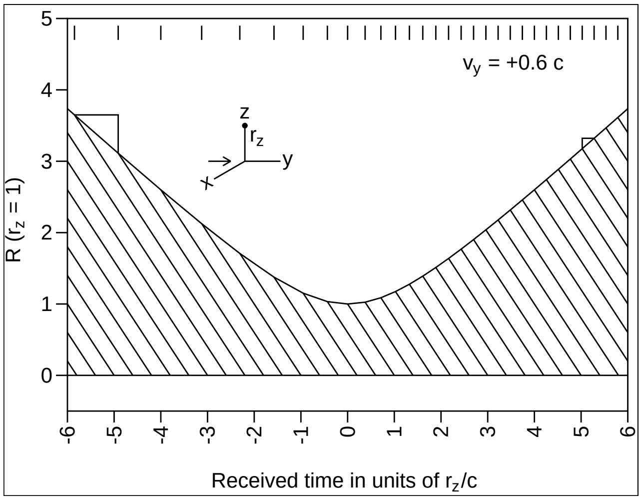

FIG. 4. This sketch shows the same data as in Fig. 2, except that the magnitude of ΔR is shown instead of the y component of the position vector.

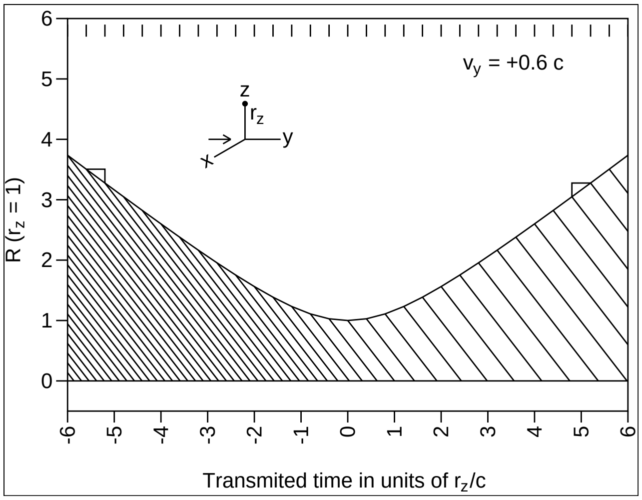

FIG. 5. This sketch shows the same data as in Fig. 2, except that the light cone equation was solved for t_s as the independent variable. It represents the solution from the perspective of the other observer. The pulses are transmitted at a uniform rate, but they are doppler shifted to a higher frequency when the source is approaching the field point.

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The basis of the inconsistency is that the midpoint of a time interval at the particle does not map into the midpoint of the corresponding time intervatl at the field point when there is a transverse velocity component. The transfer function is not linear. The duration of the two intervals is also different. The frequency of a doppler shifted frequency is constantly changing when the motion is not radial. That makes it difficult to determine what the number displayed by the other clock was, and it is more difficult when the midpoint of one time intrval does not map into the midpoint of the other.

Since an unaccelerated clock does run at a constant rate, the time shown by the other clock should be obtainable in prolonged observations by requiring that its computed rate be constant after correcting for the propation delay.

The vector r0 in Fig 1 is easily identifiable in a global solution, but it has to be possible to apply retardaion equations without knowing when the time tf=0 will occur.

The perceived velocity is greater when the particle is nearby. The perceived angular and translational velocites are coupled, making it difficult to separate them unless the retarded velocity is already known, which is usually not the case.

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The basis of this relationship is the delay from the particle to the field point. A change in the velocity of the particle is not perceived at the field point until one light time later. In the meantime, cycles of radiation are piling up along the propogation path, and the cycles do not lie along a straight line when there is a transverse velocity component. Simplistic doppler equations are not valid when the angular relationships are constantly changing. The angular relationships causes a transverse velocity term to look like a Newtonian acceleration term that is parallel to the retarded velocity. Thus there are two different effects that determine the frequency of a doppler shifted signal, and they are difficult to separate. In being of order $(v/c)^3$, the angular terms are small enough that they can usually be neglected.

The LW equations are for the time tf=0. Other choices would be equally valid, but translating the coordinates in time has no effect on the solution, so there is no point in choosing a different value.

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In being the integral of the fields, potential solutions are arbitrary to within a constant of integration. It is the nature of retardaion equations that the constant of integration depends on where the particle is, relative to the observer. Unlike simpler equations, the derivatives of a function will not be consistent with the function if the constant is not correctly determined. Other equations also have this characterisc if they represent a double integral and the first constant of integrataion is included in the second integration.

Only the derivatives of the retarded potentials are measurable. Local physical arguments should not be applied to potential equations until they are suitably differentiated, as they do not represent locally measurable relationships.

In being representabla as a 4-vector, the 4-potential transforms in the same way as the coordintes, implying that the coordinates can be viewed as being a potential representaion. The coordinates in 3+1 space have an absoluteness that leads to contradictions if there is more than one observer in the problem. One observer evaluating a system at two widely separated times counts as two observers.

IX. THE LORENTZ TRANSFORM

The curvature term vanishes when t_f is zero. The LW retardaion equations are for the time $t_f = 0$.

In 3+1 space, we are always free to choose $t_f = 0$. Other choices would be equally valid, but choosing other values does not accomplish anything, because curvature is not representable in 3+1 space.

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In 3+1 space, the derivatives of $\boldsymbol{R}$ would be exact in all orders, because the four coordinates are mutually orthogonal. It has been established that the four dimensional space cannot be reduced to first order so easily[7]. It is doubtful that it can even be fully reduced to first order.

A more accurate solution can be obtained by representing the trajectory by two or more straight line segments. Two segments are sufficint if the v^4 terms are not carried. Subdividing the time interval at the field point does not accomplish anything. It is necessary to take the interval at the particle as being the independent variable. It is necessary to integrate in the frame of refernce of the particle before differentaing in the frame of reference of the field point, except that a full integral is not required. When working in series form, only a few straight-line segments are needed.

An observer at the field point would not know what to do with this solution, because time does not progress at a uniform rate.

In 3+1 space, this derivative would be exact. Indeed, in 3+1 space, the derivatives in all orders would be exact, because the four coordintes are mutually orthogoanl.

If the light cone equation is solved for one specific instant, the derivatives in 3+1 space are uniquely defined in all orders, because the four coordinates are mutually orthogonal. The four dimensional space is not so easily reduced to first order[7]. The derivatives in 3+1 space are degenerate.

Being degenerate does not have the same meaning as being wrong. The Coulomb solution is degenerate, but it is not wrong.

An observer watching a moving particle does not glimpse it at one instant. It is continuously visible. It is permanently on the light cone.

The light cone equation is $(\Delta\boldsymbol{R})\boldsymbol{\cdot}(\Delta\boldsymbol{R}) - c^2(\Delta t)^2 = 0$. The equation can be solved by selecting one of the roots of a polynomial. One of the roots is for a signal propagting from the particle to the field point. The other is for a signal propagaing from the field point to the particle. Depending on the application, both roots are meaningful. There are more than two roots if the $\boldsymbol{a}$ and $\dot{\boldsymbol{a}}$ terms are carried. The method of successive approximation is usually better when working in series form. The constant velocity solution is obtained in the SOM.

One of the terms of the velcity of the retarded intersection is

The velocty of the retarded intersection is not the same as the retarded velocity. The retarded velocity is

$$\frac{\boldsymbol{R}(dt_f) - \boldsymbol{R}(0)}{t_s(dt_f) - t_s(0)} = \frac{\partial \boldsymbol{R}}{\partial t_f}\frac{\partial t_f}{\partial t_s} = \boldsymbol{v}_0 \tag{7}$$

The retarded velocity cannot be measured without having an assistant near the source. The assistant could take various forms, but in most cases the retarded Newtonian velocity is not measurable from afar. It is usually only the percived velocity that is measurable, such as with a doppler shifted monochromatic signal. The perceived velociy is delayed by the propagation time from the source.

The retarded velocity is obtained by differentiting with respect to the retarded time at the source. The percevied velocity is obtaind by differentianting with respct to the time at the field point.

Assuming that the coordinates at the simutaneous point are already known, which is usually the case when working the retardation problem, the trajectory parametrized by R and its derivative can be expanded as the Taylor series

Since the location in this solutin is not where the particle is known to be, a correction is required to account for the propogation delay.

Both soluions are correct, depending on how the measurements are made. However, if the particle is known to be on an inertial trajectory, and independent measuremnts provided by a distant observer are not availale, a correction must be applied to the measurements of Rv and its derivatives to account for the propagaion delay. Since the trajectory is perceived as being curved, a more accurate soluton can be obtained by approximeating the trajectory with two straight line segments rather than just one. That has the effect of refining a constant of integration.

Eq – is a solution for a particle on the light cone, but it is not the solution for an inertial particle. An isolated and unaccelerated particle would be half way to the destination in half of the travel time. The particle is perceived as having an accelertion component in the same direction as the retarded velocity. The perception is real, but a relativistic correction is required if the objective is to reconstruct the trajectory of the particle.

We are not free to choose a coordinate system that follows us as we move about in space and time if there is, or could be, another observer in the problem. such solutions are coordinte-dependent. Our frame of reference is nevertheless the only one we can ever measure anything in, and it is possible to obtain equations that work in our frame of reference. However, they require transformation for any other frame of frame of refence. They have no absolute significance.

a correction to Rvdot measuremts at the midpoint of the trajectory is required. A similar correction is required when tf=0.

In increasing linearly with time, the $\dot{\boldsymbol{R}}$ term looks like an acceleration term, but it is present for unaccelerated particles. It vanishs when $\boldsymbol{v}$ and $\boldsymbol{R}$ are parallel or antiparallel. When $\boldsymbol{v}$ and $\boldsymbol{R}$ are perpendicular, it has the form of the angular velocity of the particle, multiplied by a $(v/c)^2$ relativistic correction. For unaccelerated particles, there is no curvature without the relativistic correction.

Due to the constantly changing propogation delay from a moving clock to the observer, the trajectory of an unaccelerated clock is perceivied as being curved when there is a transverse velocity component. The relativistic correction to the angular velocity represents the curvature. For unaccelerted particles, there is no curvature without the relativistic correction.

The curvature is more than just a visual effect. It will influence the interactions of the particle with other particles.

The particle velocity cannot exceed c, but angular velocity is unbounded. It becomes progressively more difficult to obtain accurate solutions as the size of the system becomes smaller.

The curvature term vanishes when t_f is zero. The LW equations are for the time $t_f = 0$. Once the term is lost, differntiating the potential solutions cannot reinstate it.

X. COORDINATE DEPENDENCY

The solution is for a particle on the light cone, but it is not the solution for a particle on an inertial trajectory. An unaccelerated inertial particle should be half way to the destination in half of the travel time.

For each observer, the solution is for a particle on the light cone, but it is not the solution for a particle on an inertial trajectory. The solution is observer-dependent. At time -r0/(2 c), no two observers can agree on where the particle is.

The solution is for a particle that is permanently on the light cone, but it is not for the right particle.

The solution is coordinate-dependent. Equations of physical significance have to work in the same way at different places and at different times. If C1 is zero, then after the data measurments are processed, no two observers could agree on what the trajectory of the particle was.

XI. INERTIAL PARTICLES

The solution in Eq – is for a particle on the light cone, but it is not the right particle unless it has an independent means of propulsion.

In the calculation of this section, the particle is on the light cone for an instant at the retarded intersection. It emits a signal at that point that propogates at c to the field point. In the meantime, the particle proceeds on to the simultaneous point. The particle arrives at the simultaneouls point at the same time that the signal arrives at the field point.

An observer can never know what is happening at the simultaneous point while it is happening, which poses a problem in establishing a reference point. The equations are slippery. There are no absolute reference points that are accessible for us.

An isolated inertial particle should move with a constant velocity unless it has an independent means of propulsion.

For an isolated and unaccelerted inertial particle, both halves of the journey should take the same time. The trajectory seems to be curved, but that is because the solution is coordintate-dependent. Coordinate-dependent solutions are not of physical significance, for we can never know were we are in space and time.

XII. THE SECOND INFINITESIMAL STEP

Finite difference equations are widely used in digital engineering applications, where they are used to approximate analog transfer functions. The second and third differnces are closely related to the second and third derivatives.

The extrapolation formulas for the n th differnce are elementry. They are derived at .. The extrapolation formula is simply $\delta_k(t_f)n^k$. k is 2 for the second difference, 3 for the third difference, and n is the number of straight-line segments used to approximate the trajectory. This extrapolation formula cannot be used unless all of the $\delta_{k+1}(t_f)$ differences are zero. When they are zero, integration degenerates to multiplication for the sum of the $\delta_k(t_f)$ terms. In effect, it is necessary to integrate the light cone equation before differntiating it, except that a full integral is not required. Only are few straight-line segments are needed to approximate the trajectory.

Subdividing the trajectory into more segments makes no diffrence at all in the solution, showing that the solution exists in the first order of the infinitesimal. However, the third difference would be required if the vx4 terms are carried. The extapolation formula for the third differnce is $\delta_3(t_f)n^3$.

The two distances should be the same for an unaccelerated inertial particle.

For quantities that are of order v^1, such as the magnetic field and the momentum of a particle, the refined v^3 terms represent a $(v/c)^2$ relativistic correction.

The trajectory would have to be divided into three segments if the v^4 terms were carried, but it will be necessary to venture into the four dimensional space one step at a time.

Eq – is a solution for a particle on the light cone, but it is not the solution for an inertial particle. The solution depends on where the observer is. The coordinate system for one observer does not work for a different observer.

It contains a coordinate dependency. No two observers can agree on what the trajectory is if each observer is free to choose a coordinate system that is centered upon themselves.

In reconstructing a Newtonian trajectory for an isolated and unaccelerated source from light cone events, such as a doppler shifted monochromatic signal, the quantity of interest is not where the source was when it emitted the signal. With the Newton equations, it is where it will be when the signal is received at the field point that matters. The objective is to accurately reconstruct the trajectory in 3+1 space, despite the propagation delay from the source to the field point. The calculation requires a relativistic correction.

The solution in Eq – is for a partticle that is on the light cone, but it is not the right partitcle. The computed trajectory is not that of an inertial particle. The reconstructed trajectory depends on where the observer is. No two observers can agree on what the trajectory was. Solutions that follow us as we move about, such as Eq –, contain a coordinte dependency. Equations of physical significance must not depend on the choice for a coordinate system, for we can never know were we are in space and time. We are not free to choose a coordinate system that follows us as we move about if there is, or could be, another obserer in the problem.

The invariance of the speed of light is a necessary but not sufficient condition.

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The soluton is not right, as the retarded Newtonian velocity of the particle was already known. The problem is to measure the retarded velocity correctly despite the propagation delay from the particle. *If* the location of the particle is assumed to be already known when the signal reaches the field point, which is usually the case when working the retardaion problem, then its location at the retardation intersection is computalbe.

The observer at the field point can not know what is happening at the simultaneous point while it is happening, but it is necessary to make an assumption to obtain a solution. The assumption does leave a loophole.

Equations that look like they are exact are convenient, however there would be other terms in this soluton if the vx4 terms were carried from the beginning.

The vx4 terms would have to be carried from the beginning if they are needed, which would require four nearby light cone events.

The compactness of equations that look like they are exact is convenient, but this solution is an approximation. The vx4 terms would have to be carried from the beginning if they are needed, which would require four nearby light cone events.

These relationships look like vector equations, and they are, but the underlyig relationships are actually tensor equations. As shown in the SOM, the same solution is obtainable more elegantly with tensor equatios, which is not to say that the calcuations in the SOM are elegant. The essential relationship is that the gradient of a vector is a tensor of the second rank.

We still do not know where the particle was when it emitted the signals. In computing the first derivtives of the retarded potentails, which are the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields, it is not necessarily true that we have to know precisely where it was. The constraints would be stronger if the first derivatives of the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are needed.

Four light cone events would be required to compute the differential position relationships for three points. We are always one step behind, but that is all right if the vx4 terms are not carried.

This solution is only accurate to order $(v/c)^3$. Three nearby points on the trajectory would be required if the v^4 terms are to be carried.

XIII. THE FIRST INFINITESIMAL STEP

A more complete solution could be obtained by carrying the v^4 terms from the beginning.

The light cone equation represents an undifferentiated vector function. The gradient of a vector is a tensor of the second rank. The tensor irreducibity theorm [] is in 3-space, but a space rotation in 3-space is still a space rotation in 4-space. The rotations of the Lorentz group [,] represent the angular relationships in a more general way.

The Lorentz transform takes out the velocity terms. It is the acceleration terms that represent the first derivative. We have to carry extra terms in the equations to represent the first derivative in our frame of reference.

XIV. FORM INVARIANCE

Retardaion equations have to work in the same way when applied in different places and at different times.

This solution looks like the LW solution in series form, but it is not, because the location of the particle is different at time dt.

dt has not dropped out, showing that the LW equations do not look like the same equations at time $t+t_f+dt$. The first derivative at a later time is one of the contributors to the second derivative.

Form invariance has the same meaning as the requirement that equations of physical significance should not contain a coordinate dependency. The invariance of the speed of light invariant is a necessary but not sufficient condition. For this reason, we are not free to choose a coordinate system that remains centered upon ourselves. It is necessary to consider how an observer in the other frame of reference would perceive us. We can never be sure that we are not the other observer.

Coordiante systems are not portable. They do not follow us as we move about in space and time.

XV. SOME PREIMINARIES

The second differences are all the same, so integration degenerates to multiplicaion.

The tx2 terms drop out if the vx3 terms are not carried, then the solution is the same as the LW solution. The vx4 terms do not drop out of the second difference, but the do drop out of the third difference. The extrapolation coefficnt for the third differnce is nx3.

The Lorentz transform takes out the velocity terms. The particle is initally at rest in the the seconed frame of reference, where the acceleration terms represent the first derivative. The tensor of the second rank represent the first derivatives[5]. We need additinal terms to represent the first derivative in our frame of reference.

A retardation solution has no special significance for other observers until it is transformed. For the same reason, a solution in our frame of reference does not apply to a sensor that is moving in our frame of reference until it is transformed. The equation for the transverse force on a charged particle, $\boldsymbol{F} = q\boldsymbol{v}\times\boldsymbol{B}$, is an example of a transformed solution.

There are no velocity terms in the Maxwell equations. They apply in a frame of reference where the velocity is zero, in which case we are in the second frame of reference without knowing it. How would we know?

There would be terms quadradic in t_f if the v^4 terms were carried, so they have to be set aside for future study. The first order of the infinitesimal is free of error, but that does not mean that any equation can be reduced to first order in one step.

Comparing the value of computed contants to laboratory measurements provides a way to test our understanding.

The particle is not where it would be in 3+1 space, but it is still on the light cone. The r dot r in eq – is the same as in Eq ().

r dor r is the same as in eq –, so the particle is still on the light cone.

The $\ddot{\boldsymbol{R}}$ contribution to the location of particle could be obtained with a double integral, but the Taylor theorm provides a more convienent way of integrating all of the derivatvies in this solution. There are derivatives in all orders, depending on how many powers of velocity are carried.

of integratin can be determined with a definite integral from infinity. Most potental equations can be viewed as representing the perspective of an observer at infinity, however that is a perspective that we can never know.

The particle is at the center of a sphere of radious r_0 at the time $t_s = 0$. A signal emitted by that particle at that instant reaches all points on the surface of the sphere at the same time, even if the particle is accelerated. For $k = 1$, the time at the the field point is $r0/c$ and a light cone solution is obtained without calculation. When k is 0 the time at the field point is 0 and the paricle is at the locatin Rv. The time at the source is then -—Rv + rv—/c.

The acceleration and the velocty of the particle both contribute to the perceived curvature of the trajecory, so they are not separable in observations of short duration, but they are separable in prolonged observations.

The accelereation of a particle does not affect its location at the time dt, so it is not possilbe to tell from this solution which terms are accelertion terms. The Newtonain velocity and acceleration of the particle are nevertheless separable in prolonged observations.

The number displayed by an acceleromter attached to a particle can be read by observers in any frame of reference. In the frame of refence of an accelerated particle, the partcile acquires the velociy $\boldsymbol{a}\ dt$ at time dt. There are no relativistic corrections of order v^1, so the number displayed by the accelerometer is an invariant quantity. There are relativistic corrections at time $dt_1 + dt_2$, so the Newton equations no longer apply, even in the frame of reference of the particle.

It would of course be possible to represnt the tranectory with more than two stright-line segments, in which case the avdot terms would have to be carried. In light cone probglems in the first frame of reference, and always in the second frame of reference, the ax2 terms are in the same order as the avdot terms. The avdot terms integrate to acceleration terms in the time dt, so not much should be expected of solutoins that do not include the avedot terms.

Even though the Newtonian accelertaion and velocity terms are both curvature terms, they can be separated in prolonged observations, and there is sometimes a need to separate them.

It would of course be possible to divide the path into three or more segments, and in more general solutios the acceleration terms should be carried even with only two segments.

No matter how small the interval is, the midpoint of the line element is not at the middle, and the error is of the first order. The behavior is unphysical. If a long thin wire with slowly varying properties is stretched, the elongation will not be uniformly distributed along the wire, but it will be uniform in any aritrarilly short section.

The behavior is unacceptable for physical rather than mathematical reasons. The clock in the frame of reference of a distant and detached observer runs at a constant rate.

There are two ways of viewing this problem. In our frame of reference, there is only one way of viewing it, because we have to use a clock that is at rest in our frame of reference as a time standard. We cannot use a clock in a differnt frame of reference as a standard. The distinction between the two perspectives is not important for the first infitesimal step, but it does become imporetant is subseqent steps. More generally, only first order equations can be reduced to first order in one step.

If the signal arives at the time dtf then it must have been transmitted approximatly at the time . It become necessary to start over with the better estimate.

This calculaion illustrates that only first ordee equations can be reduced to first order in one step, even though the first order of the infitesimal is free of error. This principle is not unique to the four dimensional space [].

The equaion is similar to the equation for the Thomas precession [], which occurs in the other frame of refernce. In this case the othere frame of refeence is our frame of referene. We can never know which frame of reference we are in. A distant and detached obserer would not perceive any accelertion, so a second transform is required to take it out. A third transform would be required to take out the avdot terms if the vx4 terms were carried.

.. The vector has the same magnitude as it would have in 3+1 space, but it is rotated. The rotations of the Lorentz group[,] represent the 4-space angular relationships in a more general manner.

The calculations have not used the chain rule for differentiation, but is should be possilbe to obtain the solution in a more concise form with it.

The time inteval indicated by dt is correct, but the elapsed time shown by the other terms depends on when the observation is performed. The behavior is unacceptble in the first frame of refernce, and it is difficult to obtain a solution that an observer in the second frame of reference would not object to. tf0 is a free parameter. A common meeting ground is obtinable by choosing it so that the time in the first frame of refence is 0

These relationships could be expressed more elegantly with the chain rule for differentiaron, but other methods are sometimes easier to visualize. The chain rule is required whenever the variables are not independent. The time and space coordintes are not independent variables in light cone solutions, or in more general solutions in the second frame of reference. Light cone solutions are special cases of more general solutions.

The chain rule for differentiation is required when the variables are not independent, and the retarded potentials exist only for the purpose of being differentiated. The terms of a Taylor expansion are mutually orthogonal, but the terms of the Newton series cease to be orgthogal in light cone solutions, or in more general solutions in the second frame of reference. Light cone solutions are special cases of more general solutions.

For light cone solutins only, the retarded postion vector can be expanded in a Taylor series.

This series expansion does not represent the trajectory of an inertial particle, nor does the Newton series. The terms of a Taylor series are mutually orthogoanal. The chain rule for differentiaon is not needed when the terms are orthogoanal, but it is necessary to insure that the terms in the equations actually are orthogonal.

The Newtonian acceleration and velocity terms are not independent variables in light cone solutions, making it difficult to separate them, as both terms cause the perceived trajectory of a particle to be curved. The terms are nevertheless separable.

In being quadradic in time, the curvature is in the same order as acceleration, so the solution for the perceived curvature would not be complete without the acceleration terms. But acceleration is obtained by differentiating velocity. The acceleraton soluton is not obtainable without knowing the velocity solution first. (It would be possible to integrate acceleration to obtain the velocity, but those equations would be of a different form.)

The particle is still on the light cone and the potentials propogate at c, but they were not emitted by the particle when the LW equations assume that they were.

.. is where the observer thinks the particle is. .. is where it acutally is. A field of observers would know where the particle actually is. How could one observer know? Actually, there is a way for one obsever to know. That is to observer the partcile for a long time and then integrate the trajectory. The trajectory cannot be integrated unless the derivatives of the function are computabhle from it.

?? The light cone equation is traditionally solved in our frame of reference, but it can also be solved in the frame of reference of the particle. It is possible that the way the other observer perceives us is not the same as the way we perceive ourselves.

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The solution will be shown in a more compact form in a later version of this paper.

The constant of integrataion was not determined in this calculaion, so the solution is not necessarily unique.

The thing that matters is not the constant of integration itself. The thing that matters is that the function and its derivatves are not consistent. Discovering a function that diffrentates correctly is an indirect way of refining the constant of integation.

Fortuously, dt has dropped out of the solution. Since the solution does not depend on the value of dt, it remains valid when dt is zero.

The solution will be shown in a more compact form in the next version of this paper.

In representing the integral of the fields, potential solutions are arbitrary to within a constant of integration. Retardaion equations express a realatinship between one observer and one particle. The constant of integration has no special significance for other particles or other observers. The thing that matters is that the function and its derivatives should be self-consistent. Discovering a function that differentates correctly is one way of refining the constant of integration. There is probably more than one method of discovery.

With continued use and familiarity, the potentials can seem to become more real than the fields, which is fine if it assists the comprehension of the equations, but the model can develop fractures if carried too far. Worse than that, it can cause an explorer to become lost in space and time, seeking a solution that does not exist.

It is sometimes necessary to know if an accelerometer attached to the particle would register an acceleration, and the Newonian parametrization is needed for that purpose. The accelerometr dial can be read from afar, so it is the same for observers in all frames of refrence.

The acceleration terms also vary as t^2, so they are in the same order as the angular velocity. In periodic solutions, it would not be consistent to obtain the soluton for the angular velocity without including the acceleation terms. Non-periodic solutions are included in the more general solution as special cases.

The retarded accelertion may or may not be zero in solutions containnin d2Adt2 terms. Angular velocity and acceleration are difficult to separate. The need to separate them is sometimes artificial, but it can be done.

Two consecutive infinitesimal reparamterizations would be required in order to carry the $\dot{\boldsymbol{a}}$ terms. In the second frame of refernce, the a^2 terms are in the same order as the $\dot{\boldsymbol{a}}$ terms. The a^2 terms will become important in intense field solutions.

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The Lorentz transform is for one point. It is not known to represent a smooth and continuous differentiable function without additonal processing.

When the interval is small, the first difference behaves in the same way as the first derivative. The second and third differences are closely related to the second and third derivatives, but they are different.

Discrete difference equations are widely used in digital engineering applications, but they are not covered in calculus textbooks. Some basic characteriscs are shown at ...

In being a velocity transform, the second difference is appropriate for computing acceleration with the Lorentz transform. The first derivative of velocity would be satisfactory if the Lorentz transform was an accelertion transform, but it is not. The solution can no doubt be obtained with the second derivative, and it should be done, but the second difference will be used here.

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The curvature in the infinitesimal is more than just a visual effect. It will influence the interactions of the particle with other particles.

That does not imply that curvature can be reduced to first order in two steps. A parabola can be in closer contact to a circle than a straight line tangent to it, but a cubic equation is even better.

The angular velociy of a particle depends on where the observer is. Its value does not have a meaning for other observers unless it is transformed. The particle velocity cannot exceed c, but its angular velocity is unbounded. Obtaining accurate soltions becomes progressively more difficult as the size of the system becomes smaller.

When drv is arbitrarily small, the acceleration of each particle passing through the line element vainshes in the infitesimal. The velocity of all the particles in the infitesimal line element can be assumed to be the same, so the solution of the n particles within the line eleamnt at any given instant is simply n times the solution for one particle.

However, the length of the line element, and therefore the amount of charge within it, depends on its angular velccity and the acceleration of the particles. The time dt at the field point maps into a Δts interval in the current loop, and there are $(\delta t_s)^2$ terms in the equations, even when the particle is not accelerated. An accelerated particle acquies the velocity $\boldsymbol{a}\delta ts/2$ at the midpoint of the line element, then the velociy change integrates to a position change during the second half of the traverse.

As shown in the SOM, the location of the particle at time dtf1+dtf2 also depends on the avdot and avddot terms.

This solution is only valid when the line element and the observer are in the same frame of reference. The potentials have to be transformed in other cases. The 4-potential transforms in the same way as the coordinates.

This model exists in 3+1 space, and it contains a loophole. A moving clock does not run at the same rate as an at-rest clock. The time shown by a clock, as read from afar, depends on its history.

These relationships exist in the first frame of reference. It should not matter what frame of reference the problem is worked in, but transforming incomplete solutions to the second frame of reference would not be helpful.

The calculaions of the Thomas precession [] assume that the trajectory is Newtonian. The tranformed Newton equations are the Newton equations in disguise.

The particle is not where it was thought to be. But where is it actually? The angular velocity of a particle depends on where the observer is, so there is no general answer. It is the transform that matters.

The tensor of the zeroth rank is a scalar The zeroth order retardation equation for charge is the Coulomb solution. It is often possible to obtain useful solutions with the lowest order potential equations, but they contain an error of the first order. That is because the magnetic field is of order v^1. The error resulting from the neglect of the magnetic field becomes arbitrarily small as the transverse velocity becomes arbitrarilly small, but it does not vanish quadradically, meaning that it does not vanish at all. The theorems of Euculidian calculus apply, but the chain rule for differentiation is required when the variables are not independent. No matter how low the velocity of the particle is, the zeroth order potential equations contain an error of the first order.

As shown in the SOM, the $\dot{\boldsymbol{a}}$ terms affect the location of the particle at time dt_1+dt_2. One of the velocity terms at time $dt1$ is $d\boldsymbol{r} = -3\dot{\boldsymbol{a}}r_0v_0^2/c^3\ dt_1$, but that does not imply that curvature can be reduced to first order in two steps.

appears in the solution. There are no conflicts between these solutons and the theorms of Euculidian calculus, as the chain rule for differentian is required when the variables are not independent. It it is capable of conferting first derivatiess into derivatives of any order. The terms of the Newton series not not idependet variables in light cone solutions, where the time and space coordintes are strongly coupled.

The following calculations assume that the observer and the current loop are in the same frame of reference. The calculaions would necessarily be more elaborate in other cases.

When $d\boldsymbol{r}$ is arbitrarily small the particle accelereation vanishes in the limit. The velocity of the particles in that region can be assumed to be constant throughout the interval, then the LW equaions can be applied to the amount of charge that exists in that infinitesimal line element.

The space and time coordinates are strongly coupled in light cone solutions. One of the consequences of the coupling is that acceleration and velocity not independent variables in global solutions. The chain rule for differentiaon is required when the variables are not independent, and the retarded potentials exist only for the purpose of being differentiated. The chain rule is not used in the following calculatons, but that is only because it is not needed if the terms are orthogonalized first.

In the first frame of reference, we are one step behind – sometimes two steps.

The behavior of the first derivative and the first difference are the same when the interval is small. The second difference is closely related to the second derivative, but it is not the same. In not being an accelertion transform, velocity has to be reduced to second differences when applying the Lorentz transform. The second differences sometimes work for the Newton equaions, but not always.

Eq – shows where the first observere thinks the particle is. Eq – showss wher the other observer thinks it is. So which solution is the right one? From the perspective of an observer in a third frame of reference, neither of them.

It is the transform that matters.

The t in these equations is actually delta t, with the actual time being t+delta t. In the interest of breity delta t is abbrevviated to t, however the solutions are only valid for short time intevals.

Using more than two steps has no effect at all on the soltion, showing the the locatin of the particle at time tx2 can be reduced to first order in two steps. The same is true of the Newton equaions, but the equations are not the same.

The chain rule for differentiaion is capable of converting first derivaries into derivtives higer order. In that case, the theorms of Eucludian calculus do not necessaryly apply until *after* the chain rule has been used.

Translating the coordinate sysem in space by the amount rv would not affect the final solution. The coordinates could also be translated in time. We can never know where we are in space and time, so coordinate-dependent solutions are not of physical significacne. Furthermore, coordinate systems are not portable. Portable coordinate systems lead to egocentric theories.

It is nevertheless true retardaion equations are intrinsically egocentric. There is a practical need for egocentric solutions, but those solutions are special cases of more general solutions.

There would be 1/2 av tx2 terms in the solution if the tx2 terms were carried. However, the Lorentz transform is a velocity transform and the veocity at that time is 1/2 j0 tx2. The solutin would be incomplete without the j0 terms. The Newton series is always one step behind.

avdot terms apper in the soluton after the trajectory is differnteiate in the first frame of reference. ... The solution is for a jerked particle.

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Accurate solutions become progressivly more difficult as the systems become smaller. This solution is not the last term of the series.

t0 is a constant for differenting in time. The t0x2 terms could be carried, then an accelertion transform would be required for diferentiating the solution at time t0x2. The Lorentz transform is not an acceleration transform. The location of the particle in the second frame of reference depends on both its location and velocity in the first system. Newtonian concepts do not apply. More generally, the chain rule for differentain is required when the variables are not independen. Acceleration and velocity are not independent variables in 4-space.

The vector from the particle to the field point is rotated in the fourth frame of reference. The pivoit point for the rotation is the same as the origin of the coordinate system. In this soluton the origin is at the particle, so the coordinates of the tip of the vecor are different in the fourth system. The field point would be the pivot point if the orgin is selected to be there. A vector is not affected by a translation of the coordinate system.

There is, of course, no requirement that the problem be worked in the first frame of reference, although it is as good as any.

There are several similarities between this soluton and the equations of the Thomas precession [,]. That is not surprising, as our frame of reference is not special, and it is not differnt. Indeed, it should not matter which frame of reference is used for working the problem. However, the Thomas calculions represent the transformation of the Newton equations to the second and third frames of refernce. The transformed Newton equations are still the Newton equations.

That amounts to projecting the solution into 3+1 space, then proceeding as though it were a real space. The interpretation would not be acceptable for field equations, but these are potential equations. Physical arguments should not be applied to potential equations until they have been suitbly differentiated.

The solution represents a projection of the retarded potentials into a flat 3+1 space. The equations are applied in the same way the LW equations are applied. 3+1 space is not a real space, but physical arguments should not be applied to potential equaions until they are suitably differentiated, because they do not represent locally measurable relationships. Local in a cosmological context covers a lot of territory.

There are three known ways of differentiating the solutions.

The E field transforms into a magnetic field. It is not yet known what the gravitational field transforms into.

After transfoming the Newton equations to the second frame of reference, they are still the Newton equations, but from a different perspective. It does not matter which frame of reference the problem is worked in, but the equations do have to be right in some frame of reference before they are projected elsewhere.

The expansion factor of the cosmos is not zero, so it is doubtful that the velocity terms are real either. The expansion factor does not affect the measured speed of light when the test particles are stationary []. It is not yet known if it can be neglected high order local equations, but, for now, it will be neglected.

Since the time in the equation is not the time shown by an accelerated clock, the terms in the equation are not real in a physical sense. The equation is in 3+1 space. 3+1 space is an abstract mathematical space. However, it is the space where the theorms of Eucludian calculus were discovered. The chain rule for differentiation is one of those theorms. It is capable of converting first derivatves into higer order derivativs, in which case other theorms for the first derivative do not apply until aftrer the chain rule is used.

The expansion factor is not zero inside a mass shell, but it does not affect the locally measured speed of light []. The expansion factor of the cosmos is not zero, but its space and time aspects are linearly dependent in low order laboratory measurements, so it should be all right to neglect it at first. The expansion factor is assumed to be zero in the following calculations.

The Liénard–Wiechert[8] (LW) retardation equations are for the contravariant tensor of the first rank, a vector. History skipped the gravitatioal solution for the first rank tensor. The Newtonian gravitational field is the 3-space gradient of a scalar. The equations do not contain the speed of light. The LW equations do. That is not because electrical equations are essentially different. It is because history skipped a step that is of too low an order to be of much interest. The first ordere electrical solutions are more interesting because an E field transforms into a magnetic field. The first order gravitaional field transforms into itself.

The following calculations do not assume that the location of the particle at the simultaneous point is knowable. The considerations would be different for a field of observers, but that is not the way of the retarded potentials.

The a v cross terms vanish when av and vv are parallel or anti-parallel. There are various other similarities to the calculation of the Thomas precession []. However, there is no point in transforming to the second frame of reference until the coorditates are known in the first system. The transformed Newton equations are the Newton equations from a different prespective.

The basis of these relationships is that the time and space coordintes are strongly coupled in light cone solutions. The equaions do not behave in the same way as they would in an orthogonoal coordiante system. The relationships would be representable in an orthogoanl system, but the chain rule for differentiation is required when the variables are not independent.

A second observer at a greater distance receives the signal when the velocity of the decelleating particle was higher, with the result being that the potential does not decay with distance in a simultaneous system. The behavior might or might not be unphysical, depending on how the derivatives behave. The LW equations have this characteriscic. Even less intuively, the avdot terms can actually increase with distance in a simultaneous system. However, the paticle velocity would quickly exceed c with this artifical trajectory, so the consequencs of the odd behavior are bounded.

The fields computed from the retarded potentials are for sensors that are at rest in the frame of reference of an unaccelerated observer. The advanced potentials are needed when a sensor is moving relative to a stationary charge. The advanced potentials in our frame of reference are the retarded potentials for an observer in the other frame of reference. The advanced potentials do not matter for us, but they do matter for everyone else.

The tensor irreducibility theorm[5] represents the differential 3-space angular relationships. The 4-space equations are different, but a space rotation is still a space rotation – in some frame of refernce. A space rotation does not affect the invariant quantity $\boldsymbol{r}\cdot\boldsymbol{r} - c^2t^2$.

The expansion factor also does not affect the invariant quantity[7]. The expansion factor of the cosmos is not zero, but it will be neglected in the following calclations. However, it remains to be determined if the expansion factor can be neglected in laboratory measurments of sufficently high order.

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When tfx2 is arbitrarily small, it can be replaced by the sum dtf1+dtf2. Only two steps are needed to reduce the equaion to first order unless the vxn terms are carried.

This solution differs from the LW solution in the ordere vx3. With the LW equations, the vector potential is parallel to the retarded velocity vector. This solution contans an rv dot vv term, meaning the it depends on the angular velocity of the particle.

Provided that the vx4 terms are neglected, approximating the trajectory with more than two straight-line segments does not affect the solution. The calculatons for three line segments are shown in the sOM. The solution is

The differnces from the LW solution are small, so they would be difficult to detect in the laboraty in direct comparisons. Solutions with symmetries alien to the LW equations would be easier to experimentally isolate.

The Liénard–Wiechert[8] (LW) retardation equations are contained in the representations of the contravariant tensor of the first rank, a vector. In deriving the equatons, the coodinates are transformed to the velocity of the particle at the retarded intersection. In that frame of refernce the pontential solution for a charged particle moving tangentaily to the trajectory is the Coulomb solution,

$$\begin{aligned} \psi &= q/(4\pi\epsilon_0 R') \\ \boldsymbol{A} &= 0. \end{aligned}$$

The potentails are then transformed back to the first frame of reference. In being representable as a 4-vector, the potentials transform in the same way as the coordinates. The solutions are in the Lorentz gauge, satisfying the condition $\boldsymbol{\nabla}\cdot\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t = 0$ (SI units). The potentials are then differentiated to obtain the fields with the equations

$$\begin{aligned} \boldsymbol{E} &= -\partial\boldsymbol{A}/\partial t - \boldsymbol{\nabla}\psi \\ \boldsymbol{B} &= \boldsymbol{\nabla}\times\boldsymbol{A}. \end{aligned}$$

Ill behaved solutions are mathematically possible[4], but in real problems the solutions are always solutions to the Maxwell equations.

The fields computed from the retarded potentials are for sensors that are at rest in the frame of reference of an unaccelerated observer. The advanced potentials are needed when a sensor is moving relative to a stationary charge. The advanced potentials in our frame of reference are the retarded potentials for an observer in the other frame of reference. The advanced potentials do not matter for us, but they do matter for everyone else.

The invariance of the speed of light is sometimes interpreted as meaning that our frame of reference is the only one that matters. The advanced potentials do not matter for us, but they do matter for everyone else.

Invariance is a necessary but not sufficient condition. Invariant solutions are not of physical significance if they contain a coordinate dependency. Coordinate systems are not portable. The Lorentz transform is easily misinterpreted in this respect.

The meaning of the equation is best found by relacing tf with dt. Like the zeroth order potential equations, no matter how small dt is, the solution contains an error of the first order. The chain rule for differention is required when the variables are not independent. The Newtonian vv and av are not independent variables when the speed of light mattrs. There is no point in transforming to another frame of reference until we know where the particle is in ours.

In the most general solutions, the av and avdot terms are also independent variables with the Newton equations.

With further development, the right way of telling time should be testable in the laboratory.

XVI. THE METHOD OF RETARDATION

This behavior is not contrary to the LW equaions. It is indluded in them. For a given transverse velocity, the magnetic field goes to infinity as the radius goes to zero.

For unaccel erated particles, this behavior is not in any way contrary to the solutions of the LW equations. It is rather that this is the way the LW equations work. The angular velocity terms do not appear in the solutions until the potentials are differeentiated to obtain the fields.

The following calculations do not require that the location of the particle at the simultaneous point be known before working the problem.

There are additional terms in the equations when the retarded accelertion is not zero.

From the perspective of an observer at the field point, the trajectory does seem to be curved, and it does seem to depend on the angular velocty of the particle, even though the actual trajectory is straight line unaccelerated motion.

For unaccelerated clocks, the two ways of telling time are identical.

The various observers cannot agree on where the particle is at time dt. Depending on the perspective, the solution is coordinate-dependent or observer-dependent.

These calculations are closely related to the calculations of the Thomas precession []. Those calculations are interpreted as showing that the coordinates are spinning in the frame of reference of the particle. Their actual meaning is that if we do not know where the partticle is in our frame of refrence then we do not know where the particle is in any frame of referene.

This way of telling time has a meaning in our frame of reference, however the time shown by a field of synchronzed clocks has no special significance for observers in other frames of reference.

The angular velocity terms cause the trajector of the particle to appear to be curved, even though the partile is moving in straight line unaccelerted motion.

When working the retardation problem, the particle is on the light cone for an instant at the location $\boldsymbol{R}$ and the time $t_s = -R/c$. The time at the field point is $t_f = 0$. The particle then continues it journey to the simultaineous point at the location $\boldsymbol{R}_0$. The time at the field point is then R_0/c.

An observer watching a moving particle does not glimpse it at two closely spaced instants. It is continuously visible. It is permanently on the light cone.

The coordinates of the particle are

$$\begin{aligned} \boldsymbol{r}_s &= \boldsymbol{r}_0 + \boldsymbol{v} t_s \\ t_s &= t_s. \end{aligned}$$

The coordinates of the field point are

$$\begin{aligned} \boldsymbol{r}_f &= 0 \\ t_f &= t_f. \end{aligned}$$

The trajectory appears to be curved. The curvature has the form $t_f(v/c)^2$ times the angular velocity of the particle. The angular velocity terms vanish when t_f is zero, however the retarded potentils exist only for the purpose of being differentiated.

The converse relationship exists for an observer in the frame of reference of an unaccelerated particle. Due to the continuously changing propgation delay to the field point, the trajectory of the field point is perceived as being curved. The curvature is not included in the LW equations.

The curvatue is more than just a visual effect. It will influence the interaction of a particle with other particles.

From the prespective of the observer, a trajectory that is perceived as being curved cannot be integrated in one step.

The curvature is more than just a visual effect. It will influence the interaction of the particle with other particles. Even though the trajectory is perceived as being curved, an accelerometer attached to the particle would not register an acceleration unless the retarded acceleration is not zero.

The trajectory of the particle could include acceleration terms. The retarded potentials for light cone events would not be expected to depend on the history of the particle, although the history of the particle is required for computing the location where it is on the light cone. (It can in some cases be necessary to integrate the history of the fields to obtain a potential solution, but the following calculations are not of that form.)

The equation is a polynomial in t_{s0}. It has two roots. Choosing the root for which the signal propagates from the particle to the field point,

When working in series form, the light cone equation can also be solved by the method of successive approximation.

These calculations are not in the second frame of reference, but our frame of reference is not special, and it is not different. Furthermore, it is not possible to transform to the second frame of reference until we know where the particle is in our frame of refernce. The invariance of the speed of light is a necessary but not sufficient condition. Invariant equations are not of physical significance if they contain a coordinate dependency. Coordinate systems are not portable.

The retarded velocities computed by different observers on the surface of the sphere are not for the same particle. Alternatively, in being coordinate-dependent, the soluton is not of physical significance.

That is better, but the tensor of each rank is irreducible []. We are always one step behind – sometimes two steps. The tensor of the first rank is not sufficient.

In so far as is known, the accelertion of the particle does not affect the retarded potentials at any given instant, but the accelertion is nevertheless required for computing the location of the particle at the retarded time. It is plausible that the acceleration does matter, but it is reasonable to defer the question until contradictions arise.

XVII. THE TWO WAYS OF READING CLOCKS

The basis of this relationship is that the location of a partile in the second frame of refernece depends on both it location and its velocity in the first frame of reference. In the first frame of reference, we start out one step behind, and can never catch up.

The tensor of each rank is irreducible []. We are always one step behind – sometimes two steps.

All of the observers on the surface of the sphere are not performing calculations for the same particle. ??Like the zeroth order potential equatios, the LW equations contain an error of the first order. It is of order vx2 instead of the vx1 of the zeroth order equatios, but it is first order in dt. The tensor of each rank is irreducible []. We are always one step behind – sometimes two steps.

There are two ways of computing the retarded velocity. In eq –, the particle moves along a marked course. The location of the particle is the nearest grid intersection and the time is the time shown by the nearest stationary clock in a field of clocks. The retarded time is the same for all stationary observers at different angular locations. Eq – implies that this method of calculation is only usable for observers at infinity.

The time at the particle can also be obtained by reading the number displayed by a moving clock. With this method, the time read is delayed by the light time from the clock to the observer. The time read is different for other stationary obsercvers with diferent angular relatinships relative to the velocity vector, but if the retarded location of the particle is known, the delayed time read by the observer can be corrected for the propogatoin delay. After applying the correction, both ways of telling time are the same in 3+1 space, as is illustrated by the following calculations. The coordinates are orthogonal in 3+1 space.

With this parameterization, the LW equations [] become

The identitis .. and .. can be applied as appropriate. These equations are in all ways equivalent to the solution with Newtonian parameterization. The labor required to obtain the solutions to practical problems is about the same either way.

These relationshis can be extended to the retarded acceleration. If that is done, rvddto is generally not zero for straigh line unaccelerated motion, so the occurence of avddot terms in a solution does not necessarly mean that the rerarded acceleration is not zero.

Similar relationships exist for the advanced intersection. The advanced potentials tend to be neglected, but neglecting them assumes that our frame of reference is the only one that matters. While it is true that our frame of referene is the only one we can ever measure anything in, we are nevertheless free to send messengers elsewhere. Egocentric theories have an irresistable appeal, but they do make simplifying assumptions.

****

The constant has a meaning similar to that of pi, except that it can only exist when the particles are in motion. Comparing the computed value of constants with the values measured in the laboratory provides a means of evaluating the depth of our understanding.

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The calculaions in the SOM do not use tensor notation, but tensor relaionships are sometimes representable by carrying vectors in component form in a Cartesian coordinate system. The essential relationship is that the gradient of a vector is a tensor of the second rank.

****

If a particle is approaching an obsever and decellerating, the retarded velcity for a second observer further away from the paricle is higher. Consesuenly the potaentials do not decay with distance. The LW equations have this characteristic. Depending on the global behavour of the derivatives, it is not necessarily unphysical.

Terms that do not decay with distance, when combined with the 1/rx2 terms of Eq –, decay as 1/r. 1/r terms are radiative. However, the near field 1/rx2 terms in quasistatic solutions are experimentally more accessible.

For quantities that are of order v^1, such as the magnetic field and the momentum of a particle, v^3 terms represent a $(v/c)^2$ relativistic correction.

In the solutions for rotating electrical equipment, the quantity of interest is not the velocity of the conduction electrons, but rather the sum of their velocity and the rim velocity. The velocity of the conduction electrons is not necessarily in the same direction as the rim velocity. The protons must of course be included in the calculation.

We canot tell which partile is which. The Lorentz transform is not helpful in obtaining the soluton for an inertial particle.

The basis of these difficulties is that coordinate systems are not portable. Choosing one that follows as we move about in space and time has a certain appeal, but it leads to contradictions if there is, or could be, another observer in the problem.

The solution depends on where the origin of the coordianate system is. It is coordinate-dependent. coodinate-dependent solutions are not usable, for we can never know where we are in space and time.

Similarly, if we choose a coordinate system that follows us as we move about in space and time then the soluton is coordinate-dependent if there is, or could be, another observer in the problem. All things measurable are relative, but it is not us that they are relative to.

Potential equations reperesent the view from infinity. That is a view we can never know, so it is not our perspective.

For the purpose of reconstructing the trajectory of the particle with subsequent processing of the delayed measurements, both observers have to be able to agree on where the particle was.

A correction must be applied to account for the propogation delay from the souce to the field point when using the measurements of Rv and its derivatives to reconstruct a Newtonian trajectory.

The basis of this discrepancy is that the perceived values of Rv and its derivatives are delayed by the light time from the source to the field point. If the objective is to reconstruct a Newtonian trajectory from the perceived values then a correction is required to account for the delay.

... is zero, showing that the particle is on the light cone, but it is not the same particle as in Eqs ().

Three steps would be required if the v^4 terms are carried or if the solution needs to be differentiated three times.

The inconsistency vanishes when vv and rv are parallel or anti-parallel at the simulteneous point. It is only when there is a transverse velocity component that the problem occurs.

In 3+1 space, we are free to choose a coordinate system that follows us as we move about in space and time. We still are with this solution, but only by a tiny amount. This solution would not work if there was a third navigator at the location ... .

$\dot{\boldsymbol{R}}$ requires a relativistic correction for inertial particles.

While it is possible to determine where the particle was, distant observers cannot know where it is.

Actually, we do not know where the particle was, but in working the retardation problem it might be necessary to assume that the location is already known. The assumption will require further evaluation. If the assumption is valid, then the $\dot{\boldsymbol{R}}$ in Eq () should be replaced by $\dot{\boldsymbol{R}}'$.

Another way of working the problem would be to obtain the location by integrating $\dot{\boldsymbol{R}}'$. In 3+1 space, the solution could be obtained by integrating $\dot{\boldsymbol{R}}$.

The time ticks along the trajectory of the particle will be evenly spaced if t_s is taken to be the independent variable in the light cone equation. But then the time ticks are not uniformly distributed for an observer at the field point. There is a conflict.

The invariance of the speed of light is a necessary but not sufficent condition. Invariant solutions are not necessarily of physical significance. Laboratory evaluation is required to be sure of which invariant solutions are the right ones. However, when there is a conflict between a function an its derivatives, it means that the solution should be obtained by integration.

A distant assistant reporting the times that the moving clock is nearly coincident with a field of at-rest clocks would not agree with this equation. The assistant is usually not available, although it could be done. In any case, it is the transform that matters.

The virtual acceleration occurs because the solution is coordinte-dependent. Coordintate dependent solutions are not of physical significance, because we have no way of knowing which coordinte system we should use.

drv has dropped out of the solution, showing that it does not matter where the navigator is. However, this solution would not work if the navigator was at the location ... .

In 3+1 space, we are free to choose a coordinate system that follows us as we move about in space and time. In 4-space, such solutions are coordinte-dependent if there is, or could be, another observer in the problem.

Suppose that the perceived velocity of a particle, as evaluated by a distant observer, is $\gamma \boldsymbol{v}$, with ... . With this corretion, the radius vector from the observer to the particle appears to be spinning, relative to where it would be in 3+1 space. Similarly, the radius vector appears to be spinning in the calculaitons of the Thomas precession[6]. The angular relationships of the four dimensional space are represented more generally by the rotations of the Lorentz group []. The 3-space angular relationships are represented by the tensor irreducibility theorm []. There are additonal terms in 4-space, but a space rotation is still space rotation.

Equations of physical significance also have to work in the same way when applied in different places.

The Newton equations and the LW equaions are for one navigator. Both equaions have a respectable level of accuracy and they are not wrong, but they are incomplete.

XVIII. THE THIRD TERM OF THE RETARDATION SERIES

The first term of the retardaion series for charged particles is the Coulomb solution. It is the basis of all the other terms, except that it requires adjustments in a cosmological context. The second term of the series is the LW solution. Each term of the series stands alone.

When vv and rv are perpendicular there is no first order constraint on the particle location. The light cone equation is slippery.

The second observer requires this choice, and the first observer did not know where the particle was in the fisrst place.

To first order, the transverse location of the particle is unconstrained. Equations of physical significance have to work in the same way when applied in different places. A second nearby observer is helpful in obtaining a better estimate of where the particle was. Distant observers cannot know where it is, but it is possible to obtain useful estimates of where it was.

It may be possible to obtain a more complete solution by considering an observer at ..., or in other ways. There is always more than one way of working a problem. The equations in the rotations of the Lorentz group [] are probably relevant. The calculaions of the Thomas precession [] are closely related to those equations. However, the calculaions of the Thomas precession assume that the location of the particle is computable with the Newton equations, which leads to contradictions.

drv has to drop out if the equation is to be usable without knowing where the origin of the coordinate system should be.

Two observers will not be able to agree on where the particle was unless drv drops out. The solution cannot be differentiated unless two nearby observers can agreee on where the particle was. Actually, the equation could be differentiated, but the solution would not mean anything unless both points are for the same particle. The first derivative is not sufficient unless the equation is of first order, but it is a necessary starting point.

The two observers cannot agreee on where the particle was unless drv drops out. The first derivative does not mean anything unless both points are for the same particle.

drv has to drop out. If it does not drop out, the two points needed for computing the first derivative are not for the same particle.

... The particle is on the light cone for the first observer, but its location depends on where the second observer is. Neither observer knows where the particle was.

Both observers now agree on where the particle was. The two points required for computing the first derivatvie are for the same particle.

The curvature term vanishes when tf=0. The function is not consistent with its derivatives. When there is such a conflict, it means that the solution should be obtained by integration.

XIX. INERTIAL TRAJECTORIES

Distant observers cannot know what is happening at the simulteneous point while it is happening. It is necesary to either extrapolate to that point with light cone events or wait a while to find out what happened there.

This inconsistency does not occur unless there is a velocity component in the transverse direction, which requires at least two directions in space. More generally, the gradient of a vector is a tensor of the second rank. However, $(d\boldsymbol{r}\cdot\boldsymbol{\nabla})\boldsymbol{R}$ is a vector.

The angular velocty term in Eq – can be retained by integrating in time by the amount r0/c, which cancels the 1/r0 term. The integral does not look like an angular velocty term, and it is not, however the retarded potentials exist only for the purpose of being differentiated.

Except in special circumstances, the gradient of a vector is a tensor of the second rank, which is probably the appropriate way of perfoming the integration. There appears to be a shortcut way of obtaining the solution.

... The light cone constraint is slippery in the infinitesimal.

In the first frame of reference, there are powers of velocity in all orders at time dtf, but they are a consequence of representing 1/(1+v/c) in series form. The implied accuacy of the series expansion, expecially in the transverse direction, is illusionary. It is not possible to quickly determine where the particle was with light cone equations.

$\Delta_{01} = \Delta_{12} = \Delta^2\boldsymbol{R} =$

The magnitude of $\Delta\boldsymbol{R}$ needs to be considered in relation to the magnitude of R.

$$\frac{\Delta(\Delta\boldsymbol{R})}{R^2} = \tag{8}$$

When $\boldsymbol{R}$ and $\boldsymbol{v}$ are perpendicular, the equation has the form of the angular velocity of the particle, multiplied by a $(v/c)^2$ relativistic correction.

From the perspective of the second observer, the particle is not where the first observer thought it was. The second observer is probably wrong too.

The equations would also be interpretable as meaning that the first observer was right and the second observer is wrong. The retarded potentials exist only for the purpose of being differentiated. The differnce between the solutions is more important than the value of either.

The two observers cannot agree on where the particle was. The retarded potentials exist only for the purpose of being differntiated. The difference between the two perspectives is more important than the correctness of either.

The equations could be developed further by taking Eq () as the assumed starting point instead of Eq ().

The particle is on the light cone, but it is not the same particle.

Something is amiss.

The full transform in Eq () will also halt the particle in a single step. Are we sure the solution is for the right particle?

The propogation delay from the particle to the field point, when combined with the constantly changing angular relationships, results in a virtual accelertion term that is parallel to the retarded velocity. The virtual acceleration is perceived, but it is not real. A relativistic correction for the angular velocity of the particle is needed.

XX. ONE PARTICLE, MANY OBSERVERS

The LW equations can be derived from the Lorentz transform, suggesting that the Lorentz transform is not for an inertial particle.

When computing the retarded potentials for two partilces, the retarded velocity of each particle must be computed, but that is only the first part of the problem. The contributtion from each particle must be delayed by the light time to the field point. The two signals cannot be added simultaneously. When the two partices are closely spaced, and there are only two particles in the problem, it is only the differential delay that needs to be considered when adding the two signals. There are similar consdertions for one particle at two times.

The impied curvature of the trajectory is not real. It is an observational artifact. A relativistic correction for the angular velocity of the particle is needed.

...

The derivatives of the function are not consistent with the function. That normally means that the function must be obtained by integration. But since the retarded potentials exist only for the purpose of being differented, it does not appear necessary to actually perform the integration. However, because the integral of the derivative is arbitrary to within a constant of integration, the potential solution obtained with the shortcut method is not necessarily unique. It is only the derivatives of the potentials that are measurable, so a potential equation does not have to be unique. Gauge transforms exploit this characterisc of potential equations. It is necessary to assume that we know where the particle was when working the retardaion problem, but it is difficult to be sure that we do know where it was. An observer at infinity would know, but that is not our perspective.

If Rdot were computed in three orthogoal directions and interpreted as the components of a vector, the result would be the same as the LW vector equation in Eq (). That would be the right way to differentiate a vector in 3+1 space.

It has to be possible to apply retardation equations without knowing what time it is. They have to work in the same way for any value of tf0.

The only thing that this calculation shows is that the derivtives of a function are consistent with the function, and we knew that already.

This would be the solution for the retarded potentials in 3+1 space. The problem is not in 3+1 space.

This solution is for the first time derivative. The soltion for the second derivatve will be more elaborate unless the relationships are degenerate.

There are two ways of viewing this solution. Since the particle is not accelerated, it is a visual artifact and is not real. On the other hand, it is the delayed signal that a charged particle will respond to.

The expansion factor within a mass shell stretches distances and times by the same amount, so the measured speed of light is not affected[7]. It is not yet known if the cosmological expansion factor affects laboratory electrical measurements, but it should be possible to find out if it does by including it in the retardaion equations. The location of a particle at the simultaneous point contains a hidden degree of freedom in this reqpect.

Unsurprsingly, as shown in the SOM, the Lorentz transoform is obtainable by integrating the infitesimal transform. However, the integral is arbitrary to within a constant of integration.

There can be one observer and many particles, or one particle and many observers. When there is one particle and many observers, each observer is not free to choose their own coordinate system. It is not meaningful to ask which perspective is the right one. It is the transform that matters. It is more difficult to transform from one observer to another observer than to transform from the particle to one observer.

From the perspective of an observer in the frame of refence of an unaccelerated particle, a clock in our frame of reference would appear to be accelerated in the direction of the retarded velocity, so a clock in our frame of refernce would not be a good choice for a time standard.

In Eq (), it can be seen by inspection that R=r0 when tf=0. The equations assume that the solution is known before working the problem. The formulation is overconstrained, leading to an unphysical solution.

****

Potential equations exist only for the purpose of being differentiated. In not representing locally measurable relationships, local physical arguments should not be applied to them.

In Eq (), the derivatives of the function are not consistent with the function. It is sufficient to obtain a function that differentiates correctly. Always in concept, and sometimes in practice, potential equations can be gauge transformed, so there is no requirement that the function be unique. There is also no requirement that it have local physical significance. Potential equations allow us to see ourselves as others see us, to know what we cannot see.

The solution is the LW solution when tf=0. However, the solution is for the wrong particle at time tf.

Since the coordintes appear to be spinning, the transform must be performed in two steps, first to take out the translaional velocity, then to take out the spin of the particle. A point charge cannot actually spin, but it does seem to be spinning.

****

Other choices would be equally valid, but choosing other locations does not accomplish anything, because curvature is not representable in 3+1 space. However, 4-space can be projected into 3+1 space with potential equations.

The method of retardation is so different from the methods of field equations that it is often difficult to see why there should even be a connection, but there is. Neither field equations nor retardation equations form a complete representation. Field equations constrain the fields without specifying the physical processes responsible for them. Retardation equations do specify the physical processes, but they are inept in other ways.

It remains to be determined if there are electrical terms in the solution for a hammered mass, and conversely.

Refer to Ref. 10 and `www.s-4.com/som2/som.htm` for some useful background material and equations.

It is unlikely that the second derivtive can be obtained by differentiating this solution in 3+1 space, but obtaining the second derivtive that way should be a good approximation when the velocity is low. Conversely, it would be easy to carry more powers of velocity in the solution for the first derivative, but their validity will need further investigation.

drv and dt would be orthogonal if both were defined at the field point, in which case the drv*dt cross term would vanish. In this soluion dt is at the field poilnt and drs is at the particle. The two terms are not orthogonal.

The spin of the particle must now be taken out by transforming from the tip of the vector to the tail of the vector.

The rotation causes the velocity vector in the second frame of reference to not be parallel to the vector in the first frame of reference. The rotation is not inclued in the derivation of the LW [] equations. It is also not included in the calculatinns of the Thomas precession []. It is not included in any of the calculations of the special theory of relativity [].

It is possible to interpret this solution in a different way – that simultaineouls events in the first frame of refernce are not simultaneous in the second system.

The solution can be interpreted in two ways. With one interpretaion, events that are simultaneous in the first frame of reference are not simultaneous in the second system. With the other interpretaion, the velocity vector in the second system is not parallel to the vector in the first system. Light cone solutions can be helpful in resolving the ambiguity.

.. A second transform is required to take out the virtual spin of the particle. The equations would be muddled if the angular and translational velocity terms were not separated.

The angular velocity of the particle is v=w r0. When vv and rv are perpendicular, the equation simplifies to delta $r = w(v/c)^2$ [?at time t=0].

The traditional interpretaion of this and similar relationships is that simultaneous events in the first frame of reference are not simultaneous in the second system [].

It follows from the calculations of the Thomas precession [] that two consectutive Lorentz transforms are not representable with a single Lorentz transform. Two transforms are equivaltt to a singl transform followed by a space rotation. It is the velocity of the tip of the radius vector that is relevant in the second frame of refence. Since is is not the same as the velocity of the tail of the vector, two consecutive transforms are required for transforming the potentials back to the field point.

Potential equations can usually be viewed as reprenting the perspective of an observer at infinity. The perspective can be mathematiclly convenient, but it is one that we cannot know.

This solution can be interpreted in two ways. In one way, events that are simulteneous in the first frame of reference are not simulteneouls in the second sysem. With the other interpretaion, the radius vector is spinning in the second system. The second interpretaon is used in the following calculaions. However, there is not necessarily anything wrong with the first perspective, provided that is consequences are developed. It is the transform that matters, not which perspective is the right one.

As shown in the SOM, this calculaion can be obtained by solving the light cone equation twice, first for time tf0 then for time tf0+tf1, and the solution is the same as this one.

This equation applies equally well to doppler shifted acoustic signals. although they do not transform in the same way as electromagnetic signals.

The constant of integration would evidently depend on where the observer is, relative to the particle, so it may be difficult to determine it in a general way.

In 3+1 space, this would be the retarded velocity of the particle at time tf, and the retarded acceleration would be zero. For light cone solutions, and more generally in the second frame of refernce, the time of an event depends on both the location and the velocity of the particle, so the acceleration cannot be computed until the velocity becomes known. That requires one more step. Similarly, an extra step would be needed for the $\dot{\boldsymbol{a}}$ terms. In 3+1 space, we always one step behind. There is nothing wrong with being one step behind, provided that we do not try to lead.

The solution indicates that a relativistic correction for the anglar velocity of the particle is needed. Eucludian calculus does not neglect this form of curvature, because the chain rule for differentiation is required whenever the variables are not independent. The investigation will be continued.

The trajectory is perceived as being curved. The trajectory is a straight line, but the time intevals along the line are not properly spaced.

Since the trajectory is perceived as being curved, a more accuate soltion can be obtained by representing the trajectory with two segments. The two segments are parallel, but their lengths have to be computed.

The LW equations are not form invariant for translations in time. This relationship causes the first time derivative of the LW equations to be wrong, even though the equations are correct at time tf=0. The derivatives of the function are not consistent with the function. If the time at the field point were the only availble time that would mean that the function would have to be obtained by integration. For the purpose of deriving the retardaion equations, the time at the source is also available, although it is not availbe for field equations.

The mapping of the time at the source to the time at the field point is not a linear function of the time. The midpoint of a time interval at the source does not map into the the midpoint of the corresponding interval at the field point, no matter how small the interval is. It is easy to misapply the theorms of Eucludian calculus when the midpoint of an infinitesimal line element is not at the middle. However, the chain rule for differention always required when the variables are not independent. The chain rule is not needed when the time at the source is taken as the independent variable.

The LW equatios work correctly at time tf=0 and time tf=+r0/c. The eqations have a different form at other times. It is not possible to apply the equations without knowing what time it is. We cannot know what time it is.

Our frame of reference is not special for anyone except us. Coordinate systems are not portable if there is, or could be anothere observer in the problem. One observer evaluating a system at two widely spaced times counts as two observers. It is nevertheless possilbe to obtain equations where the coodinate system seems to be portable, within limits. That is equivalent to stateing that the equations can be differentiate specified number of times. The count for the LW equations is zero.

The gradient of a vector is not a vector, but (dr vdot del) Rv is, and the equation can be applied recursively. It is possible to obtain vector equaios that behave like vector equaions, within limits.

The LW equations are also form invariant for translations in time, provided that they are not differentiated.

The solution shows that the LW equations cannot be differentiated without knowing what time it is. We cannot know what time it is. Coordinate dependencies can exist in either space or time.

The propogation delay from the particle to the field point casuses the angular velocity of a particle to become coupled to the perceicved translational velocity. This relationship is included in the LW equations. The retarded velocity is still v0.

The basis of this relationship is that the midpoint of a time interval at the source does not map into the midpoint of the corresponding time interval at the field point. The durations of the two intervals are also different.

An observer in the frame of refernce of an unaccelerted clock would insist that the clock runs at a constant rate, as would an observer at the field point. Dynamically adjusting the clock rates is not an option. Distorting the mapping by adjusting the midpoint of one or the other is not possible either, because it would affect the propogation velocity.

However, the time shown by the other clock was not known in the first place, so now we have the opportunity to synchronize it, to the extent that it is possible with the retarded potentials. By displacing all of the events associtated with the other clock in time, the two midpoints can be brought into alighment.

The midpoint is now at the middle, and the false acceleration term has vanished. Or is it that a curvature term has been added? That depends on the perspective. It could be that the charged particle is at rest in our frame of reference and the other observer is moving, so both perspectives are needed.

It can seen in Eq – that the closest approach occuas at time tf1=0. Something is amiss.

The retarded potentials exist only for the purpose of being differentiated, so it is not necessarily true that we have to know where the particle actually was if the derivatives are consistent. But when applying retardation equations to specific configurations, it is necessary to assume that we know where it was. However, potential equations are subject to gauge transformations, so the assumed potentil solution is not necessarily unique. In any case, potential equations do not represent locally measurable relationships, so local physical arguments should not be applied to them.

This adjustment seems contradictory, but perhaps not. In 3+1 space, the time of an event depends only on its location. For light cone solutions, and more generally in the second frame of referece, the time of an event depends on both the location and the velocity of the particle. It is difficult to be sure that all of the events in Eq () are for the same particle.

The perceived transverse velocity is parallel to the retarded velocity, but there is a realtivitic correction for it when the particle is nearby.

Either Eq – is not for an inertial partcile, or all of the events are not for the same particle. It is difficult to be sure which particle is the right one when the angular and translational velocites are coupled.

There are no longer any angular velocity terms in the solution. The angular and translational velocites have been decoupled.

There are still angluar velocity terms in the solution, but they are no longer coupled to the translational velocity. That is how it would be if there were no propogation delay.

The constantly changing angular relatioships, when combined with the propogation delay, cause the other clock to not be perceived as running at a constant rate, even though it does. The rate for an unaccelerted clock is not subject to dynamic adjustments, but it can be that the time shown by the clock is not what it was thought to be.

The perceived transverse velocity is not the same as the retarded velocity when the particle is nearby. It is the same when the particle is at a great distance.

The nonlinear mapping, when combined with the propogation delay, causes the rate of the other clock to seem to be variable. An unaccelerated clock will run at a constant rate, so the perceived variation in the rate can be used to derive a constant offest to the times that were incorrectly assumed to be the time shown by the other clock in Eq ().

The other clock is now perceived as running at a constant rate.

However, taking the Thomas precession out of the retardaion equations puts it in to the solutions at the field point. From the perpective of an observer in the frame of refernce of the particle, the LW equations contiain terms that do not belong there. From the persective of the observer at the field point, the LW equations are missing terms.

The nonlinear transfer function, when combined with the propagation delay, causes the clock to appear to run at a non-uniform rate. An unaccelerated clock does run at a constant rate, but it is difficult to determine the time shown by the clock when there is a transverse velocity component. Since the clock is known to run at a constant rate, it is possible to synchronize it by choosing a time offset from the times assumed in Eq () that causes the rate to appear to be constant. The times assumed in writing Eqs() seem reasonable, but they lead to subtle contradictions.

From the perspective of an observer at the particle, the correction removes a visual abbertion. From the perspective of an obervrer at the field point, it adds a minor correction to the much larger curvature terms that are already present in the LW equations. The LW equations are not in the same order as the Newton equations. The static Coulomb solution is in the same order as the Newton equations.

??The problem could be worked in the first frame of reference, and the equations would be interesting, but it turns out that it is computationally easier to synchronize the other clock by solving the light cone equation in the second frame of reference.

It has to be possible to apply retardation equations without knowing what time it is, because we do not actually know the time. Coordinate dependencies can exist in either space or time.

The magnitude of $\Delta \boldsymbol{R}$ needs to be considered in relation to the magnitude of R. The second difference is

For the purpose of applying the retardation equations to specific configurations, a suitable reference point can be obtained by requireing the the time of closest approach in Eq () coincide with the time of closest approach in Eq (), even though we cannot know where the particle actually is at the simultaneous point until after waiting a while.

Our frame of reference is not special or different, so it should also be possible to work solve all of the equations in the other frame of reference.

There is a very old belief that we know the time in our frame of reference, and it is true that our frame of reference is as good as any.

If our frame of reference were the only one that matters then the effet of the correction would be to add a minor correction to the much larger curvature terms that already exist in the LW equations.

Potential equations are arbitrary to within a constant of integration. It is the nature of retardaion equations that the constant of integration depends on where the observer is, relative to the source, so it does not look like a constant. That makes the chain rule for differentianon necessary, because the variables are not independent. Potential equations that do not require the chain rule are more convenient, but they do not repersent locally measurable relationsips, so local physical arguments should not be applied to them.

It should be possible to work the problem in either frame of reference, but the calculations are simpler if the light cone equation is solved in the second frame of reference.

Retardation equations have to work in the same way when applied at different times, because we have no way of knowing when the time tf=0 should be. The time tf=0 can be chosen arbitrarily, but once selected, it cannot be readjusted.

Eq () assumes that we can know what is happening at the simultaneous point while it is happening. That is not possible. There is no physical basis for asssuming that the location of the particle at time $t_f = 0$ in Eq () is knowable. There is a particle at that point in space and time, but it is difficult to be sure that it is the right particle.

the the particle is at the location $\boldsymbol{r}_0$ when the time at the field point is r_0/c. There is a particle at that point in space and time, and it is on the light cone, but it is difficlut to be sure that it is the same particle as when the time is $-r_0/(2c)$. It might be possible to determine if the two particles are the same if the transfer function were linear, but it is not linear. It does become linear if the location of the partice on its trajectory is displaced in time.

The retardaion equations now look the same at three different times. The quadradic terms have been taken out. The equations will be more difficult to linearize when the velocity is high enough that the t_{so}^2 have to be carried.

The perceived velocity contains vv/R vsq terms, which represent a relatiisic correction to the angular velocity.

This is the LW solution when the coordinates are first-known at time tf=-r0/c. It has the same form as when they are first-known at time tf=0.

This is the LW solution for the time tf=-r0/(2 c). We have no way of knowing when the time tf=-r0/(2 c) is.

This relationship causes the derivatives of the function to contradict the function. That can mean either that the derivatives are being computed the wrong way or that the function is wrong. The two perspectives are not necessarily mutually exclusive, but the equations do need to be self consistent.

Form invariance has about the same meaning as the requirement that the equatons should not contain a coordinate dependency. Coordinate dependencies can exist in either space or time.

The time at the source is

Differentiating with respect to tf

The clock rate will be constant if the first derivative of ts is 0.

Orthogonalizing the angular and translationl velocites has the effect of taking out the Thomas precession. However, taking it out of the light cone equation has the effect of putting in to the solutions of the retarded potentials, where it appears in clearly recognizabe form as a rotion of the retarded vector potential. The Thomas precession is of order ax1 vx1. The vector potental is of order vx1, so the rotaion is of order ax1 vx2 when the particle is accelerated.

The potential equations do not contain acceleration terms, but they are the integral of the fields, so the acceleration terms do not appear until the solutions are differentiatd.

This transform takes out the quadradic terms, but there are other corrections needed if the vx4 terms are to be carried.

No matter how short the time intervals are, the midpoint does not map into the middle. That would complicate the theorms of Eucludian calculus if the chain rule for differentation is not applied first. The chain rule is not needed if the equations are orthogonalized. It is doubtful that complete orthogonalizaion is possilbe, but the problem can be developed as a series expansion.

It is for this reason that the LW equations do not work the same way at the midpoint of the interal as they do at either end. The midpoint of an interval at the particle does not map into the midpoint of the corresponding interval at the field point. The transfer function is not linear in time. The location of the particle is linear in the time at the source, so the time at the source should be taken as the independent variable for inertial particles.

The time at the field point then does not progress at a uniform rate. There are circumstances when we should care about what time it is at the field point, but approximate solutions for the retarded potentials can be obtained without knowing what time it is there. That is because the retarded potentials exist only for the purpose of being differentiated. The only requirment is that the eqations be linear for a short time at the field point.

There is no upper limit for the anglular velocity of the partile, so this approxiation will become unusable if the particle is too close. It is possible for the interval at the particle to be far longer than the interval at the field point, in which case the model is not accurate.

The observer at the field point cannot know where the midpoint of a line is, causing the equations to be nonlinear in a way that Eucluid neglected. In 3-space, straight lines do not have tic marks at regularly spaced intervals along their length.

The retarded potentials exist only for the purpose of being differentiated, so we do not need to know where the midpoint of a long line is. There are circumstances when we should care about what time it is in our frame of reference, but approximate solutions for the retarded potentials can be obtained without knowing the time.

The tic marks are almost equally spaced along short lines, but the observer at the field point probably does not understand what short lines are. In Min– space [], the distance between light cone events is zero.

The other observer already knows the time, so it is easier to work the problem from the other perspective. The retarded potentials exist only for the purpose of being differentiated, so the actual time in our frame of refernce does not matter. It is only the derivtives that matter.

From the perspective of an observer at the particle, this soluton removes a false visual abberation. From the perspecive of an observer at the field point, it adds curvature term. It is the transform that matters.

The dt interval would have to be subdivided into three intervals when the vx4 terms are needed.

The other observer already knows the time, so it is easier to work the problem in the second frame of reference. The retarded potentials exist only for the purpose of being differentiated, so the time in our frame of refernce is a secondary consideration if the derivatives are correct. In being the integral of the fields, potential solutions are arbitrary to within a constant of integration. The potential solution obtained this way is therefore not necessarly unique, and there is no requirement that it be unique.

As was first realized by Einstein [], and developed from a different perspective with the equations of the Lorentz group [], the Lorentz transform contains hidden degrees of freedom. We cannot tell the difference between angular and translational velocity in observations of short duration, but we need to know the difference in global solutions, as we could not otherwise tell if we are orbiting a particle, or if the particle is orbiting us. It could be either way.

This equation is the appropriate equation when we do not care what time it is, although it there are circumstances when we should care. But in obtaining retardation solutions for one observer and many particles, it is only necessary to know the solution for a short time. However, the solution will not be valid for a long time unless more terms are carried in the equations. The solution has to be accurate for longer times if it is to be differentiated more times. The LW solutin is accurate enough to compute the first derivatves. The Maxwell equations are in terms of the second derivatives.

This solution satisfies the equation $\boldsymbol{r}'\cdot\boldsymbol{r}' = \boldsymbol{r}\cdot\boldsymbol{r}$. That is possible, because a space rotation does not affect the magnitude of a vector. Rotations are mathematically simpler in the complex number domain, but they are harder to understand for most of us. There is an extensive and relevant literature on the subject []. No attempt has been make to find equivalent equations in the literature, but they almost certainly exist, and in a more elegant form.

In the first frame of reference, the vector from the field point to the particle rotates as the particle moves. The observer in the frame of refence of the particle perceives the vector as rotating in the other direction. For one choice of the coordinates, right handed rotation becomes left handed rotation. For unobvious reasons, the other observer does not think the particle is where we think it is. The potential solution is first-known in the frame of reference of the particle, so the other perspective is the appropriate one.

Which of us is right? Since we do not know our own velocity, an arbitrer at infinity might be able to resolve the question. Potential equations can usually be viewed as representing the perspective of an observer at infinity. The 4-potenital transforms in the same way as the coordinates, it is implied that the coordinates can be viewed as being a potential representaion.

Potential equations do not represent locally measurable relationships, so local physical arguments should not be applied to them.

If the coordinates are not viewed as being a potential representaion, then the chain rule for differention is always required when the variables are not independent. The time and space coordinates are not independent variables in light cone solutions or in the second frame of reference. It is not sufficent that light cone solutions specify the function. They must also specify the derivatives of the function, which can require integrating the derivatives in order to obtain mathematical self consistency.

There is an extensive literature on invariant equations []. There is a good possibility that a solution equivalent to this one exists somewhere in it, perhaps in an abstract form that is difficult to recognize.

The chain rule for differentiaon is required when the variables are not independent, so there should be a way of working this problem with the chain rule. The chain rule is not needed if the relationships are orthogonalized. In this case, orthogonalization appears to have the same meaning as linerizing the equations by using the time at the source as the independent variable in the light cone equation.

It should also be possible to linarize the tajecory of the particle with the chain rule for differention . If the chain rule is used, the valuc of a vaector at a nearby point is Rv + (dRv vdot del) Rv. The thing that matters is that the time shown by a clock in our frame of reference is not the right time base for the kinematic relatinships of the particle. In containang velocity, eq – makes it look like there is a connection, but the equation is misleading. The equation represents a sequence of static light cone events having no connection to the trajectory of an inertial particle. The eqation looks like it is in 4-space, but it is in 3-space. The equally spaced tick marks along straight lines are not visible in 3-space, but they are there.

The mapping is linear when the motion is radial, and a simple correction can be applied. The mapping is not linear when there is a transverse velocity component and the partilce is nearby. The time shown by a clock in our frame of reference is not a satisfactory time standard for computeint the trajectory of an inertial partcile unless the angular velocity is zero. Corrections could be applied for the nonlinearity, but they are not simple, and they become more elaborate when the particle is closer.

It is for this reason that the solution to the LW equaions in Eq - looks different. The midpoint of the three soloutions is not at the middle in the sense that it canot be obtained by linarly interpolating the other two solutions. The midpoint would be much closer to the middle if a short time span for the three solutions were used, but the solution would have to be integrated, and the large discrepancy would re-appear after integration. There is no known way of reducing the four dimensional space to first order, although it can be linearized in various stages.

The propogation delay, when combined with a transverse velocity component, causes the transfer function to be nonlinear in time, making it difficult to use the clock at the field point as the time standard, although it could be done.

Each of the events in Eqs () is for a particle on the light cone, but there is no physical basis for assuming that all of the events are for the same particle. The function is technically correct, but the derivativs of the function contradict it. That means that either the function has to be obtained by integration or a differet function computed. The time and space coordintaes are not independent variables in light cone solutions, so the chain rule for differention would be required if the function is to be obtained by integrating the derivatives.

It would be possible to use the clock in our frame of refernce by apply a corrction for the nonliear transfer function. The location of the partcile is already a linear function of the time at the source, so it is easier to use the time at the source as the time standard.

It is for this reason that the LW solution in Eq – at the midpoint of two times cannot be linerly interpolated from the other two solutons. The transfer function is not linear in time when there is a transverse velocity component. Since the particle veocity cannot exceed c, the nonlinaritey is only important for observations of short duration when the particle is nearby. But even when it is not nearby, it is not where it was though to be.

The retarded potentials exist only for the purpose of being differentiated, so the time in our frame of reference is not very important. We do not even need to know the time in our frame of reference to compute the derivatives of the retarded potenials. The location of the particle is a linear function of the time at the source, so it is easier to use the other clock as the time standard than it would be to apply a nonlinear correction for the time in our frame of reference.

A short time interval in our frame of reference maps linearly into a short interval in the frame of refernce of the particle. The retarded potentials exist only for the purpose of being differentiated, so it is not necessary to know what time it is in our frame of reference. There is therefore no need for nonlinear corrections to the time shown by our clock.

The location of the particle is already a linear function of the time at the source, so it is easier to use the other clock as the time standard. We do not need to know the time in our frame of refernce to compute the retarded potentials, so it is then no longer necessary to apply a nonlinear correction for the time shown by our clock.

There is no upper limit to the angular velocity of a paricle when it is nearby, so there are probably limitations to this model, with the neglect of the acceleration terms being one of them. It is doubtful that large powers of velocity can be carried in these calculations without additional considerations. There may be additional considerations for the v^3 terms.

The following calculations neglect the acceleration terms. They will be considered in a later version of this paper, but velocity terms are of lower order.

The value of R0 in our frame of reference has no meaning for the other observer.

Taking out the false acceleration term has the effect of applying a relativistic correction to the curvature terms that already exist in the LW solutions.

The propogation delay causes a false acceleratin term to appear in the light cone solution when there is a transverse velocity component. This difficulty does not occur when the motion is radial.

The Newtonian parameters are obtained by differentiatig with respect to the time at the tip of the radius vector. The perceived quanatiaties are obtained by differentiang with respect to the time at the tail of the vector.

The particle is perceived as moving along a straight line, but it does not appear to move at a uniform rate along the line when it is nearby and there is a transverse velociy component. The trajectory seems to be curved in a way that has no meaning in 3-space. Straight lines in 3-space do not have equally spaced tick marks along their lengths. 4-space can be projected into 3-space, but the tick marks are not a part of 3-space geometry. The curvature causes the angular and translational velocites to become coupled when the particle is nearby. The two kinds of velocity are easily distinguishable in global solutions, but it takes a while to determine which is which when the observations are delayed by the light time accross the system. The perceived velocity depends on both the distance to the particle and the angle of the trajectory. The angular and translational velocites are coupled. It is not possible to quickly determine which is which unless the retarded velocity is known by independent means.

The propagaion delaty results in a virtual acceleration parallel to the retarded velocity. It is not possilbe to quickly distinguish between the virtual acceleration and real acceleration unless the retarded velocity is known by independent means. (An accelerometer in free-fall does not register an acceleration, so the meaning of real acceleration is negotiable.)

Translational and angular velocites are easily distinguishable in global solutions, and in any solution when the propagaion time accross the system is zero. When the observations are delayed, it is not possible to quickly determine which is which unless the retarded velocity is known by independent means.

The Lorentz transform has the same problem, and for the same reason. The midpoint is not at the middle. The particle appears to be accelerated in a direction parallel to transform velocity. For this solution, there is no connection between the transform velocity and the kinematic behavior of the particle. The false acceleration terms need to be taken out if the other observer is to concur with what we already know – that the particle is at rest in our frame of reference.

There is no ambiguity in the reading shown by an accelerometer dial attached to the particle. The dial can be read from any frame of reference. The other obsrver should read the same dial.

It has been established that the Lorentz transorm is not the right transform for inertial particles[7]. There is an extensive literature on invariant equations []. The equations tend to be abstract and difficult to interpret. It is possible that a satisfactoy transform has been discovered but has not been recongnized.

The calculations of the Thomas precession[] do not distinguish between the virtual acceleration terms and the acceleration measured by an accelerometer attached to the particle.

The midpoint is at the middle when the motion of the distant observer is radial, but the special case is not of much interest. The solution is otherwise for a ficticous particle. On the other hand, our frame of reference is the only one we can ever measure anything in. The best that can be done is to understand the idosyncyry of our perspective.

Eqs – represent a family of light cone events. The events are not all for the same particle. The midpoint of a time interval at the field pont does not map into the midpoint of the corresponding time interval at the particle. The nonlinearity of the transfer function is shown in Eq () and in Fig. 1.

The coupling is a consequence of the non-orthogonality of the time and space coordinates. They are not independent variables in light cone solutions, so the chain rule for differention is required. The tenosor of each rank is irreduciblre [], so it is not possible to fully orthogoanlize the coordintes with the chain rule, but it can be done in stages. The zeroth rank tensor is a scalar, and the Coulomb solution is the retardation equation in that order. The solutons can be differentited, but the derivatives are degenerate. The LW equatons are for the tensor of the first rank, a vector. The solutions can be differentiaed, but the derivatives are degenerate, becasue the gradient of a vector is a tensor of the second rank. The contravariant tensor of the second rank represeents the first derivatives. Its decomposition products are a scalar, a vector, and a quadupole []. The scalar is the Lorentz condition, ... The scalar is always zero in the solutions of the LW equations. The vector is the E field, ... According to the LW equations, the magnetic field is a transformed E field, so it does not have an independent role. It is possible that the magnetic field has a more general interpretaion.

There is no known need for the quadurpole in 4-potential equations, but it is there unless the Lorentz condition is zero. The analogs of the elongation and shear terms contribute to the quadupole. They are not mathemarically separable [], but they do seem to have distintuishable intuitive meanings, and their meanings would be more clear in the decomposition products of the tensor of the third rank, which are a scalar, three vectors, two quadupoles, and an octopole. As will be develped later, the scalar is $\boldsymbol{\nabla}\cdot\boldsymbol{E}$. The scalar is always zero in the solutions of the LW equations. There are too many zeros in those solutions.

Since the time and space coordinates are not independent variables in light cone solutions, the chain rule for differentaion is required. The location of a particle at time dt_f is

$$\boldsymbol{R} = \boldsymbol{R}_0 + \frac{\partial \boldsymbol{R}}{\partial t_f} dt_f + \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{R}_0} \cdot \frac{\partial \boldsymbol{R}_0}{\partial t_f} dt_f$$

The second term in this equation can be dropped when the velocity is low. $\partial\boldsymbol{R}/\partial\boldsymbol{R}_0$ stands for a tensor of the second rank. The equation reduces to the form used for the solutions of the LW equations when the tensor has no signifiant off-diagonal terms, which is the appropriate form at low velocites. The equations could be applied recursively if the potential solutoin needs to be diffrented twice, which is frequently the case, but it will be better to proceed one step at a time. The second derivatives of the first step can still be computed after the solution is obtained, but they will be incomplete.

It is likely that a more appealing representaion exists. Obtaining more terms in the series should be helpful in inferring its form.

3+1 space exists only in the mind, but it is a convenient space.

The Rvdot in Eq – would be the correct value in an orthogonal coordinate system, and the coordinaes are indeed approximately orthogonal at low velocites.

The angular and translational velocities have been decoupled. Whether or not they need decoupling depends on the perspective.

It is for this reason that the solution for the midpoint of the time interval in Eq – cannot be linearly interpolated from the other two solutions. The three solutions are not connected by a linear equation. It is not a linear eqation becuse the solution is not for an inertial particle. The location of an unaccelerated particle is linear in time.

The coordinates are not orthogaonal in light cone solutions, but one small step in space, along with one small step in time, is the same as one small step in space-time. The space-time cross term vanishes in the infinitesimal of the first frame of reference. It does not vanish in the second frame of reference. The second frame of refernce could be our frame of reference next time, so it also vanishes in the second system when the coordinates are first-known there.

The equations are linear for small steps in our frame of refernce. There is a space-time cross term in the second frame of reference. However, the equations are also linear for one small step in space-time in the second frame of reference if the coordinates are first-known in the second frame of reference. The retarded potentials are first-known in the other frame of refernce. It is the perspective of the other observer that matters.

The second step works in the same way as the first step, but it is not possible to take the second step in either frame of referene without the chain rule for differentiaon.

Now that the coordinates are first-known at time dt, the second infitesimal step would work the same way as the first step. The second step is not needed when the vx4 terms are small enough that they can be neglected, but the solutions will contain multipole terms that are not present in the solutions for the first step.

A clock in our frame of reference is not a satisfactory time base for separating the angular and translational velocities of light cone events. On the other hand, our clock is the only one that we have.

In our frame of reference, the radius vector to the particle rotates as the particle moves. From the perspective of the other observer, it rotates at a different rate than we think it does. There are irreconcilable differences. The retarded potentials are first-known in the other frame of refernce, so it is the perspective of the other observer that matters for them. That would not be the appropriate perspective for most calculations.

The acceleration terms can be neglected when only the first derivatives are needed. The Maxwell equations are in terms of the second derivatives, so there will be some inconsistencies.

The midpoint of a short time interval is closer to the middle when the interval is short, but it is never exactly at the middle. That can mean that the chain rule for differntaion is needed.

The meaning of these calculations is that the chain rule for differention is required when the variables are not independent. The chain rule was not used in the calculations, but there is usually more than one way of obtaining a solution.

t_s has dropped out of the solution. We do not need to know the time in either frame of rererence to compute the derivatives of the retarded potentials. They are special in this respect. However, in being the integral of the fields, they are arbitrary to within a constant of integration. This solution is therefore not necessarily unique. We do not know where the particle was yet, but when applying retardation equations to specific configurations, it is necessary to assume that we already know where it was.

Rudot dot ru is 1 when the motion is radial. Substitutng Rudot ru = 1 into eq –

The equatons are the same when the motion is radial. The diffrence between eqs – and – occurs because the gradient of a vector is a tensor of the second rank.

Equations have to free of first order error before they can be integrated, as the error will otherwise grow without bounds when the equation is integrated. Only first order equations can be reduced to first order in one step, but it a necesary step in the progression. The order of the four dimensional space is not yet known.

tf0 does not drop out of the solutin, so there is an error that exists in the first order of the infinitesial. The error will grow without bounds when the equations are integrated, but it will be small in observaions of short duration. tf0 can be set to zero in those solutins.

These relationships could be experessed more elegantly with the chain rule for differention. The chain rule is always required when the variables are not independent, although it does not necessarily need to appear explicitly in the equations. The essential consideation is that, after extrapolation to a nearby point, a vector is still a vector, but the two vectors are connected by a tensor of the second rank. Similarly, three closely spaced vectors are connected by a tensor of the third rank []. The tensor of the third rank is irreducible in the sense that the solutions will contain multipole terms that are not representable with the first derivative.

Discussion

As is more clear in the form shown in Eqs (), the solution mostly represents the first derivative, even though it does not look like a first derivative. We do not yet know where the particle is, but it is necessary to assume that we already know where it is when applying retardaion equations to specific configurations.

While the transfer functionn from the time at the source to the time at the field point is not linear, any short section of it is approximately linear. Indeed, even quite a long path is linear enough for the LW equations, and there is nothing wrong with that when the solutions are accurate enough. A shorter section is not perfectly linear, and the problem becomes harder when the particle is closer. No matter how short the path is, the midpoint is not perfectly at middle. There is no known way of reducing the four dimensional space to first order, although it can be orthogonalized in stages.

While the mapping is not linear, a short inteval at the source maps into a short interval at the field point, and the mapping is approximately linear for short sections. We do not need to know the time in either frame of refernce to compute the derivatives of the retarded potentials. But because we do not yet know the time shown by either clock, the equations of the retarded potentials are inept in many ways. They are just one perspective of a much larger problem.

The light cone equation has to be solved twice so that both the function and its first derivatives will be defined.

dru dot ru0 is $plusminus1$ for radial motion. dru drops out altogether in that case.

The retarded potentals exist only for the purpose of being differentiated, so the terms multipled by tf0 are in the same order as the solution for the fields. It is assumed in the following calculations that the error introduced by setting them to zero is small. They are set to zero, but it is indicated that the solution for the second derivaties is needed.

The light cone equation must now be solved a second time so that both the function and its derivatives will be defined. The retarded potentials exists only for the purpose of being differentiated, so the derivatives are more important than the function.

After differentiaoon to obtain the fields, the tf0 terms in the solution are in the same order as the fields, so they should be carried, but they cannot be carried when applying the equaions to specific problems. It is indicated that the second derivatives are needed. Since the LW equations have the same problem, it is likely that the error from setting the tf0 terms to zero is small.

This should be a linear equation. The quadradic terms make it seem that the particle is accelerated when it is not. In some sense, the quadradic terms are real, but they do not belong in the equations for the retarded potentials.

Sadly, we have never known where the particle was. We have always assumed that the particle is either approaching or receding from us. That is not true. The transverse velocity matters, expecially in tiny systems.

That is computaionally convenient, but it is also a weakness, because there are many calculations where we do need to know what time its.

The first derivative at a displaced point is normally one of the contributors to the second derivative. But in this case, since the equaions must be form invariant, it must not matter when the frirst derivative is computed, so it is only what it looks like – a first derivative.

The error at the midpoint can be neglected when the v^3 terms of the vector equation are not needed. There is nothing wrong whe the LW equations when they are applied within their range of validity.

We are always one step behind in 3+1 space. A calculaion that would normally be for the second derivtive is only for the first.

The LW equations have the same problem, so there is a precident for neglecting the residual, even though it does not vanish properly. First order terms do not vanish at all, but they can be small enough to neglect.

Unsurprisingly, the LT has the same problem. That is beause the LW equations can be deriived with the LT. The basis of the problem is that we cannot tell the differenc bewewen angular and translaions velocitits in obsertions of short duration if the signal is delayed by the light timie accorss the system. There are two velocities in a problem where there would be only one if the speed of ligth did not matter.

The basis of this relationships is that the midpoint of time interval in the second frame of reference does not map into the midpoint of the corresponding interval in the first system. That represents a form of curvature that has no meaning in 3-space. The midpoint is closer to the middle in short time intervals, but is is never perfectly at the middle. The decrepancey is small, but of first order, no matter how short the interval is. The behavior is different than in 3-space, where curvature vanishises in the limit. However, the 3-space curvature equation is singular for straight lines.

The infitesimal transform can halt a particle in a single step, no matter how high the velocity of the particle is. The equations should not work this way. The infitesimal transform is only valid at low velocites.

There is not necessarily anything wrong with being one step behind, but the consderations are not the same as for the leader. The equations have to look like second derivatives to compute the first derivative. They have to look like third derivatives to compute the second derivative.

To order vxx, this solution reduces to the Lorentz transform when the motion is radial. The midpoint is at the middle of the time interval in both frames of refrence for radial motion.

The trajectory seems to be cureved in a way that has no meaning in 3-space. The trajectory of an isolated an unaccelerated particle is not actually curved. The solution is not for an inertial particle. As has long been known [], the Lorentz transform is not for inertial particles.

The basis of this abberation is that the midpoint of a time interval in the first frame of refernce does not map into the midpoint of the coorespoinding interval in the second frame of reference when there is a transverse velocity component. The abbrration is real in some sense, as an observer would not perceive a partile as moving at a uniform velocity when there is a transverse velocity component and the particle is nearby. While it is possible to view the abberation as being real, that is not the perspective of the retarded potentials.

As has been known from the beginning, the Lorentz transform in not for inertial particles [].

In many ways, retardation equations have an inverse meaning to field equations. They do have one thing in common. The solution should be for an inertial particle.

No matter how high the velocity of the particle is, the infinitsemal transform can halt it in a single step. The solution looks like it is exact, but of course it is not.

For observations of short duration, the composite of an angular and translational velocity can be viewed as being the sum of two translational velocites, although we are not yet sure of which is which.

There is no upper bound for angular velocity, so the problem becomes harder when the particle is closer.

The solution reduces to the Lorentz transform when the motion is radial. The midpoint is at the middle in those solutions.

Like the Lorentz transform, this solution holds the quantiy r dot r - cx2 cx2 invariant. There is an extensive literature on invariant coordiante transforms []. It is unlikely an invariant transform exists that was not discovered long ago, although the literature does tend to be abstract and difficult to interpret.

The velocity of the tip of the vector from the other observer to the field point is v0 and its acceleration is zero. It is from the perspective of an observer at the tail of the vector that the motion of the particle is not perceived as being uniform along the straight-line trajectory. These two perspectives are not distinguishable when there is no propation delay, but 4-space equations without a propagaion delay are not actually in 4-space.

Thus, the Lorentz transform is not for an inertial particle, and for the same reason that the light cone solution is not. The midpoint in one frame of reference does not map into the midpoint of the other frame of refernce. It has been known from the beginning that the Lorentz transform is not for an inertial particle [].

It is not sufficient that equations hold the speed of light invariant. They must also not depend on the choice of a coordinate system. Since the transfer function is not linear in time, projecting the time shown by a clock in our frame of refernece onto the second frame of reference causes the solutions to become coordinate-dependent if there is a transverse velocity component. The midpont remains at the middle if the motion is radial.

The curvature vanishes when only the v^1 terms are carried. The infinitesimal Lorentz transform represents the v^1 terms, and it can be applied recursively.

The solution is the LW solution when tf=0. However, the solution depends on what time it is, and we have no way of knowing what time should be used. Retardaion equation have to work without knowing the time.

There is an extensive literature on invariant coordiante transforms [], and it is likely that there is a satisfactory transform for obtaining the equations of the retardaion potentials, but it is not the Lorentz transform if the solution is to be for inertial particles.

There are no longer any tvx1 terms in the solution. That would be acceptable if only the first derivatives were needed. The Maxwell equations are in terms of the second derivatives.

The Lorentz tansform is not for inertial particles, and for the same reason that the light cone equation is not. The midpoint is not at the middle when there is a transverse velocity component.

We have no way of knowing where on this curve an isolated light cone event is. Retardaion equations have to work in the same way at any point on the curve.

The Lorentz transform has the same problem, and for the same reason. The midpoint of a straight line is not at the middle. .. The transfer function is linear for radial motion, but that subset of the solutions is of limited interest.

Due to the nonlinar transfer function from the source to the field point, the equations of the retarded potentials have to be usable without knowing which region of the nonlinear curve the solution is for. The rigth region would be identifable in global solutions, but the global solutin is not yet known. The retarded potentials exist only for the purpse of being differentiated, so it is sufficient the the derivatives be correct for any region of the curve. Conversely, in being the integral of the fields, potential equations are arbitrary to within a constant of integation. The potetial solution obtained below is therefore not necessarily unique. The solution obtained does not specify where the particle actually is, wheras an invariant coordinate transform should be able to specify its location, at least in differental form.

It can be inferred from Fig 3 that the second derivative become smaller, relative to the first derivative, when the time t=0 is further in the future. That makes it possible to predict when the time when the particle will be at its closest approach. There are obviously other considerations for accelerated particles, but the velocity terms come first.

It would be easy to carry more poweres of velocity in this calculaion, but it might be that the third derivative should not be neglected, so more accurate soltions will not be considered at this time.

This solution is for one space dimension and one time dimension. With this simplificalion, the trajecrory of particle is always conincident with the field point at some point in the future. That is not true when there are two space dimensions, so the solution for two space dimensions will need further analysis.

Fig .. The geometry of light cone events. This figure represents the light cone events from the perspective of an observer at the particle. The construction is not usable at the field point because time does not progress at a uniform rate at the field point in this construction.

In the construction as shown, Rvdot is the same of all of the events, but R is longer at earlier times. The Rvdot/R term in Eq – is therefore not an invariant equation. There is an extensive literature on invariant equaions [], and it may well be that a satisfactory cooreinate transform exists, but it is not the Lorentz transform.

The particle emits a signal at time t=-r0/c. The light ray arrives at the field point at the time tf=0. The particle arrives at the tip of the vector r0 at the same time that the signal arrives at the field point. When there is only one space coordiate, the particle is conincident with the field point at some point on the trajectory. That is rarely the case when there is more than one space coordiante.

The figure assumes that the location of the partile at time t=0 is already known. There is no simple way of knowing where the particle actually is then, but when applying retardation equations to specific configurations it is necessary to asume that the location is already known. However, in being the integral of the fields, potential equations are arbitrary to within a constant of inegation. The soluton obtained is therefore not necessarily unique, nor is it necessarily true that we know where he particle actually was.

? tf0 and tf1 have dropped out of the solution, so it does not matter what time it is.

Despite the different angles and lentgths of R for the events in the figure, the equation for the retarded poententials has to look the same for all of them. It has to be an invariant eqquition. The LW equations are invariant, but only at low velocites.

These calculations can be extended to three or more virtual particles.

discussion

At time $(\Delta t)^2$, the real $at^2/2$ terms are in the same order as the v^2 terms, so they should be carried in periodic solutions for small systems. The virtual $\dot{\boldsymbol{a}}$ terms are small but in the same order as the real acceleration terms, so they should be taken out when good accuracy is needed. The Newton series is not an orthogonal series when the speed of light matters. Individual terms of the series, considered in isolation, do not have a meaning.

In many ways, retardation equations have an inverse meaning to field equations. A false term from one perspective can be a neglected term from a different perspective. The perspectives do have one thing in common. The solution should be for an inertial particle.

Since the trajectory is perceived as being curved, it is necessary to approximate it as a series of stratigh-line segments. The segments are all parallel to each other, but their lengths vary. A low velocities one segment is sufficiently good approximation, and the solution in that order is the LW solution.

There are quadradic terms in Eq –. No matter how small dt is in our frame of reference, it is not small enough to satisfy the other observer. Dividing dt into two steps is enough to represent the lowest order curvature terms.

it is also not zero when the retarded Newtoian acceleration is not zero. When obtianing solutions for the retarded potentials, we do not need to know which case is which. There are other circumstances where we do need to know the difference. Retardation equatins are just one perspective of a much larger problem.

Since the time and space coordiantes are not independent varialbes in light cone solutions, it should be possilbe to subdivide the infitesimal with the chain rule for differentiaion, as it is capable of converting terms that look like first derivatives into second derivatives. There is always more than one way of working a problem.

The transfer funcion is linear for radial motion, but the special case is not of much interest.

These calculations are based on the premise that it is not possible to transform to the second frame of reference until the coordintes are known in the first system. There is a possibity that the same relationships can be interpreted as meaning that the Lorentz transform is not the right transform for inertial pariciles. There is an extensive literature in invariant coordinte transforms []. In any case, it is the transformations amongst the perspectives that matters, no which one is the right one.

The segments are all parallel to each other, but their lengthts vary in a way that has no meaning in a pure 3-space.

While the transfer function is linear in time, the linearity is misleading. A radially approaching source will eventually coincide with the field point. Assuming a near miss, the higher doppler shifted frequency of an approaching transmitter instantly switches to the lower but still constant doppler frequency for a receding transmitter. The LW equations are form invariant for either portion of the trajectory, but they are not capable of bridging the discontinuity between the two segments.

There is no discontinuity when the motion is not radial, but then there is a transverse component to the velocity.

The retarded potentials depend on both the velocity and the location of the particle, and two closely spaced points on the trajectory are sufficint to defefine them. However, the retarded potential exist only for the purpose of being differntiated, and three points on the trajetory are required to represent the first time derivative of the potential solution. It will be computationally convenient if the differentions can be performed without having to consider the fact that the midpoint of the three points on the trajectory is not at the middle. The differentaions could be performed anyway, but the chain rule for differention would be required. Potential solutions that do not require the chain rule for computing the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields will be easer to apply.

More than two straigh-line segments would be requird if the second derivatives are needed. The Maxwell equations are in terms of the second derivatives, but it is sometimes best to proceed one step at a time.

At low velocities, it is only the radial velocity that is important. The doppler shift depends mostly on $\dot{R}$.

As shown in Fig –, at high velocities, the differnce between a scalar and a vector becomes important. $\dot{R}$ is no longer a good indicator of the doppler frequency. $\dot{\boldsymbol{R}}$ is now required in the equations. The assymetry between negative and positive times is due to the propagation delay from the source to the field point. The quantity that matters is where the transmitter was, not where it is. The assymetry between negative and positive times is responsible for the transverse doppler effect[12]. The assymetry is of order v^3, but the doppler shift is of order v^1, so the transverse correction is of order v^2. (The figures are based on exact calculatins. The v^3 calulations shown elsewhere are not accurate enough.)

These figures would apply equally well to doppler shifted monochromatic acoustic signals. Acoustic signals are not in 3-space. They are in 4-space.

The LW eqations are form invariant for either portion of a radial trajectory, but they do not provide a means of connecting the two portions.

The basis of this inconsistency is that the LW equations are not the right equations for inertial particles unless the motion is radial.

The shift in the midpoint is small when the three points are closely space, but it is in the same order as the retarded Newtonian accelertion of the particle.

The time and space coordinates are not independent variables in light cone solutios, and the chain rule for differentiation is generally required when the variable are not independent. The chain rule could be used to apply a correction for the midpoint shift.

When the velocity is low, the scalar $\dot{R}$ is a good indicator of the doppler shift, even when there is a small transverse velocity.

The LW equations are form invariant for either portion of the trajectory. There is a discontinuity between the two segments that no differentiable equation can bridge.

The equaions obtained this way represent a projection of 4-space into 3+1 space. While 4-space can be projected into 3+1 space, the reverse projection is incomplete. 3+1 space is not a real space, so the equations are unphysical.

It will be computaionally convenient to obtain approximate potential equations that do not require the chain rule for computing the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields.

The doppler shift is too small to be easily visible in the figure, but if pulses are transmitted at regular intervals by the source, they will by doppler shifted to a slightly higher frequency when the particle is approaching on the left side of the figure, then shifted to a lower frequency when it is receding on the right side. The LW equations are a good approximation in this regime. They do in fact represent the transverse velocity terms. The transverse terms are responsible for the magnetic field, but they are only accurate to the extent that the rate of change of a doppler shifted frequency can be represented with the rate of change of three scalar $\dot{R}$ terms in three orthogonal directions, considered separately. In 4-space, the rate of change of a vector is not necessarily representable with vector equations.

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While the terms of the series in in Rv are mutually orthogonal in the first frame of reference, they are not orthogonal in the frame of reference of the particle. The lack of orthogonality results in an unfamiliar coupling between the orders. The dt interval has to be subdivided into two steps if the potential solutions are to be differentiated once. It would have to be subdivied into three intervals if the second derivative is needed. When working in 3+1 space, we are always one step behind.

In more elaborate problems, where there are both real and virtual accelearation terms, the virtual terms usually mean that we do not yet know where the particle was. This perspective would be appropriate for orbit detrmination. Orbit determination will be more difficult when the retarded virtual jerk is significant.

There are other problems where the trajectory of the particle is artifically constrained, such as in rotating electrical equipment. When the location of the particle is already known by independent means, the virtual acceleration terms need to be taken out. In more extreme circumstances, the retarded virtual jerk must also be taken out.

The light cone equation can be solved with t_s being the independent variable. If that is done the pulses would be transmitted at regular intervals but received at irregular intervals. The LW equations can be derived either way. The retarded potentials depend on both the velocity and the position of the particle, so their first derivative is the second derivtive of position. It should be possible to work the problem either way, but the irregularly spaced pulses at the field poilnt would need consideration in computing the second derivative. In 3+1 space, we are always one step behind. What looks like a first derivative is actually a second derivative.

Being one step behind is not a serious condition, but the methods are not the same as for the leader.

No matter how how short the inteval is, the midpoint at the field point does not map into the midpoint of the interval at the particle. The midpoint shift is small when the interval is small, but it is in the same order as the retarded Newtonian acceleration.

The retarded potentials are not physical quantities. It is only their derivatives that are of interest. That makes it possilbe to append a virtual velocity term to the retardaion equatins, which, after differentiation, takes out the virtual acceleration terms.

An approach better aligned with physical concepts would be to use the chain rule for differention to correct for the midpoint shift. The calculations have not been carried through, but it should be possilbe to work the problem that way.

It is possible to solve the light cone equation with t_s being the independent variable. The pulses would then be transmitted at regular interavals, but they would be irregularly spaced at the receiver, and the same problem would be present from a different perspective.

This behavior seems contrary to the theorms of Eucldian calculcus, but it is not, because the chain rule for differention is one of those theorms. The chain rule is required when the variables are not independent, and they are not independent in light cone solutions. The chain rule is capable of converting terms that look like first derivatives into derivatives of any order.

The time shown by a clock at the field point is obviously not a good choice for a time standard when evaluating the kinematic behavior of the particle, but the observer has no other option unless the retarded velocity of the source is known by independent means.

The $\dot{\boldsymbol{a}}$ terms are degenerate with the Newton equations. The $\dot{\boldsymbol{a}}$ terms are not zero in the solutions, but it is not possible to carry them when integrating numerically. There is no known way of reducing the four dimensional space to first order, so it follows that the $\dot{\boldsymbol{a}}$ terms, whether virtual or real, are not degenerate in 4-space.

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Since the potentials are the integral of the fields, a more accurate solution can be obtained by integrating before differentiating. The simplest possible form of integration consistes of approximateing a curve with two straight-line segments. While the midpoint shift is not representable with the first derivative in the first frame of reference, it does influence the integral. That makes it possible to subdivide the infinitesimal. The subdivision is only possible with potential equations, as the subdivisions would otherwise be linearly dependent. The midpoint is always at the middle when the relationhips are linearly dependent.

The actual meaning of the infinitesimal is that the relationships are of first order. Infinitesimal quantities can be of substantial magnitude, yet not be capable of subdivision because the relationships are linearly dependent.

The following calculations do not explicitly use the chain rule for differentiation, but recursive applications of the chain rule is the actual mathematical basis for the calculaltions. The chain rule is capable of converting terms that look like first derivatives into derivatives of any order.

The midpoint shift results in a virtual retarded Newtonian accelertaion term that does not represent the behavior of an inertial particle.

The retarded potentials are not physical quantities. They exist only for the purpose of being diferentiated, so the potential equations should be compensated for the midpoint shift so that they will differentiate correctly.

In the frame of reference of the field point, the retarded position vector represents three copies of the scalar $\dot{R}$ in three orthogional directions. The same is true for the advanced position vector in the frame of refence of the particle. However, the retarded and advanced position vectors are not connected by a vector equation.

The retarded potentials are first-known in the frame of reference of the particle. It is the perspecive of the other observer that is relevant for their derivation. From the perspective of the other observer, the advanced potententials at our location would exist without our presence.

The differences between the two doppler curves are subtle and not important at low velocites. At low velocites, the connection between the advance and retarded position vectors can be aproximated with vector equations, and the LW equations are a satisfactory approximation in that regime. Indeed, the velocity of conducton electrons in wire is so low that there is no possibility of detecting the difference in those configurations. The behavior of electron beams near the wires is a different problem.

All of the data shown in the figures is based on the exact root of a polynomial. The series calculations shown elsewhere are not accurate enough for the figures.

The midpoint shift introduces a false acceleration term into the solutions for the E and B fields. Four points on the trajectory would be required to compute the second derivatives of the potentials. The Maxwell equations are in terms of second derivarives.

The midpoint shift does not affect the retaded velocity of the particle. After diffeentaion to obtain the the fields, vecocity terms become acceleration terms, then the midpoint shift introduces false Newtonian acceleration terms.

The basis of the inconsistency is that the LW equaions are not for an inertial particle. They assume that an unaccelerated particle is half way to the destination in half the travel time. For light cone solutions, that is only true for radial motion.

It is for this reason that the solutin in Eq cannot be obtained by linearly interpolating the other two solutins. The three solutions are not connected by a linear equation, but the LW equations assume that the connection is linear time.

By dropping the quadradic terms, the time intervals become of first order in both frames of reference.

The LW equations are only exact in 1+1 space. When the transverse velocity is low, 3+1 space can be approximated with the three scalars ..., but there are cross terms in the derivatives that require the chain rule for differentiaon when the velocity is high.

It would obviously be necessary to take more steps if more powers of velocity were carried.

The velocity of a particle can be decomposed into a component v ru parallel to the line of sight and a component dv in a different direction.

The solution shows that the time and space coordinates of the doppler equation are difficult to separate.

The solution shows that the perceived acceleration of a particle depends on both its distance and the angle of the trajectory.

The basis of the inconsistency is that the LW equations assume that the particle reaches half way to the destination in half the travel time. Isolated and unaccelerated inertial particles do behave that way, but light cone solutions do not when there is a tansverse velocity component.

Since the trajectory is perceicved as being curved, it needs to be represented by a series of straight-line segments. Two segments are better than one segment.

This solution assumes that the location of the particle is already known. That is the appropritate perspective when the trajectory is artifically constrained or otherwise independently obtained, as in the soltuions for rotating electrical equipment. It would not be the appropriate perspective for orbital calculations.

As with the Lorentz transform, there are cross terms in the light cone solution for the sum of two velocites. Vectors do not add as vectors. In 1+1 space, the sum of two velocites is still one velocity. The cross terms of 2+1 space are not representable in 1+1 space. There are other cross terms in the solutions.

The solution for a low transverse velcity is shown in Eq –. There is some doppler shifth in the figure, but it is barely visible. Even though the LW equations belong in 1+1 space, they are a usable approximation in this regime.

A time interval at the field point maps linearly into the time interval at the particle when the motion is radial. Equivalently, a doppler shifted frequency is constant for radial motion.

None of the solutions are affected by translating the coordinates in either time or space.

The angular velocity and the perceived Newtonian acceleration are coupled in light cone solutions. It is not possible to quickly determine which is which unless the retarded velocity is known by independent means.

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There are essential differences for the light cone equaton in 2+1 space. There are related differences for the doppler frequency. The doppler shift is constant for radial motion, but it is constantly changing when there is a transverse velocity.

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The cross terms between radial and transverse velocites are visible in the 3+1 space derivatives. Light cone solutions in 2+1 space are not representable with two copies of the 1+1 space equations.

Solutons to the light cone equation contan ... cross terms in the first frame of reference. The terms are similar in form and meaning to the Thomas pdrecession []. The solution shows that light cone solutions in 2+1 space are not representable with two copies of the 1+1 space equations. The rotations of the Lorentz group [] represent the 4-space angular relationshops in a more general way.

The calculations of the Thomas precession[6] do not have to be for light cone events, but they can be, in which case transforming the wrong equation to the second frame of reference confounds the problem. The $av\times\boldsymbol{v}$ terms of the Thomas precession exist in the first frame of reference when the speed of light matters. That is as it should be unless our frame of reference is special. It does seem special, and it actually is special in some ways, because it is the only one we can ever measure anything in. However, the retarded potentials are first-known in the other frame of reference.

At low velocites, the vector sum of the rate of change of the three scalars $\dot{R}_x$, $\dot{R}_y$, and $\dot{R}_z$,condsidered one at a time, is a good indicator of the rate of change of a doppler shifted signal. The cross terms can be neglected at low velocities. The chain rule for differentiation would be required at higher velocites. The chain rule is not needed when there is only one space coordinate.

Due to the presence of cross terms in the derivatives of the light cone equation, the chain rule for differentiaon would be required at higher velocites.

At low velocites, the doppler shift can be accurately computed with the three scalars The relativistic doppler equation[12] is a good approximation in this regime. For the same reason, the LW equations are also a good approximation in this regime.

The midpoint shift does not affect the retarded velocity. However, after differentiating the potential solution to obtain the E and B fields, velcoity terms become acceleration terms, and three points on the trajectory are required to represent them. The midpoint shift introduces false acceleration terms into solutions for the retarded fields.

The retarded potentials are not physical quantites. They exist only for the purpose of being differentiated. The solutions should show the retarded Newtonin acceleration to be zero when it is already known to be zero. The false accelereation terms can be canceled by appending virtual acceleration terms of the opposite sign.

The correct interpretatin depends on the circumstances. When the conditions are mild the distrance in the equation stands for itself, and equations in that ordee are the LW equations. When the solution is more demanding, the qantity in the solution is int Rdot dt. Under still more exterme coneditions the solution shoud be ineterpreted as int (int Rddot dt) dt.

The constant of integration has to be determined from physical principles. The retarded Newtonan acceleration should be zero in solutions where it is known to be zero.

When the solution is a double integral, the first constant of integration is not a constant after the second integration.

While angular and translational velocities do not need separate consideation with vector equations in the first frame of reference, the distinction is important for the other observer.

This solution is for the first derivative. The Maxwell equations are in terms of the second derivatives, but it will be best to proceed one step at a time.

One of the considerations is that the first constant of integration in a double integral is not a constant in the solution. There appears to be a connection to the rotations of the Lorentz group ... . Those equations may provide a more reliable basis for the next step.

It is not normally necessary to distinguish between angular and translational velocities in the first frame of reference. Indeed, there is really no point in it when the speed of light does not matter, but the distinction can be importaint in more general solutions.

It is not necessary to distinguish beween angular and translational velocites in 3+1 space. There is only one velocity associated with a particle. But because the two kinds of velocity are coupled in light cone solutins, it is necessary to formulate the problem in such a way that is is possible to determine which is which in order to separate them. The chain rule for differentaion is not necessary when the variables are independent. For short times, angular and translational velocites can be represeted as two translational velocites. The pivot point for the angular term cannot be at the field point. One point is not a satisfactory angular reference.

The solutin in this form looks like it is exact, and its compactness can be convenient, but it is no more accurate than Eq ().

In the calculations for the Thomas precession in Ref – contain an error. The calculations assumed that velocities add as vectors in the light cone equation. That is not true. There are cross terms the the soluiton. The error is corrected in the following calculaions.

There is a close connection between this solution and the calculations of the Thomas precession []. The Thomas calculations project 3+1 space into 4-space. This calculation projects 4-space into 3+1 space. The equations are essentially the same either way, but the meanings are different.

Fig. 3 shows the same data as Fig. 2 with the same scales but half the velocity. The false acceleration varies as $(v/c)^3$, so it is only signifiant at very high velocites. On the other hand, since the doppler shift is of order v^1, it requires a v^2 correction to the doppler shift.

?At very low velocities the false acceleration term is insignificant but at a maximum at the point of closest appoach. The maximum moves to progressively earlier times at higher velocites.

Figs. 4 and 5 show the data from other prespectives.

The Newtonian retarded av and avdot terms are in the same order as the calculaions of this section. The velocity terms are of lower order, and it will be best to proceed one step at a time.

This equation applies to the solution at the tip of the radius vector $\boldsymbol{R}$, as interpreted from delayed observations. An observer at the other end of the vector would see the world differently.

The cross terms between the radial and transverse velocites depend so strongly on the velocity of the particle that they can usually be neglected.

The false acceleration terms are in the same order as the Newtonian $\dot{\boldsymbol{a}}$ terms, indicatating that they could be cancelled by appending false $\dot{\boldsymbol{a}}$ terms. However, the false terms should not be in the solution in the first place.

There is an extensive literature on invariant coordiante transforms []. It is likely that this solution is obtainable in other ways.

The terms of the Taylor series look the same as those of the Newton series, but the Taylor theorm has a more general meaning.

This solution is in 4-space. The Newton equations are in 3+1 space. 4-space can be projected into 3+1 space, but the reverse projection is incomplete.

Direct calculaton shows that the vxn terms should not be carried without also carrying the avddot terms.

The particle is on the light cone, but it is not the same particle as in Eq –.

No matter how small Δt_f is, it is not small enough to cause the midpoint to be at the middle, as there are still terms quadradic in Δt_s in the light cone solution.

A smaller interval could be selected by dropping the quadradic terms, but then the solution would missing terms at time dt_f, because the dt_s interval is not linear in dt_f. Light cone solutions cannot be reduced to first order in the same way that the equations of 3 space are.

Straight lines in 3-space do not have tick marks at regularly spaced intervals, making it impossible to determine when they have been streteched. In 4-space, we know when they have been stretched.

It becomes necessary to subdivide the infinitesimal dt_f interval. The corresponding segments of the trajectory are all parallel if the retarded Newtonian acceleration is zero, but their lengths vary. In effect, it is necessary to integrate before differentiating. The midpoint of the curve represented by several straight-line segments is still not at the middle, but the equations are otherwise of first order, and the first order relationships are recovered after the potential solutions are differentiated.

At low velocites, it is not necessary to subdivide dt_f in order to obtain satisfactory accuracy, and the equations of that order are the LW equations.

To the extent that the curvature of the trajectory can be approximated by two straight-line segments during the dt_f interval, the particle is now on the light cone at time dt_f.

The discontinuity goes away when the motion is not radial, as the particle will always miss the field point when there is a transverse component to the velocity.

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The solution is of first order in dt_s, but it contains terms quadradic in Δt_f. From the perspective of the other observer, the equations are not of first order, and there would be no point in subdividing dt_s, as the equations would be linearly dependent. The retarded potentials are first-known in the other frame of referene, so the perspective of the other observer is important.

The equation can be reduced to first order by dropping the terms quadradic in dt_s. But from the perspective of the other observer, the solution would be missing terms, because the quadradic terms belong in the solution, no matter how small dt_s is. The equations can be reduced to first order from both perspectives by integrating over the dt_f interval.

In performing the integration, the curved trajectory can approximated with just a few straight-line segements. When the equations are of first order from both perspectives, the midpoint is at the middle of each segment. One segment is sufficient when the velocity is low enough. The equations of that order are the LW equations. No segments at all are needed at the still lower velocites where the Coulomb soltuion is adquate.

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The potentials are the integral of the fields. The integral is already known, so there is no point in differentiating and then integrating again. The considerations would be different for orbital calculations.

In orbital calculations, the derivatives are integrated to obtain the loction of the particle. Retardaion equations assume that the location is already known, then the derivatives are computed. There are many case where the location can be independently obtained, such as in the solutions for rotating electrical equipment.

Potential equations are arbitrary to within a constant of intgration, so the solution obtained this way is not necessrily unique. It is the nature of retardaion equations that the constant of integration depends on where the particle is, relative to the observer, and we are not sure yet of where it is. Alternatively, potential equations can sometimes be gauge transformed. Local physical arguments should not be applied to potential equations, because they do not represent locally measurable relatinships. They exist only for the purpose of being differentiated.

An isolated and unaccelerated inertial particle will reach half way to the destination in half of the travel time. The LW equations assume that light cone equations behave the same way. That is not true unless the motion is radial.

The solution represents the value of Rv from the perspective of an observer who cannot tell the difference between angular and translational velocity. The observer would know the difference if the retarded veloctity were known by independent means, but that is frequently not the case.

The cross terms causes the Taylor theorm to behave differerently in 4-space than it does in 3-space.

The Newton series, ... looks like the terms of a Taylor expansion, but the problem is more fundamental than that. The problem is that the Taylor theorem exists in 3-space. Due to the presence of cross terms, it is necessary to carry more terms in 4-space than would be necessary in 3-space. Vector equations do not have cross terms, so the calculations cannot be performed with vector equations.

This behavior is a fundamental property of the four dimensinal space. Perceiving it is an everyday experience. It occurs when the doppler shifted tone of a passing vehicle is constantly changing. The equations for it are not simple.

The midpoint shift is small when the interval is small, but it is in the same order as the retarded Newtonian acceleration. The particle is perceived as being accelerted in the same direction as the retarded velocity.

The midpoint shift does not affect the retarded velocity. But after the potential soltuion is differentiate to obtain the E and B fields, velocity terms become acceleration terms. Three points on the trajectory are required to represent accelertion terms. The midpoint shift introduces false accelerations terms into the solutions for the retarded E and B fields.

In being of order $(v/c)^3$, the midpoint shift depends so strongly on velocity that the LW equations remain usable even at moderately high velocites.

That does not mean that the midpoint shift does not belong in the equations. It is rather that its meaning is ambiguous.

Since Eqs (1) can be derived with the Lorentz transform[8], all of these calculations could be performed equally well with the Lorentz transform. However, the light cone equation should be included in the calculations if the solution is to be for light cone events. (The calculations in Ref.[10] contain an oversight in this respect.)

The retarded Newtonian acceleration terms are in the same order as the calculaions of this section, so they should be included. In being of lower order, the veclcity terms are more important. In the interest of simplicity, the acceleration terms are neglected in this section, but they are included in the calculation of the next section.

For observations of short duration, the signature of a retarded Newtonian that is parallel to the retarded velocity would be the same. The light cone equation does not specify the behavior of inertial particles. The interpretaion of the solutions is ambiguous on short time scales.

The first term of the retardaion series is the Coulomb solution. It is usable in quasi-static solutions when the magnetic field is weak enough that it can be neglected. The second term is the LW solution. It is usable when the symmetric terms are small enough that they can be neglectd. The Lorentz condition, $\boldsymbol{\nabla}\cdot\boldsymbol{A} + 1/c^2\ \partial\psi/\partial t$, is a symmetric vector equation, but is is always zero in the LW solutions.

This behavior is one of the signatures of a four dimensional space. The same effect occurs when the doppler shifted tone of a passing high speed vehicle is perceived as constantly changing. The v^3 term depends so strongly on velocity that it is usually not noticeable, althought it is occasionally obvious, as the acoustic location of a low flying high speed aircraft is not the same as the visual location. Similarly, the minimum of the $\ddot{R}_y$ term in Fig. ... occurs before the closest approach. At low velocites the minimum would occur nearly at the closest apprach.

Since we have no way of knowing where we are in space and time, each observer is free to choose a coordinate system that is centered upon themselves. But when there are two observers in the same problem, both observers must use the same coordinate system. One observer evaluating a system at two widely separated times counts as two observers.

The particle is on the light cone at time tf, but there is no solution for the time tf+dtf. Tbe calculation assumes that the angular velocity does not matter. The assumption leads to a contradiction when the particle is nearby.

The difference between two closely spaced vectors is a vector, but the vector is not computable unless a global vector solution is already known.

This result can be interpreted in two ways. It could be taken to mean that the the Lorentz transform is not sufficiently general, and more genearal invariant coordinate transforms are known. With the other intrpretaion, there is no point in transforming to the other frame of reference until the coordintaes are known in the first system.

It is for this reason that Eq () is inconsistent. The LW equations are not for an inertial particle.

The particle seems to be on the light cone at time t+dt, however the calculation is only accurate to first order. Only first order equations can be reduced to first order in one step. The second derivative is required for computing the first derivative at a displaced point.

The solution is also invariant when Rv and vv are perpendicular. However, the light cone condition does not provide a first order constraint on the magnitude of R at that point on the trajectory. The transverse location of the particle at time $t_s = -r_0/c$ is not accurately constrained.

When $\hat{\boldsymbol{v}} and \hat{\boldsymbol{r}}$ are perpendicular, the point of zero doppler shift does not occur at the point of closet approach. It occurs slightly earlier. The wavelenght stretch is of order dR^2, but $d\boldsymbol{R}$ is $v_0 t_s$, so it is of order $(vt)^3$.

The transverse doppler term[12] has no first order effect on the light cone equation, but it causes the solutions for the LW equations to be for a particle that is not on an inertial trajectory. There will be additional transverse doppler terms when the retarded Newtonian acceleration is parallel to the velocity vector. They will not have a first order effect on the solution, but transverse doppler is not of first order either.

The particle appears to be accelerated. An accelerometer attached to it would not register an acceleration, and the accelerometer dial can be read from any frame of reference. The LW equations are not for an inertial particle.

The first derivative at time t+dt is always different than the first derivative at time t unless the second derivative is zero.

Only first order equations can be reduced to first order in one step. Higher order equations can be reduced to a system of first order equations[2], but not in one step.

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The particle seems to be on the light cone at time t+dt, however the equation is only accurate to first order. Only first order equations can be reduced to first order in one step. The second derivative is required for computing the first derivative at time t. The Maxell equations are in terms of the second derivtives of the potentials.

The retarded potentials exist only for the purpose of being differentiated. It is not sufficient that the particle be on the light cone at time t. It must also be on the light cone at time $t + dt$, but it is not.

$\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{v}}$ is ± 1 for radial motion and the inconsistency vanishes. It also vanishes when $\hat{\boldsymbol{R}}$ and $\hat{\boldsymbol{v}}$ are perpendicular, or when the v^3 terms are dropped. At low velocites, the particle is on the light cone for any angle.

The solutin in Eq could be obtained for the time tf, then the substitution tf=tf+dt made, and the particle would seem to be on the light cone at time tf+dt. To first order, it would in fact be on the light cone, but only first order equations can be reduced to first order in one step. For more general equations, computing the first derivatve at a displaced point requires the second derivative. The second derivative has to be free of second order error. The first order of the infitessmal is free of error, yet differential equations of all orders exist.

The solution becomes invariant if the t_1^2 terms are dropped, suggesting that an invariant solution could be obtained by integration. The solution would require a constant of integration.

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Coordinates do not have an absolute significance. It is only coordinate differences that are relevant in either frame of refernce. The origin of the coordinate sysem in our frame of reference has no special significance for an observer in the other system, and conversely.

The perceived acceleration of the particle depends on both its distance and the angle of the trajectory. The angular and translational velocites are coupled and difficult to separate unless the retarded velocity is known by independent means.

As shown by the dashed line in the top panel of Fig.–, the midpoint shift depends so strongly on velocity that it can be neglected even at moderately high velcites. At low velocities, the magnitude of $\ddot{R}$ is at a maximum at the point of closest approach. The maximum occurs earlier at high velocites.

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The retarded velocity at time $t + dt$ should be $\boldsymbol{v}$, but it is not. The LW equations are not for an inertial particle, which is why Eq – is inconsistent.

There is no mathematical contradiction in this calculation, because only first order equations can be reduced to first order in one step. The equation has to be reduced to first order if it is to be integrated.

The solution seems to be for the wrong particle, and it may be. On the other hand, since we cannot know what is happening at the simultaneous point while it is happening, it is difficult to be sure of which particle is the right one.

Even though dts/dtf containts quadradic terms, choosing a smaller value for dtf does not accomplish anything, because the solution for a smaller value of dtf is linearly dependent. But from the perspective of the other observer, no matter how small dtf is, it is not small enough to reduce the problem to first order.

The third derivative would be required to compute the first derivative at a twice displaced point.

Eq () becomes invariant if the t_f^2 terms are dropped. The $-r0/c$ term can still be arbitrarily large, but the particle will only be on the light cone for a short time.

The midpoint shift only occurs when the clock at the field point is used as the time standard. In Fig. ..., the light cone equation was solved with t_s as the independeent variable. From the perspective of the other observer, the midpoint shift occurs at the field point.

Equations of physical significance must not depend on the orientaion of the coordiante system. The principle is much older than the theory of relativity.

The acceleration terms should be carried in equations that are quadradic in time, but they will be neglected in these calculaions.

The acceleration terms are in the same order as the transverse doppler terms, so they should be carried. Due to the lenght of the expressions, only the velocity terms are considered here. Similarly, the $\dot{\boldsymbol{a}}$ terms are in the same order as the v^4 terms, but they can be neglected when the conditions are mild.

The $-r_0/c$ term can still be aribtrarily large, but the particle will only be on the light cone for a short time – just long enough to compute the second derivatives.

The light cone equation does not provide a first order contraint on the transverse location of the particle.

The equation reprsents the transverse doppler effect. When .. is zero, the point on the trajector for which the doppler shift is zero does not occur at the particle's closest approach. It occurs slightly sooner.

Differentiating Eq – with respect to tf0

This solutin is wrong. The transverse doppler when ... is zero is half of that value.

From the perspective of the other observer, this is the right equation for the transverse doppler. Sometimes we need to see ourselves as others see us. It is too easy to write the equations as though we were at the center of the universe, while denying that we are doing what we are doing.

From the perspective of the other observer, this is the equation for the transverse doppler. It is probably our equation too.

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The time and space coordinates are not independent variables in light cone solutions, making the chain rule for differentiaion necessary. The equations do not work in the same way as they do for one independent variable.

This relationship indicates that the chain rule for differention is required.

dR=paretial rv/dt dt + (drv dot del) rv drv

del rv is a tensor of the second rank. rv is the Newtonian location of the particle. Rv is the perceived location.

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There is a close connection bewteen the doppler shift of a moving monochromatic source and the retarded potentials. Flux is emitted by a charged particle at a constant rate. When the time interval at the field point is shorter than the interval at the source, the flux becomes concentrated into a shorter time, enhancing the field strength. (A source of power would be required for a stationary particle to emit anything. This is just a model.) The model could be interpreted as applying to either the potentials or the fields, so it is not necessarily precise.

dtf can be the period of a monochromatic signal. There is no doppler shift at the point of closest approach.

Since the speed of light is the same in all frames of refrence, it would be easy to conclude that we can always consider events in relation to ourselves. The retarded potentials would exist if we were not here.

When we are at one fixed point, angular relationships in relation to ourselves have no meaning. On the other hand, there is a very old question as to whether an event without an observer has a meaning.

The doppler shift seems to be zero at the point of closest approach.

This is the right equation for transverse doppler.

It may seem that we have always known where the prticle is when k is zero, but it is difficult to be sure, because the time at the field point is then +r0/c. The location of the particle at that point on the trajectory could be determined by an assistent near there. The assistant could observe the time that the particle is adjacent to a nearby clock in a field of synchronized clocks. However, the observer at the field point could not know the outcome of the observation until a later time. We cannot know what is happening at the simultaineous point while it is happening.

The solutions conain fluctuating virtual charge. Preliminary indications are the the global sum of the virtural charge is zero.

The light cone equation does not impose a first order constraint on the location of the partcle at time tf=+r0/c. A more accurate solution can be obtained by extrapolating to that point from a region of the trajecory where the constraints are of first order.

k has dropped out of the solution, so it does not matter when the coordinates are first-known.

There is no transverse doppler term if the location of the partice is the same as the one assumed in the derivation of the LW equations. Transverse doppler is of order v^3, but doppler is of order v^1, so it reprsents a $(v/c)^2$ correction.

When integrating numerically in n steps, it is not sufficent that the error vanish in the limit. The term must either vanish as $1/n^2$ or be carried. After integration, terms that vanish as $1/n$ are in the same order as the solution.

dt_f could be the period of a doppler shifted monochromatic signal. This is not the right equation for radial doppler.

... There is no transverse doppler at the point of closest approach.

For unaccelerated particles, $\dot{R}$ is zero when $\boldsymbol{R}$ and $\boldsymbol{v}$ are perpendicular. The light cone equation does not impose a first order constraint on $\dot{R}$ at that point on the trajectory. It is difficut to be sure of exactly where the particle is then. That does not matter for the retarded potentials, as they are free of first order error. However, they exist only for the purpose of being differentiated twice.

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The particle is on the light cone, but it is not the same particle that the LW equations are for.

We cannot know what is happening at the simultaneous point while it is happening. We can find out what happened there, but not until after waiting a while. Determining exactly where the partile was then is not easy, and it cannot be done quickly.

In Eq (), dts/dtf is 1 at the point of closest approach. There is no transverse doppler.

An assistant near the source could record the time that the particle passes the nearest clock in a field of synchronized clocks, but the assistant is usually not available. A distant observer must infer the number that the assistant would read by extrapolating to that point with several delayed observational data points.

The solutions are the same as those of the relativistic doppler equation.

There are two ways of viewing this solution. In one way, the Lorentz transform in not the right transform for the problem, and other invaraint coordinate transforms are known. In the other way, there is no point in transforming to the other frame of reference until the coodintes are known in our frame of reference.

The particle is on the light cone, but it is not the same particle that the LW equations are for. The LW equations are not for an inertial particle.

This solution represents a projection of 4-space into 3+1 space. The location of the particle appears to be where it is thought to be in a conventinal Newtonian light cone calclation. This statement represents an interpretation, one that may need further evaluation. The considerations for computing the orbit of a free particle would be different.

A more accurate solution can be obtained by extrapolating to the poorly constrained portion of the trajectory from the solution for other times where the light cone constraint is of first order.

The particle is on the light cone. To first order, it is also on the light cone in Eq (), but this solution is more accurate. There is no transverse doppler term[11,12] in Eq ().

dt_f can be the period of a doppler shifted monochromatic signal. There is no transverse doppler at the point of closest approach.

The sign of $\hat{\boldsymbol{r}}$ can be inverted if the usage is consistent.

There is a particle at the origni of the coordinate system when tf=r/c, and it is on the light cone, but it is difficult to sure that it is the right particle.

The observes cannot both assume that they already know where the particle is. The retarded potentials are first-known in the other frame of refernce. The opinion of the other observer is more relevant.

There can be many particles and many other observrs. It is not clear that the location of the particle has an absolute meaning, but its location can be bounded.

In a simultaneous system, the location of the particle is

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When extrapolating around a circle, the first infitesimal step is free of first order error. If the equations are then reparameterized by coordinates first-known after the first step, the the second infitesimal step works in the same way as the first step.

There are necessarily other ways of working the problem. Some of the equations of the Lorentz group[5,9] are probably relevant to the retardation problem. Retardation equations should not contain time as a parameter, because we do not know what time it is.

In being the integral of the fields, potential solutions are arbitrary to within a constant of integration. The retarded potentials exist only for the purpose of beingf differentiatd, so the constant is probably superflous, but it could be important from other perspectives. In any case, in being subject to gauge transformations, potential equations are not necessarily unique.

The solution would have been the same as the LW equations if the integration had been performed in one step. There does not appear to be anything wrong with the LW equations when they are of sufficient accuacy.

We have no way of knowing what time it is. Retardation equations have to work in the same way at time t+dt as they do at time t.

When a paticle is moving parallel to the vector pointing from one observer to another observer, either observer should be able to solve the light cone equation for the other oberver. The solution should not depend on how far apart the two observers are unless a cosmological term is inclded.

The expansion factor within a mass shell is not locally detectable[7], but it is responsible for the gravitaional redshift for distant observers. The expansion factor will be assumed to be zero in these preminінary calculations.

The acceleration terms will need development, but velocity terms do come first.

XXI. DISCUSSION

Attempts at obtaining this solution with the undifferentiated Lorentz transform have not thus far been successful. A traditional approach is to reduce the problem to first order, then obtain the solution by integration. It is doubtful that the four dimensional space can be fully reduced to first order, but it should be possilbe to develop the problem as a series expansion. Other invariant coordeinates tranforms are known, and they may be relevant to the retardaion problem.

When computing an orbit, the derivatives are integrated to obtain the location of the particle. When applying retardation equations, the location of the particle is assumed to be already known, then the derivatives are computed. The location of the particle has to be known by independent means before retardation equations can be applied.

The expansion factor within a mass shell stretches the time and space coordinates by like amounts, so the locally measured speed of light is not affected[7]. It is not yet known if the cosmological expansion factor affects laboratory electrical measurements. It should be possible to find out if it does by including it in the retardation equations. There is no possible way of recognizing cosmological terms without the appropriate equations, as they have been there from the beginning. The expansion factor is already known, which would faciliate evaluation of the equations.

I will submit this paper to a journal after some more development. I cannot provide a conventional reference yet, but please acknowledge the source if you make use of the material. Comments and suggestions will be carefully considered.

These relationships need further evaluation when the retrded Newtonian acceleration is not zero. Some useful background material and equations are available in Ref.[10].

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